MATHEMATICAL MODEL FORMULATION FOR SUBMERGED AQUATIC CANOPY

BY

MANKO, Ayesha
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DEPARTMENT OF CIVIL ENGINEERING
FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA

SEPTEMBER, 2021

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A THESIS SUBMITTED TO THE POSTGRADUATE SCHOOL, FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA, NIGERIA IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF THE DEGREE OF MASTER OF ENGINEERING IN WATER RESOURCE AND ENVIRONMENTAL ENGINEERING

SEPTEMBER, 2021

ABSTRACT

The Hydraulics of flow in an open channel/waterway with flexible vegetation is studied. Vegetation along waterway has an ecological advantage; it enhances biodiversity, reduces erosion, and traps sediment. However, it has a hydraulic impact on flow. This study reviews hydrodynamics of vegetation along waterways (a concept to promote a sustainable green environment). It applies a modified one-dimensional (1-D) hydraulic model to replicate the vegetative velocity profile and Reynolds stresses using several laboratories experimental and field dataset found in the literature. Using this concept, a synthetic velocity profile is generated under varying hydraulic conditions. Using the concept of dimensional similarity, the vegetative parameters and flow resistance equation which relates these vegetation parameters, flow depth, and the zero-displacement parameter is proposed. The findings gave clear and comprehensive deduction, that mathematical model could to an extent replicate the rigorous experiments, as evaluations were parallel to the estimated laboratory values. It was concluded that at <10% slope, the flow channel can be construction without the use of concrete and other rigid materials.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

Contrary to popular opinion, the vegetation in flow channels to a large extent does more good than harm to hydraulic systems. In the cause of achieving significant innovative solutions to the problems faced as a result of environmental conditions, the water infrastructures have become some of the key resources that require constant development and subsequent overhaul. The flow of fluid in systems is a phenomenon that is dependent on certain factors, some of which includes but not limited to, Velocity of flow, configuration of the system (that is size, shape and structure), geographical conditions, state of the fluid, and the likes. Out of the numerous benefits of Vegetation in flow Channels, the following are benefits derived by the aquatic habitats; to improve its ecosystem, to stabilize the bed slope and as a result curbs the likelihood of erosion, a reasonable amount of improved water quality, accumulation of sediment which may further enhance growth of vegetation. In any flow field, velocity is the most important characteristic to be identified, at any point (Featherstone & Nalluri, 1998). This has been the basis upon which all other flow conditions have been measured and studied. The effect of Velocity could affect a system in variety of ways (positively or negatively), when the flow is constant or gradually increases with pressure along the pathline, the next point of check is Erosion and Flood Control.

In a flood control system, for every (open) channel flow, the requirement for flood control is the fundamental necessity that governs its development. The design of flood-control systems will usually include a variety of conveyance channels that must behave in a stable and predictable way to ensure a known flow capacity will be available for an unplanned flood event. As soil erosion always occurs for a flood flow, channel linings are required either temporarily or permanently to attain channel stability. These linings may be classified as Rigid or Flexible. Rigid Linings such as, Channel Pavements of concrete or asphaltic concrete, a variety of precast interlocking blocks and articulated mats are encountered. Flexible Linings includes loose stones (ripraps) Vegetation, manufactured mats of lightweight materials, fabrics or a combination of these materials. The selection of a particular lining is a function of the design context, involving issues related to the consequences of flooding, the availability of land, and environmental needs.

A rigid lining can withstand high discharge and high velocity flow. Flood-control channels with rigid linings are often used to reduce the amount of land required for a surface drainage system. However, a flexible lining on the other hand can respond to a change in channel shape, and therefore, it is not so easy to subject it to local damage. It is used as temporary lining for control of erosion during construction or reclamation of disturbed areas. From environmental considerations, flexible linings are inexpensive, permit infiltration and exfiltration, and allow growth of vegetation. Flow conditions in the channel lined with flexible materials generally can be made to conform to conditions found in a natural channel, thus provide better habitat opportunities for local flora and fauna.

Rigid and flexible lining materials, Channel roughness is affected by the relative height of the roughness compared to the flow depth. Consequently channel roughness increases for shallow flow depths and decreases as flow depth increases.

Vegetative Lining; channel roughness varies significantly for vegetative linings, depending on the amount of submergence of the vegetation. As vegetation is flexible, the amount of submergence will increase as the drag force bends the plant stems toward the channel bed. The Manning's coefficient can be determined practically by the Kouwen's method (to be elaborated in Literature.

Vegetative canopy occurs in riverine environment such as in channels or on the flood plain, rivers and wetlands. It has a significant influence on the behaviour of the fluvial system. It increases the hydraulic resistance and reduces the inflow velocity, thereby causing problem on flood control. Meanwhile, it benefits as a storm surge protection, providing habitat for aquatic animals, reduce erosion (causing bank/channel stabilization) and water quality improvement motivate research of vegetated flows.

Several studies had been done on the resistance of vegetation to flow leading to established empirical relationships between vegetation parameters and the flow hydraulics (Cowna, 1956). Upon development of measuring devices, research interests have been extended to the study of velocity distributions and Reynolds stress. Based on this, theoretical analysis, Lopez and Garcia (2001) experimental study Stoesser *et al.* (2006) and mathematical model (Su Xiao-hui, 2003) have become the adopted study scheme. The classical issue of different vegetation type and hydraulic conditions of flows has restricted the generalization of experimental results. Hence, several mathematical models are been put forward based on the experimental results.

Many approaches have been proposed to develop the models. For example Naot, Huang (2002) used the continuity equation, energy equation and momentum equation in 3 dimensions to establish mathematical models. Among other 3-D models include two-equation $k - \omega$ turbulence model (Huai *et al.,* 2009), large Eddy Simulation (LES) (Wang *et al.,* 2009; Kubrak *et al.,* 2008) etc. these models yielded results of high accuracy, but their limitation is based on complexity with large computation quantities and time.

Huai *et al.* (2009) developed a simpler mathematical model with restriction of rigid cylindrical stem of low vegetation density and not applicable to compound channel. Based on this, for sea grasses and other blade-type vegetation of high flexibility (that is, large deflection) and vegetation density, valid mathematical models are needed. Another approach regards vegetation layer and soil layer as homogenous and isotropic media, and applies the theory of turbulent flow and Biot's poro-elastic theory to study the vegetated flow (Ghisalberti & Nepf, 2004). The method is, however, useful for soil of large porosity.

Recently, a one-dimensional numerical that combined the continuity equation and momentum equations with Spalart- Allmaras model with a modified length scale which is dependent on the vegetation density and vegetation height to water depth ratio as turbulence model (Busari & Li, 2015). The vegetation flexibility is accounted for using a large deflection analysis Based on the synthetic data an inducing equation is derived which relates the Manning roughness coefficient to the vegetation parameters, flow depth and a zero-plane displacement parameter. The predictions of the equation depend on the accuracy surrounding the estimation of plant's drag coefficient.

In this study, the analysis of the turbulence structure in the vegetation region of the flow with submerged vegetation employs the mixing length approach and modifies its expression using Karman similarity theory base on the concept of zero-plane displacement to distinguish the regions in the vegetated area. The main force acting on a lining composed of large particles is the drag force. The effect is to increase the shear parameter and consequently Vegetation. Two major types of Vegetation are widely known Emergent Vegetation and Submerged Vegetation (Mazda *et al.*, 1997).

The plant life, such as trees, grass, and bushes, always grows in the channels, rivers and wetlands. The vegetation can increase the resistance and reduce the velocity in flow, which has the negative influence on the flood control. However, the vegetation in flow can promote sediment deposition, reduces the river bed erosion, improves water environment and restores the river ecological systems. Therefore, it is important to study the influences of vegetation on the flow. At earlier time, researchers focused on the resistance of vegetation to flow and established some empirical relationships between the vegetation and the flow. With the development of measuring equipment, researchers became more interested in the distributions of the velocity and the Reynolds stress. They adopted three different study schemes, namely experimental study, theoretical analysis, and mathematical model. But the difference of vegetation types and experimental conditions restrict the generalization of the experimental results. So at present researchers put forward several mathematical models based on the experimental study. Many approaches have been proposed to construct the models. For example Naot *et al.* (2000) combined the continuity equation, energy equation, and momentum equations in three dimensions and established mathematical models, and other 3-D models include two-equation k - Z turbulence model and Large Eddy Stimulation (LES). The 3-D models can give relatively accurate results, but these models are complex with large computation quantities. So, simpler, valid mathematical models are needed. Another method is to derive the momentum equations regarding the flow with vegetation as a 1-D one. By adopting the mixing length expression, the model can give the vertical distributions of the stream-wise velocity and the Reynolds stress. To analyze the turbulence structure in the vegetated region of the flow with submerged vegetation, this work applies the mixing length approach and improves its expression according to the Karman similarity theory, and adopts the conception of penetrated distance to distinguish the regions in the vegetated area.

1.1.1 Mathematical modeling and simulation; *numerical methods*

In simple terms, a Mathematical Model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed **mathematical modeling**.

Numerical Methods are techniques by which mathematical simulations/models are formulated so as to obtain their solutions using arithmetic operations. They usually comprise of large number of tedious calculations. Numerical solutions are often approximate values which may be the exact solutions, thus, are generally acceptable and valid. They can be determined either experimentally or analytically, the analytical method is applied in the case of this study as experimental methods have become more cumbersome and less economical, whose results are easily prone to error due to environmental conditions and human factors. A simple principle of numerical method is discretization, which is dependent on time and space of flow considered which gives an approximate result. It is also important to note, that the smaller the time interval, the more accurate the approximate result obtained.

In Hydraulics, Computational Fluid Dynamics is the use of computer aided design to determine, suggest and analyze fluid flow. It is the study of complex fluid flow, by solving the equations of flow velocity and motion, known as Navier-Stoke's Equations (also referred to as Momentum equation, is used for the complete set of equations solved by numerical methods, which also includes the energy and continuity equations.), in a certain geometry and physical environment. Such flow environments could be considered waters channels with free surfaces, porous packed beds or porous concrete/metallic structures.

The presence of vegetation significantly have impact on flow conditions, while increasing flow resistance by (highly reducing erosion and stabilizing the earth through the plant root system), thereby improving the general purity of water.

Turbulent flows in such channels with submerged vegetation evidently have their structures depending on the nature of vegetation, its density and how it is arranged. Application of Computation Fluid Dynamics is of high significance as compared to physical experiment, due to its operation within the evolving Computer Aided Design/ Information Technology structure in conjunction with drawing and manufacturing tools, making it more accessible than experimental methods. The major issue in river modeling is the uncertainty in the predictions of resistance (Galema, 2009).

1.2 Aim and Objectives of the Study

1.2.1 Aim

The aim of this research is to obtain the Mathematical Model formulation for Submerged Aquatic Canopy (Flexible Vegetation).

1.2.2 Objectives of the study

This is guided by the following objectives;

1. To formulate the required theory of vegetated flow.
2. To evaluate significant hydraulic parameters using dimensional analysis.
3. To develop a Mathematical model for aquatic canopies.
4. To simulate mean velocity profile and Reynolds stress for different hydraulic conditions.

1.3 Statement of the Research Problem

In the cause of this study, it is likely to encounter some hindrances (not necessarily challenges) which may affect the expected outcomes of this research but not damaging to a viable result. The expected limitations may result from lack of access to accurate experimental findings, and hence the incorporation of Computational Fluid Dynamics to investigate the types of flow and further solve the flow equations. The Samples to be considered are restricted to blade-type flexible vegetation, whose model may have a certain level of variation with other types of vegetation (flexible or rigid). There is a likelihood of error due to experimental set-up, which may be experienced during physical sampling and hence may not be noticed immediately. Vegetation is observed to cause problem of flood control in a channel, which has further reinforced the lapses of vegetative cover in a flow channel.

1.4 Justification of the Study

With the recent intensity of discussions on flooding as a result Climate Change, the common acceptance that Vegetation is a cause of hindrance to flow or a cause of flow resistance is in high contrast to the findings by research and studies. This has further influenced the possibility that vegetation along a flow path or path line does not only have environmental benefits, but might also be a means of regulating excesses as a result of certain flow conditions.

The use of Mathematical Model shall further elaborate the significance of the need for more results and evidences to reinforce this theory.

The 3-D models can give relatively accurate results, but these models are complex with large computation quantities. So, simpler, valid mathematical models are needed. Another method is to derive the momentum equations regarding the flow with vegetation as a 1-Dimension. By adopting the mixing length expression, the model can give the vertical distributions of the stream-wise velocity and the Reynolds stress, though these models always involve some unknown parameters which are difficult to estimate.

1.5 Scope of the Study

With maximum accuracy, this Research is targeted at covering a wide range of situations and conditions; hence, it is not limited to a particular type or location of Vegetation. For the course of this research, we shall focus on the submerged aquatic canopy, study velocity, reconfiguration of flow channels to conserve aquatic life, alongside flood/erosion control by the application of Mathematical Model. This research shall be taken through the following stages

Stage one; The introduction and elaboration of aim and objectives

Stage Two; Detailed review of Literature available

Stage Three; Method of research process

Stage Four; Analysis and evaluation of results obtained

Stage Five; General observations and recommendations

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Hydrodynamics (Fluid Flow)

Water Systems world over have been considered to have one major effect or the other on its surrounding life and properties (Galema, 2009). It is regarded as a major element of human existence hence its availability cannot be overemphasized, which has brought the need for how to maximize its use with minimal or no danger. In the concept of fluid flow in river or water bodies, the velocity may be regarded as the key cause of high or low discharge which either ways may result in a disaster or the other, some of the effect of flow could be but not limited to Flooding or Erosion.

Hydrodynamics simply, is the study of fluid in motion. It is a branch of physics that deals with the motion of fluids and the forces acting on solid bodies immersed in fluids and in motion relative to them.

In the event of a disaster such as flooding, it is essential to have a foresight about the water levels and to also deduce the effect of appropriate measures to guard its environs against the occurrence of flood. To be able to carry out such operations effectively with as high as possible accuracy, the behavior of water bodies are predicted with the use of computational flow models.

2.1.1 Types of fluid flow

There are two major types of fluid flow

The Laminar and Turbulent flows, but, in major advance situation there is interchange that occurs between these flow stages, this referred to as Transitional flow.

a. Laminar Flow: It is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.

b. Transitional Flow: The change from laminar to turbulent flow depends on: the geometry, surface roughness, flow velocity, surface temperature, and type of fluid etc.

c. Turbulent Flow: The intense mixing of the fluid in turbulent flow as a result of rapid fluctuations enhances momentum transfer between fluid particles, which increases the friction force on the surface and thus the required pumping power and the friction factor reaches a maximum when the flow becomes fully turbulent.

2.2 Channel Linings

In a flood control system, for every (open) channel flow, the requirement for flood control is the fundamental necessity that governs its development. The design of flood-control systems will usually include a variety of conveyance channels that must behave in a stable and predictable way to ensure a known flow capacity will be available for an unplanned flood event. As soil erosion always occurs for a flood flow, channel linings are required either temporarily or permanently to attain channel stability.

2.2.1 Types of lining in a flow channel

These linings may be classified as **Flexible or Rigid.**

Rigid Linings such as, Channel Pavements of concrete or asphaltic concrete, a variety of precast interlocking blocks and articulated mats are encountered. Flexible Linings includes loose stones (ripraps) Vegetation, manufactured mats of lightweight materials, fabrics or a combination of these materials. The selection of a particular lining is a function of the design context, involving issues related to the consequences of flooding, the availability of land, and environmental needs.

A **rigid lining** can withstand high discharge and high velocity flow. Flood-control channels with rigid linings are often used to reduce the amount of land required for a surface drainage system. However, a **flexible lining** can respond to a change in channel shape, and therefore, it is not so easy to subject it to local damage. It is used as temporary lining for control of erosion during construction or reclamation of disturbed areas. From environmental considerations, flexible linings are inexpensive, permit infiltration and exfiltration, and allow growth of vegetation. Flow conditions in the channel lined with flexible materials generally can be made to conform to conditions found in a natural channel, thus provide better habitat opportunities for local flora and fauna.

Flexible and Rigid Lining Materials; Channel roughness is affected by the relative height of the roughness compared to the flow depth. Consequently channel roughness increases for shallow flow depths and decreases as flow depth increases.

2.3 Vegetative Lining

Channel roughness varies significantly for vegetative linings, depending on the amount of submergence of the vegetation. As vegetation is flexible, the amount of submergence will increase as the drag force bends the plant stems toward the channel bed. The Manning's coefficient can be determined practically by the Kouwen's method.

Vegetative canopy occurs in riverine environment such as in channels or on the flood plain, rivers and wetlands. It has a significant influence on the behaviour of the fluvial system. It increases the hydraulic resistance and reduces the inflow velocity, thereby causing problem on flood control. Meanwhile, it benefits as a storm surge protection, providing habitat for aquatic animals, reduce erosion (causing bank/channel stabilization) and water quality improvement motivate research of vegetated flows.

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Many approaches have been proposed to develop the models. For example Naot *et al.* (2002) used the continuity equation, energy equation and momentum equation in 3 dimensions to establish mathematical models. Among other 3-D models include two-equation $k - \omega$ turbulence model Huai *et al.* (2009), large Eddy Simulation (LES) Wang *et al.* (2009); Kubrak *et al,* (2008) etc. These models yielded results of high accuracy, but their limitation is based on complexity with large computation quantities and time.

Huai *et al.* (2006), developed a simpler mathematical model with restriction of rigid cylindrical stem of low vegetation density and not applicable to compound channel. Based on this, for sea grasses and other blade-type vegetation of high flexibility (i.e large deflection) and vegetation density, valid mathematical models are needed. Another approach regards vegetation layer and soil layer as homogenous and isotropic media, and applies the theory of turbulent flow and Biot's poro-elastic theory to study the vegetated flow (Ghisalberti & Nepf, 2004). The method is, however, useful for soil of large porosity.

Recently, a one-dimensional numerical that combined the continuity equation and momentum equations with Spalart- Allmaras model with a modified length scale which is dependent on the vegetation density and vegetation height to water depth ratio as turbulence model (Nepf & Ghisalberti, 2008). The vegetation flexibility is accounted for using a large deflection analysis (Nepf, 1999). Based on the synthetic data an inducing equation is derived which relates the Manning roughness coefficient to the vegetation parameters, flow depth and a zero-plane displacement parameter. The predictions of the equation depend on the accuracy surrounding the estimation of plant's drag coefficient.

In this study, the analysis of the turbulence structure in the vegetation region of the flow with submerged vegetation employs the mixing length approach and modifies its expression using *Karman similarity theory* base on the concept of zero-plane displacement to distinguish the regions in the vegetated area.

The Manning's coefficient; is determined as follows, the coefficients a and b are based on a classification of the three types of flow conditions with vegetation: Erect, Submerged (bent), and Flattened and is determined from a table. The initial shear stress that bends the vegetation from an erect position is referred to as the vegetative critical shear stress.

For cohesive material, the variation in critical shear stress depends on the concentration of the clay particles within the soil. The mean boundary shear stress is already given by the local boundary shear stress varies within a river reach as a consequence of the non-uniform distribution of velocity in the cross section. For design, it is important to assess the maximum shear stress occurred at specific locations in the reach. The maximum boundary shear stress is then given by; where Ka=boundary shear-stress adjustment factor, depending on the conditions of channel bed, channel bank, and flexible lining.

Steep-gradient channel design (Channel gradient greater than 10%)

The maximum permissible shear stress is less for channels on steep slopes. As velocity increases and flow depth decreases the exchange of momentum between portions of the channel becomes more efficient. The channel boundary away from the zone of maximum shear stress receives increased shear stress that approaches the maximum. Localized shear zones are created near irregularities in the lining. This requires linings with larger particle sizes.

To determine the flow resistance in channels with large scale roughness the form drag of the roughness elements and the distortion of the flow as it pass around roughness elements should be accounted for. The flow-resistance formula must take account skin friction and form drag. The Manning's form of equation for this flow condition is given by Steep-gradient channel design

Where the function f (Fr) accounts for the free surface drag of the elements, f (REG) accounts for the roughness geometry; f (CG) accounts for the relative roughness area. The main force acting on a lining composed of large particles is the drag force. The effect is to increase the shear parameter and consequently.

2.3.1 Types of vegetation

Two major types of Vegetation are widely known

1. **Emergent Vegetation and**
2. **Submerged Vegetation (**Mazda *et al.*, 1997)

2.4 Composite-Section Channel

Highest shear stresses occur on the bed of channel or on the channel bank near bed. Composite lining is generally used with higher strength lining used selectively in high shear areas. Low-flow channels are within the main channel. Channel banks are vegetated for environmental or ecological considerations.

2.4.1 Channels with sediment transport

If there is no lining, sediment will generally be transported in flood control channel. Routine maintenance will be required to prevent excessive accumulation. Sediment is transported in a stream channel as a combination of bed load (sediment that is in frequent contact with the bed of the channel) and suspended load (sediment from the bed that is mixed with the water flow by turbulence).

Sediment transport begins when the critical shear stress of the bed sediment is exceeded. As the mean shear stress and flow velocity increase, the total rate of sediment-transport increases. The sediment particle must be transported by the flow to the cross section. If the supply of sediment from the catchment exceeds the sediment transport at a cross-section, sediment will accumulate, that is aggrade (Naot, 2000).

The plant life, such as trees, grass, and bushes, always grows in the channels, rivers and wetlands. The vegetation can increase the resistance and reduce the velocity in flow, which has the negative influence on the flood control. However, the vegetation in flow can promotes sediment deposition, reduces the river bed erosion, improves water environment and restores the river ecological systems. Therefore, it is important to study the influences of vegetation on the flow. At earlier time, researchers focused on the resistance of vegetation to flow and established some empirical relationships between the vegetation and the flow. With the development of measuring equipment, researchers became more interested in the distributions of the velocity and the Reynolds stress. They adopted three different study schemes, namely experimental study, theoretical analysis, and mathematical model. But the difference of vegetation types and experimental conditions restrict the generalization of the experimental results. So at present researchers put forward several mathematical models based on the experimental study. Many approaches have been proposed to construct the models. For example, Naot combined the continuity equation, energy equation, and momentum equations in three dimensions and established mathematical models, and other 3-D models include two-equation k Z turbulence model, Large Eddy Stimulation (LES) etc. The 3-D models can give relatively accurate results, but these models are complex with large computation quantities. So, simpler, valid mathematical models are needed. Another method is to derive the momentum equations regarding the flow with vegetation as a 1-D one. By adopting the mixing length expression, the model can give the vertical distributions of the stream-wise velocity and the Reynolds stress. But these models always involve some unknown parameters which are difficult to estimate. Recently, a new and interesting approach regards vegetation layer and soil layer as homogeneous and isotropic media and applies the turbulent flow theory and Biot's poroelastic theory to study the vegetated flow. The method may be useful when the porosity of soil is large, while its application in the situation of small porosity still needs to be affirmed. To analyze the turbulence structure in the vegetated region of the flow with submerged vegetation, this article applies the mixing length approach and improves its expression according to the Karman similarity theory, and adopts the conception of penetrated distance to distinguish the regions in the vegetated area. In the case of emerged vegetation, the vertical distribution of stream-wise velocity is almost uniform, which is especially evident when the vegetation is stiff (Huai *et al.*, 2009).

2.4.2 Theory of vegetation flow

Vegetation flow generally exists in nature, and plays a significant role in flood control and sediment transport. In addition, Aquatic plants can purify sewage and provide habitat for microorganism and aquatic animals, which is beneficial to the river ecosystem. Therefore, the research on vegetation flow has become a focus in environmental hydraulics.

Previous scholars have made numerous achievements on open-channel flow through rigid vegetation, but they assumed vertical porosity to be constant during calculation and selected rigid cylinder to simulate vegetation in their experiments. For example, Ghisalberti and Nepf, (2004), applied a one-dimensional numerical model to predict the vertical velocity distribution of submerged vegetation flow by assuming a single mixing length above the vegetation. The model was verified with experimental data. Cui and Neary (2008) investigated fully developed turbulent flows with submerged vegetation by using Large Eddy Simulation. In addition, their study analyzed the role of coherent structures on the momentum transfer across the water-plant interface. By placing a typical cylindrical stem in the middle of a vegetation zone, Kothyari *et al.* (2009), used strain gauge to directly measure drag force. To investigate the effects of vegetation on flow structure, Gao *et al.* (2011) applied a model with the two-layer mixing length turbulence closer to a physical model channel. Based on the Navier-Stokes equation, Guo *et al.* (2000) focused on velocity distributions for laminar and turbulent flows through emergent and submerged vegetation.

Their study reported that brief Jacobi elliptical functions can exactly describe laminar flow through both emergent and submerged vegetation, whereas turbulent submerged vegetation flow was approximated by a hyperbolic sine law. Recently, some scholars paid attention to the influence of spatial variation of vegetation on channel flow. For example, Berger *et al.* (2011) measured flow in a channel with spatially varying distribution. The variation was achieved by changing longitudinally the stem area number density and stem distribution. But they still did not consider the varying vertical porosity. In nature, practical porosity varies vertically with stem thickness and the leaf density. If a circular cylinder is selected to simulate rigid vegetation in the numerical simulation and experiments, the research results may cause deviation in practical engineering.

Many previous researchers modeled vegetation flow in channels as porous media flow because of the similarity. For example, Hsieh and Shiu, (2006) investigated the vertical velocity profile of flow passing over a vegetal area by applying Boit's theory of poroelasticity and discussed five factors' effect on vegetation flow. In their researches, permeability is the most important factor in affecting flow characteristics. However, there was not an accurate expression of permeability in submerged vegetation flow. Recently, Xu, have summarized some modifications of the permeability and presented an analytical expression for the permeability based on the fractal characters of porous media and capillary model. Nevertheless, none of them can be used in submerged vegetation flow directly.

In order to obtain more practical results, a truncated cone is selected to simulate rigid vegetation to explore the influence of the varied vertical porosity on submerged vegetation flow. A three-layer model (upper free water layer, interface layer and vegetation layer) for submerged vegetation flow with variation of vertical porosity is proposed to predict vertical distribution of velocity. The governing equation for velocity in the whole area of submerged vegetation flow is presented by applying the poroelastic media flow theory.

The fitting expression of permeability (*k)* in submerged vegetation flow is also obtained from experimental data. With the finite analytic method, the new finite analytic solution for velocity in the vegetation layer and the interface layer is presented with high accuracy. Furthermore, the calculated velocity distribution agrees with experimental data in a flume experiment. The model and approach presented in this publication can predict velocity distribution of submerged vegetation flow in river ecological restoration. Even if aquatic plants have complex shapes, the velocity distribution can be obtained accurately with the finite analytic method because of considering the influence of varying vertical porosity on vegetation flow. Therefore, the research provides a new vision for researching submerged vegetation flow in a complex environment.

2.4.3 Methods of solving fluid flow problems in presence of vegetation

In most ideal conditions, flows are modeled using the Navier-Stokes equation, this due the fact that the equation gives the most unbiased and total description of flow of fluids. Though, being that the Navier-Stokes equation is not linear, it becomes complicated to have a comprehensive computation of flow. This has therefore made it unavoidable to obtain a more simplified flow description for practical applications, as illustrated by drag-dominated flows, simple shear flows, numerical solution techniques or a simple combination of two out of the three. The following major techniques may be employed for this process,

1. Direct Numerical Simulation, DNS (Solving the Navier-Stokes Equation)
   a. Large-Eddy Simulation (LES)
   b. Incorporating Vegetation Flow Resistance
2. Solving the Reynolds-averaged Navier-Stokes equation
   a. Incorporating Vegetation flow resistance
3. Using Spatially-averaged flow descriptions.

2.5 Dimensional Analysis

In engineering and science, **dimensional analysis** is the analysis of the relationships between different physical quantities by identifying their base quantities (such as length, mass, time, and electric charge) and units of measure (such as miles vs. kilometers, or pounds vs. kilograms) and tracking these dimensions as calculations or comparisons are performed. The conversion of units from one dimensional unit to another is often easier within the metric or SI system than in others, due to the regular 10-base in all units. Dimensional analysis, or more specifically the **factor-label method**, also known as the **unit-factor method**, is a widely used technique for such conversions using the rules of algebra.

The concept of **physical dimension** was introduced by Joseph Fourier in 1822. Physical quantities that are of the same kind (also called *commensurable*) (for example, length or time or mass) have the same dimension and can be directly compared to other physical quantities of the same kind (that is, length or time or mass, respectively), even if they are originally expressed in differing units of measure (such as yards and meters). If physical quantities have different dimensions (such as length versus mass), they cannot be expressed in terms of similar units and cannot be compared in quantity (also called *incommensurable*). For example, asking whether a kilogram is larger than an hour is meaningless.

Any physically meaningful equation (and any inequality) will have the same dimensions on its left and right sides, a property known as *dimensional homogeneity*. Checking for dimensional homogeneity is a common application of dimensional analysis, serving as a plausibility check on derived equations and computations. It also serves as a guide and constraint in deriving equations that may describe a physical system in the absence of a more rigorous derivation.

2.6 Mathematical Modeling and Simulation; Numerical Methods

In simple terms, a Mathematical Model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed **mathematical modeling**.

Numerical Methods are techniques by which mathematical simulations/models are formulated so as to obtain their solutions using arithmetic operations. They usually comprise of large number of tedious calculations. Numerical solutions are often approximate values which may be the exact solutions, thus, are generally acceptable and valid. They can be determined either experimentally or analytically, the analytical method is applied in the case of this study as experimental methods have become more cumbersome and less economical, whose results are easily prone to error due to environmental conditions and human factors. A simple principle of numerical method is discretization, which is dependent on time and space of flow considered which gives an approximate result. It is also important to note, that the smaller the time interval, the more accurate the approximate result obtained.

In Hydraulics and the likes, we have what is referred to as Computational Fluid Dynamics, is the use of computer aided design to determine, suggest and analyze fluid flow. It is the study of complex fluid flow, by solving the equations of flow velocity and motion, known as Navier-Stoke's Equations (also referred to as Momentum equation, is used for the complete set of equations solved by computation fluid dynamics, which also includes the energy and continuity equations.), in a certain geometry and physical environment. Such flow environments could be considered waters channels with free surfaces, porous packed beds or porous concrete/metallic structures. Application of Computation Fluid Dynamics is of high significance as compared to physical experiment, due to its operation within the evolving Computer Aided Design/ Information Technology structure in conjunction with drawing and manufacturing tools, making it more accessible than experimental methods. The major issue in river modeling is the uncertainty in the predictions of flow resistance (Galema, 2009).

For a *Vegetated Region*, there are four major zones apparent in its Model.

1. Clear-view Zone
2. Top Vegetated Zone
3. Transition Zone and Finally
4. The Viscous Zone. These shall all be considered in the cause of this Research.

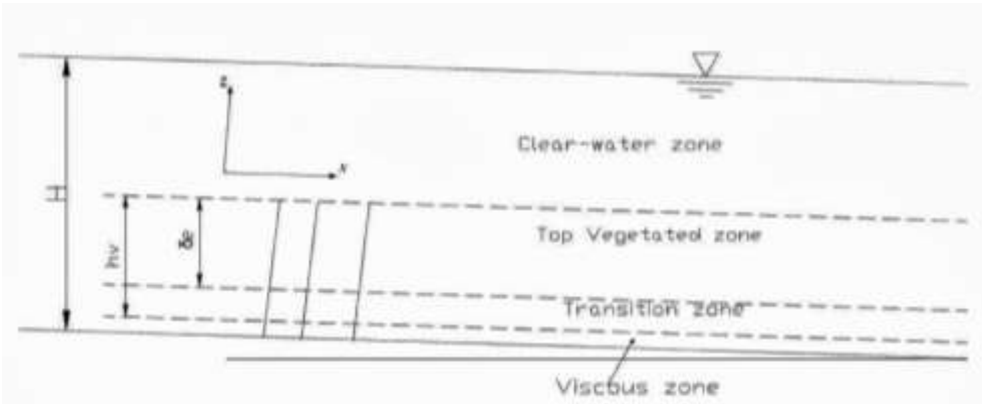

Figure 2.1: Sketch of four layer model

For highly flexible blade or under high discharge, the vegetation along the river channel can be submerged partially or completely depending on the flow magnitude. The flow can be analyzed as a uni-directional fully developed uniform turbulent flow (Figure 2.1).

With the recent intensity of discussions on flooding as a result Climate Change, the common acceptance that Vegetation is a cause of hindrance to flow or a cause of flow resistance is in high contrast to the findings by research and studies. This has further influenced the possibility that vegetation along a flow path or path line does not only have environmental benefits, but might also be a means of regulating excesses as a result of certain flow conditions. The use of Mathematical Model shall further elaborate the significance of the need for more results and evidences to reinforce this theory.

The 3-D models can give relatively accurate results, but these models are complex with large computation quantities. So, simpler, valid mathematical models are needed. Another method is to derive the momentum equations regarding the flow with vegetation as a 1-Dimension. By adopting the mixing length expression, the model can give the vertical distributions of the stream-wise velocity and the Reynolds stress, though these models always involve some unknown parameters which are challenging to estimate.

Advantages of 1-D models

1. Accurate hydraulic description in river/channel which behave predominantly one – dimensional.
2. Automatic access to sub-models such as the advanced structure operation module and flood forecasting module.
3. Less computational points than 2-d model, i.e less CPU time.
4. Easy to overview and comprehend reports.

Disadvantages of 1-D models

5. Flow paths must be known beforehand in cases where floodplain branches and link-channels are established.
6. Generally more effort in measuring schematization than in 2-D model.
7. No detailed result (for example, velocity profile) on flood plain.

CHAPTER THREE

3.0 RESEARCH METHODOLOGY

3.1 Preamble

To attain significant outcomes, one or two major procedures/stages are likely to be taken, for high degree of accuracy. Here, the manner and method of approach that shall be implemented in achieving this research result and documentation are stated explicitly.

This study is expected to be conducted through the following stages,

i. Dimensional Analysis
ii. Theory of Vegetated flow path
iii. Derive analytic relationship between velocity, flow and vegetation parameters.
iv. Application of numerical methods to the equations that relates these parameters.
v. Develop a Model and use the model to generate Velocity Profile and Reynolds stresses.

3.2 Modeling of highly flexible submerged dense vegetation (Buckingham Pi method of dimensional analysis)

3.2.1 Simulation procedure

Using the experimental dataset of Busari and Li, (2014), the simulation consists of five (5) categories. For each category, the number of vegetation per unit area, (N) and water level, (h) is fixed. Modulus of elasticity, (E) was constant throughout the simulation. The plant height (h_v) varies from 0.18 to 0.08m for the categories. The range of (h/h_v) is between 2 and 4.5 for the considered categories. For each category, vegetation stem size (b_v) varies from 0.002 to 0.006 at 0.0005 intervals. The moment of inertia (I) also varies, $I = f\{b_v\}$.

3.2.2 Model outputs and computed variables

The output of the model are the mean streamwise velocity (U), Reynolds Shear Stresses ($-/u^2w^2/$) and deflected height of plants, whereas mean velocity, U_{av}, shear velocity (U*) and mean streamwise velocity in the vegetation layer, U_{veg} are the computed variables.

3.2.3 Effect of flow on EI

The flexural rigidity, R is denoted as

$$R = EI \tag{3.1}$$

By defining R as a function of flow characteristics and vegetation parameters, we have

$$R = f\{g, \rho, \mu, U, A_p, h_v, E, h, N\} \tag{3.2}$$

Where $A_p \cong b_v h_v$ cross section area of submerged vegetation. Taking E = constant when the b_v, perpendicular to flow is unchanged, therefore equation (3.2) becomes

$$R = f\{g, \rho, \mu, U, b_v, h_v, h, N\} \tag{3.3}$$

Applying dimensional analysis to equation (3.3), with $U, h_v \, and \, \rho$ as repeated variables, the relationship between the flexural rigidity, flow parameters and vegetation parameters is given by

$$\frac{R}{\mu U k^3} = f\left\{Re, Fr, \frac{h}{h_v}, \frac{D}{k h_v}, N b_v h_v\right\} \tag{3.4}$$

For dense vegetation of high flexibility, equation 3.4 can be expressed as

$$\frac{\boldsymbol{R}}{\boldsymbol{\mu U k_d}^3} = f\left\{\boldsymbol{Re_{veg}}, \boldsymbol{Fr}, \frac{\boldsymbol{h}}{\boldsymbol{k_d}}, \frac{\boldsymbol{D}}{\boldsymbol{k h_v}}, \boldsymbol{N b_v h_v}\right\} \tag{3.5}$$

3.3 Theory of Vegetated Flow Path

In most cases, a flow path is subjected to the growth of lining/vegetation in its route, as a result natural habitats and the ecosystem in the flow path are enhanced. Vegetation flow generally exists in nature, and plays a significant role in flood control and sediment transport. In addition, Aquatic plants can purify sewage and provide habitat for microorganism and aquatic animals, which is beneficial to the river ecosystem.

Therefore, the research on vegetation flow has become a focus in environmental hydraulics.

For highly flexible blade under high discharge, the vegetation along the river channel can be submerged partially or completely depending on the flow magnitude. The flow can be analyzed as a uni-directional fully developed uniform turbulent flow. By considering a control volume in vegetated flow, equation based on the balance of active forces can be derived as follows:

$$\frac{\partial \tau}{\partial z} + \rho g S_o - F_{cd} = 0 \qquad (3.6)$$

As the stress τ is made up of the Reynolds stress τ_{xz} and the viscous stress τ', the density of water ρ, the bottom slope S_o , which serves as an alternate to energy slope due to the uniform nature of the flow, and f_{cd} is termed drag as a result of vegetation which is obtained using:

$$f_{cd} = \frac{1}{2}\, \rho N C_d b_{\,v} u^2 \qquad (3.7)$$

Where N governed by

$$N = \frac{1}{S_x S_y} \qquad (3.8)$$

S_x and S_y, are the vegetation spacing in the x-y direction respectively,

$b_{\,v}$ = blade width (m)

N = number of vegetation

u = mean flow velocity

C_D = the drag coefficient of the stem stated by Schlichting.

To adequately solve the equation (3.6) in the drag zones, we are required to proportion the entire flow area into just two zones of external and vegetated zones, as in (Figure 2.1), since there is no case of drag in any zone above vegetation. The Reynolds stress at the subordinate part of the vegetated zone is so little and significantly negligible, considering the outcome of the experiments, hence, we may further partition the vegetated zone into two to further make a total of four zones namely:

1. *Clear-water zone*, where h_d < z < H, as h_v is deflected height of the vegetation with the depth of water as H. Here, drag due to vegetation is not seen and the shear stress due to viscosity is too small to be considered, there the gravity and the turbulent shear stress balance:

$$\frac{\partial \tau_{xz}}{\partial z} + \rho g S_o = 0 \tag{3.9}$$

2. *Top vegetated zone*, the zone governed by $h_v - \delta_e < z < h_d$ with δ_e signifying extent of penetration or downward reach, representing the effect of instability close to the upper part of a vegetation. In this zone, there is a consideration of drag force, as such, the gravity, Reynolds stress and the drag are balance, as illustrated in the proceeding equation:

$$\frac{\partial \tau_{xz}}{\partial z} + \rho g S_o - \frac{1}{2} \rho N C_D b_v u^2 = 0 \tag{3.10}$$

3. *Transition zone*, it states here that $\delta_0 < z < h_v - \delta_e$, where δ_0 is the thickness of the zone of viscosity. This zone shows that the whirlpool prompted by the stems is little, hence the Reynolds stress can be ignored as well the viscous stress, since the zone is a bit far from the bed:

$$\rho g S_o - \frac{1}{2} \rho N C_D b_v u^2 = 0 \tag{3.11}$$

4. *Viscous zone*, in this zone, here $0 < z < \delta_0$ and is ruled by the viscous stress.

$$\frac{\partial \tau\prime}{\partial z} \rho g S_o - \frac{1}{2} \rho N C_D b_v u^2 = 0 \tag{3.12}$$

The viscous stress here is determined by the Newton internal friction law, stated as,

$$\tau' = \mu \frac{\partial u}{\partial z} \tag{3.13}$$

3.4 Analysis of Flow Structures

3.4.1 Mix length approach

The study applies Prandtdl's mixing length theory to calculate the Reynold's stress as shown in Equations. (3.9), (3.10):

$$\tau_{xz}=\rho l^2 \left|\frac{\partial u}{\partial z}\right| \frac{\partial u}{\partial z}$$

(3.14)

With l representing the mixing length, from the Karman similarity theory Nazarenko (2000), it is known that the mixing length can be determined by the actual velocity distribution, that is,

$$l = k\frac{\left|\frac{\partial u}{\partial z}\right|}{\frac{\partial^2 u}{\partial z^2}} \tag{3.15}$$

Where k is the Karman constant, k = 0.41. Therefore, based on the velocity distribution, we can qualitatively analyze the mixing length in the clear water zone and top vegetated zone. While we have no past knowledge of the theoretical expression of velocity, this article uses the polynomial fitting method to understand the variation trend of velocity. According to Equation (3.15) and the polynomial, we obtain the variation of mixing length with water depth.

The mixing length in the vegetated zone is approximately a constant, and in the clear-water zone is proportional to the water depth. Huai *et al.* (2009) pointed out that the flow in the entire area can be regarded as the compressed flow on the new riverbed which is made up of vegetation, and presented a river compression coefficient to express the compression. From the conception of compressed channel, we give the mixing length as:

$l = \eta kz,\ h_v < z < H$ (3.16a)

$l = l_0,\ h_v - \delta_e < z < h_v$ (3.16b)

Where η is the compression coefficient of watercourse, which is = $(H - h_v)/H$, l_0 is the constant determined by the continuity of mixing length at $z = h_v$, hence $l_0 = \eta k h_v$, δ_e is the thickness of clear water zone, h_v is the height of blade.

3.4.2 Thickness of each zone

It has been stated that the array of the Clear water zone varies from h_v to H, while that of the proceeding zones has not been specified yet. This is because the viscous zone is constantly thin, hence we adopt that its thickness δ_e = 0.005m. Making the distance of blade penetration, δ_e , the only problem left, which gives the boundary between the top vegetated zone and the transition zone.

The top vegetated zone is also known as vegetation shear layer, Finnigan, (2000) suggested that numerous continuous vortices, induced by the K-H instability in the flow field, control the exchange of momentum and have a significant effect on the turbulence features, the vortices in the clear water zone grow repeatedly downstream, while that in the vegetated zone only enter through a fixed depth (δ_e in Figure 2.1) from the top of vegetation. The distance of entrance δ_e splits the vegetation zone into the top layer and the lower layer having heavy turbulence and weak turbulence respectively Nepf and Vivoni, (2000), which are correspondingly named as the top vegetated zone, the transition zone. Nepf, (2004) gave a calculated result for δ_e in vegetation by experimental study,

$$\frac{\delta_e}{h_v} = \frac{0.23 \pm 0.06}{C_D \lambda h_v} \quad (3.17)$$

Hence, the zone of transition thickness is, $h_v - \delta_e - \delta_0$

3.4.3 Velocities in vegetation and clear water zones

For the *Clear water zone*, combining equations (3.14), (3.16) and then write the governing equation (3.9) as

$$\frac{\partial}{\partial z}\left(z\frac{\partial u}{\partial z}\right)^2+\frac{gS_o}{(\eta k)^2}=0 \tag{3.18}$$

The effects of surface tension and wind at z = H is negligible, so one boundary condition is fixed in the calculation as

$$\left(\frac{\partial u}{\partial z}\right)_{z=H}=0 \tag{3.19}$$

Applying the boundary condition (3.19) to equation (3.18), we find the analytical solution to the stream-wise velocity in the clear water zone:

$$u=2\sqrt{\frac{gS_o}{(\eta k)^2}}H\left\{cos\left(arcsin\sqrt{\frac{z}{H}}\right)+ln\left[tan\left(\frac{arcsin\frac{z}{H}}{2}\right)\right]\right\}+C \tag{3.20}$$

As C stands for the integral constant that is equal to the velocity at boundary of the water surface.

For the *Top Vegetated zone*, we put together equations (3.14) and (3.16) and rewrite the governing equation (3.10) as,

$$\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2+\frac{gS_o}{{l_0}^2}-\frac{C_D N b_v u^2}{2{l_0}^2}=0 \tag{3.21}$$

It is challenging to obtain an analytical solution of Equation (3.19), so the finite difference methods are adopted for this purpose,

$$2\frac{u_1-u_{i-1}}{\Delta z}\ \frac{u_{i+1}-2u_i+u_{i-1}}{{\Delta_z}^2}+\frac{gS_o}{{l_0}^2}-\frac{C_D N b_v u^2}{2{l_0}^2}=0 \tag{3.22}$$

Where, Δz is the spatial step, and here Δz = 0.0001m, u_1 is the instantaneous velocity at the *i*th node, the boundary condition of the variation in pattern (3.22) is the velocity at *z* = $h_v - \delta_e$, which can be determined from equation (3.11) as shown below:

$$u_z = h_v - \delta_e, = u = \sqrt{\frac{2gS_o}{C_D N b_v}} \tag{3.23}$$

The velocity in the transition zone is said to be constant, according to equation (3.11), which is also obtainable by equation (3.23).

In the case of the *viscous zone*, the governing Equation (3.12) can be interpreted into the finite difference scheme as,

$$\tau' = \mu\frac{u_{i+1} - 2u_i + u_{i-1}}{{\Delta_z}^2} + \rho g S_o - \frac{1}{2}\rho C_D N b_v {u_i}^2 = 0 \tag{3.24}$$

Also, the scheme needs two different boundary conditions z = 0 and at z = δ_e respectively:

$$u_{z=0} = 0 \tag{3.25}$$

$$u_{z=\delta_0} = \sqrt{\frac{2gS_o}{C_D N b_v}} \tag{3.26}$$

A Summary of expected findings are as stated in the aim and objectives of this study.

3.5 Mathematical Model; Direct Numerical Simulation

The use of DNS in designing a model as adopted in this study, the Navier-Stokes equations are resolved to the smallest details of fluid flow (Huthoff *et al.*, 2005). The associated Kolmogorov micro-scales, given by the equation 3.27 and 3.28 below

$$\eta_k = \left(\frac{\upsilon^3}{\epsilon}\right)^{1/4} \tag{3.27}$$

$$\tau_k = \left(\frac{\upsilon}{\epsilon}\right)^{1/2} \tag{3.28}$$

These prescribe the required spatial and temporal resolutions to capture all relevant patterns of flow. At smaller scales, forces on molecules governing energy losses, that for common flow situations are well represented by an energy sink based on viscosity.

For turbulent flows with relatively small Reynolds numbers and in simple geometric domains, DNS has already been applied successfully (Vincent & Meneguzzi 1991; Moin & Mahesh 1998; Breugem 2004; Williams & Singh 2004). However, clear disadvantages of DNS are the large computational cost, which increases with Re^3, Pope

(2000) and the amount of memory capacity needed. A simple calculation reveals that the passage of a flood wave over a distance of 1km in a typical lowland water body could easily require tens or hundreds of years of DNS computation time.

The mean velocity and its instabilities were measured by 3-D Micro Acoustic Doppler Velocimeter (Micro ADV). The sample frequency was 50 Hz and the time of testing was 30s, so one measurement could collect 1500 samples. Carollo *et al.* (2002) had indicated that more than 700 samples data could obtain a constant velocity and guarantee the accuracy of the measurement. We selected two typical locations for the measurements. One (Point B) situate behind the rod and the other (Point A) was on the centre-line between two cable ties.

3.5.1 Theoretical analysis

Entire flow analysis

Mix length

The study applies Prandtdl's mixing length theory to calculate the Reynold's stress as shown in Equations. (3.09), (3.10):

With *l* representing the mixing length, from the Karman similarity theory l, Nazarenko, (2000) it is known that the mixing length can be determined by the actual velocity distribution.

Where *k* is the Karman constant, K = 0.41. Therefore, based on the velocity distribution, we can qualitatively analyze the mixing length in the clear water zone and top vegetated zone. While we have no past knowledge of the theoretical expression of velocity, this article uses the polynomial fitting method to understand the variation trend of velocity. According to Equation (3.15) and the polynomial, we obtain the variation of mixing length with water depth.

The mixing length in the vegetated zone is approximately a constant, and in the clear-water zone is proportional to the water depth. Huai *et al.* (2009) pointed out that the flow in the entire area can be regarded as the compressed flow on the new riverbed which is made up of vegetation, and presented a river compression coefficient to express the compression.

The compression coefficient of watercourse, which is = $(H - hv)/H$, is the constant determined by the continuity of mixing length at $z = hv$, hence = ηkhv, is the thickness of clear water zone, hv is the height of blade.

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Preamble

This chapter presents the results as obtained from the procedures stated in chapter three; it also discusses the results hence giving a meaningful interpretation to the entire process. The output of the model as illustrated in a test analyses are shown the figures.

From dimensional analysis,

Geographically, aquatic vegetation appears in numerous varieties of geometrical features. Out of the various parameters, here are some standard ranges, enlisted. For a normal foliage,

$0.001 \leq h \leq 0.01m$ (Leonard and Luther, 1995; Lightbody and Nepf, 2006);

For sea grasses$0.003 \leq h \leq 0.01m$; $t \approx 10^{-3}m$; $10 \leq \lambda \leq 100\ m^{-1}$(Luhar *et al*., 2010; Nepf, 2012);

For grasses,$10 \leq h_v \leq 20\ cm$; $5 \times 10^{-6} \leq EI \leq 1 \times 10^{-3} Nmm^2$ (Nepf & Vivoni, 2000; Ghisalberti & Nepf, 2002).

These were applied to the available data set for the simulations, data as shown in Table 4.1

4.2 Analysis of Results of Application of Buckingham pi (π) Method to the Modeling of Highly Flexible Vegetation

This is as illustrated in a test analysis is shown in the sections below.

4.2.1 Variation mean vertical streamwise velocity with flexibility parameters

In section 3.1, dimensional analysis was applied to vegetation and flow parameters for highly flexible, dense vegetation in submerged states. Based on the results (equation 3.5), numerical simulations has been carried out relating the mean velocity and vegetation parameters for different submergence ratio and results are as shown in Figure

4.1. The graphs follow log-law and the coefficients of correlations are approximately unity ($R \cong 1$).

Based on the fitted equations from the curves, a generalized equation that related mean vertical flow velocity, mean vegetal velocity, plants flexibility and average deflected height of plants is proposed in equation 4.1. The deflection increases with increasing flow velocity for higher submergence.

$$\frac{U}{U_{veg}} = A' \ln\left(\frac{EI}{\mu U {k_d}^2}\right) + B' \tag{4.1}$$

Where A' and B' are approximately equal to 1.6 and 4.8 respectively.

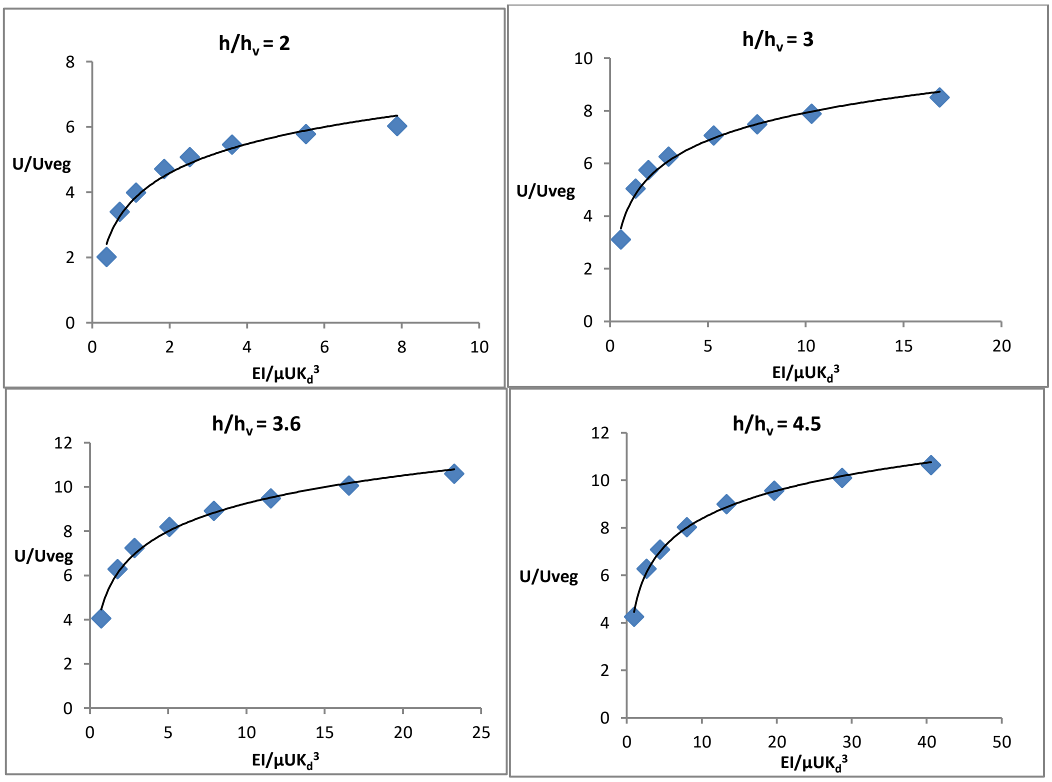

Figure 4.1: Normalized mean vertical streamwise velocity against normalized plant's flexural rigidity for varying submergence ratio

4.2.2 Variation mean vertical streamwise velocity with induced drag coefficient

Figure 4.2 display the graph of mean streamwise velocity and the vegetation induced drag for different submergence ratio. Increasing flow velocity increases the induced drag from the vegetation in order to balance gravity force and the vegetal drag force. The fitting equation from the graphs yield equation 4.2 as follows:

$$\frac{U}{U_{veg}} = \varpi ln\left(C_d A_p\right) + \xi \tag{4.2}$$

Where ϖ and ξ are 4.9 and 52.8 respectively for $(2 \leq {}^{h}/_{h_v} \leq 5)$

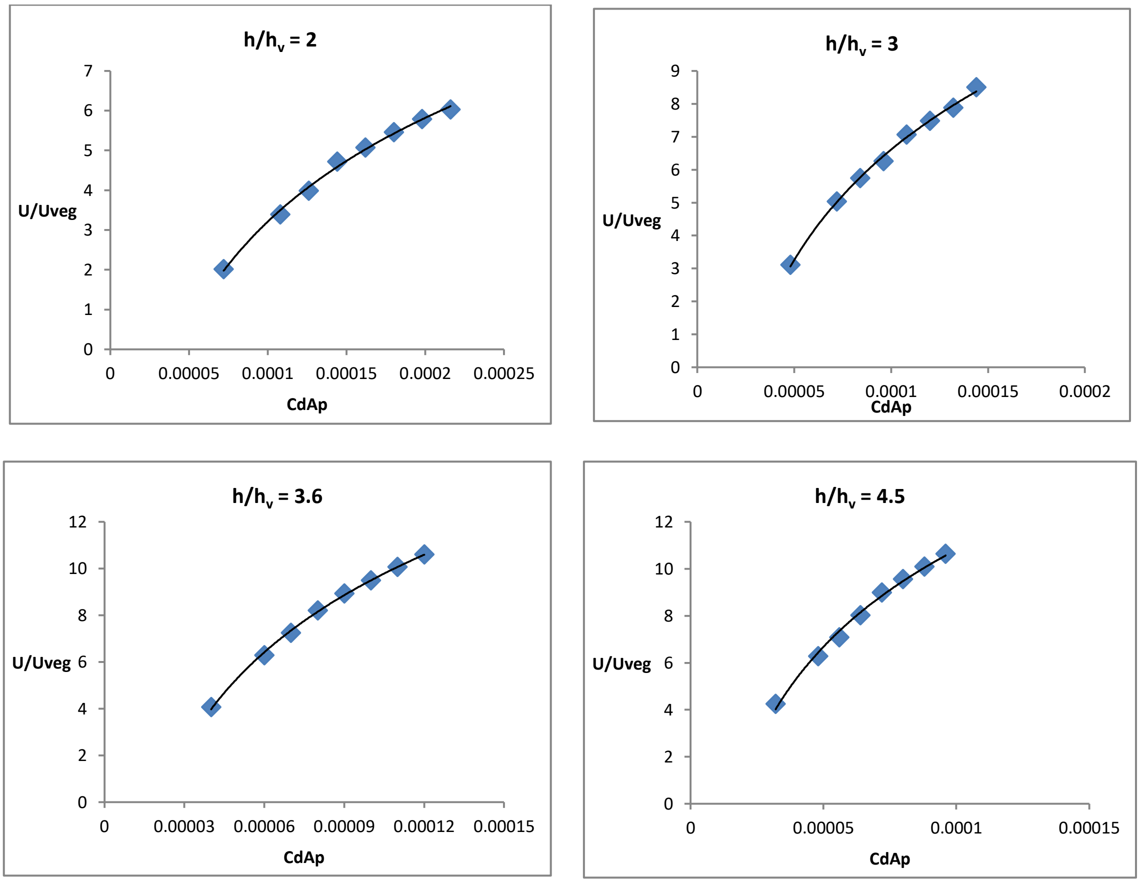

Figure 4.2: Normalized mean vertical streamwise velocity against induced drag

4.2.3 Variation mean stem Reynolds number with drag coefficient

Figure 4.3 shows the relationship between stem Reynolds number and drag coefficient under different submergence ratio. It can be observed that the drag coefficient decreases with increasing submergence ratio due to streamlining of the vegetation subjected to deflection. The stem Reynolds number also decreases with increasing submergence ratio due to the diversion of more flows over the vegetation. The Reynolds number increases with the induced drag due to low flow in the vegetation zone and corresponding increase in flow velocity in the clear water zone.

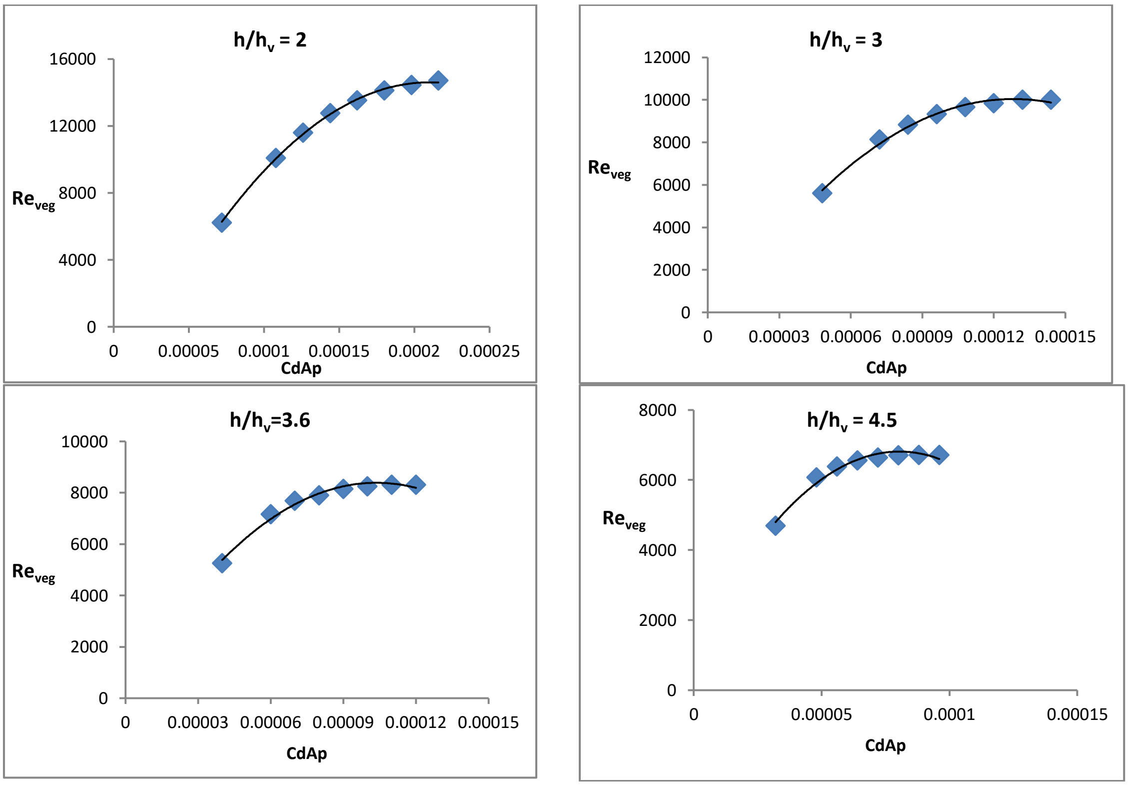

Figure 4.3: Variation of Reynolds number with induced drag for different submergence ratio

4.3 Velocity Profiles and Reynolds Shear Stresses

The parameters in the laboratory experiment by Busari, (2016) is as shown in Table 4.1.

Table 4.1: Parameters in the experiments (Busari, 2016)

Run	bv (m)	N (m $^{-2}$)	hv (m)	Cd	h (m)	u (m/s)	gSo	h/hv
1.	0.005	5000	0.1	3	0.151	0.03	0.0051	1.51
2.	0.005	5000	0.1	3	0.253	0.11	0.0100	2.53
3.	0.005	5000	0.085	3	0.382	0.367	0.0300	4.49
4.	0.005	5000	0.1	3	0.152	0.098	0.0500	1.52
5.	0.005	5000	0.1	3	0.151	0.143	0.1001	1.51
6.	0.005	5000	0.05	3	0.242	0.560	0.0939	4.84
7.	0.005	5000	0.1	3	0.350	0.205	0.0100	3.5

From Figure 4.4, a careful examination shows that the model predicted the velocity profiles and the Reynolds shear stresses perfectly well for low submergence ratio due low vortex shedding at the interface between the top of the deflected vegetation and the clear water zone (Figures 4.4a, 4.4d and 4.4e). However, the prediction became bias above the vegetation layers in the other figures due high submergence ratio resulting to large diverted flows above the vegetation and large scale of eddies formulation (Figures, 4.4b, 4.4c, 4.4f, and 4.4g). Generally, the model replicated the velocity well within the vegetation zone. The maximum Reynolds shear stresses increases with submergence ratio. The zero shear stresses in the vegetation layer resulted from a balance between the gravity and vegetation drag forces.

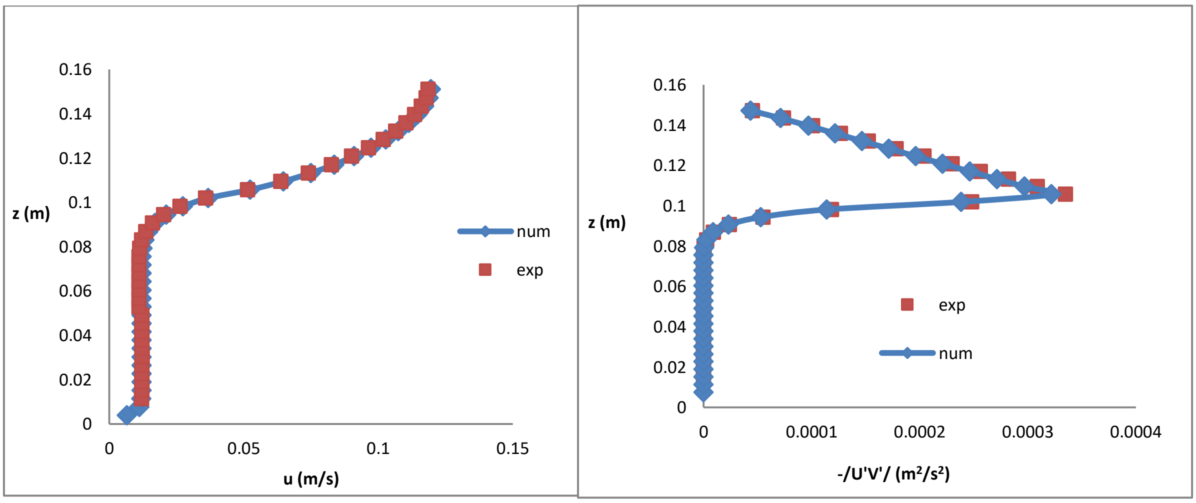

Figure 4.4a: Plot of mean vertical streamwise velocity profile and Reynolds shear stresses (RUN1)

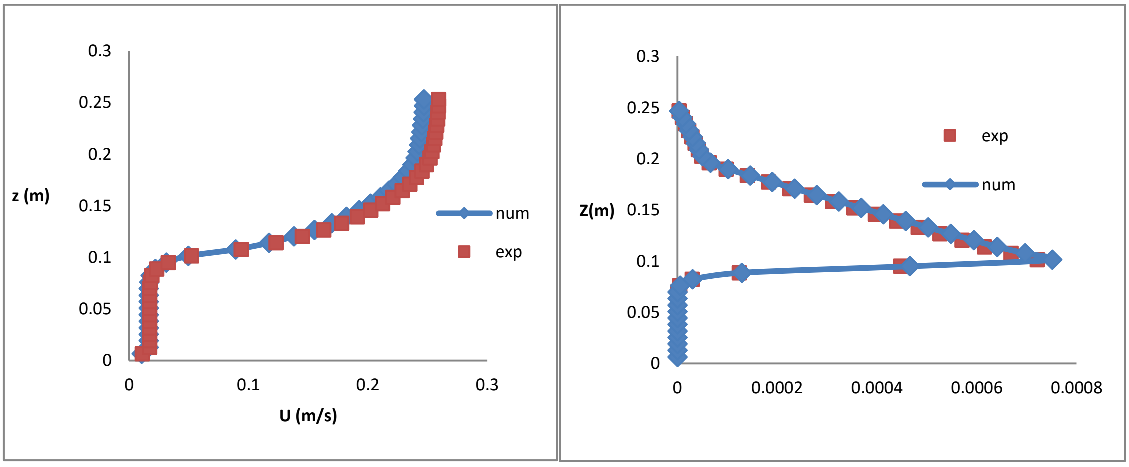

Figure 4.4b: Plot of mean vertical streamwise velocity profile and Reynolds shear stresses (RUN2)

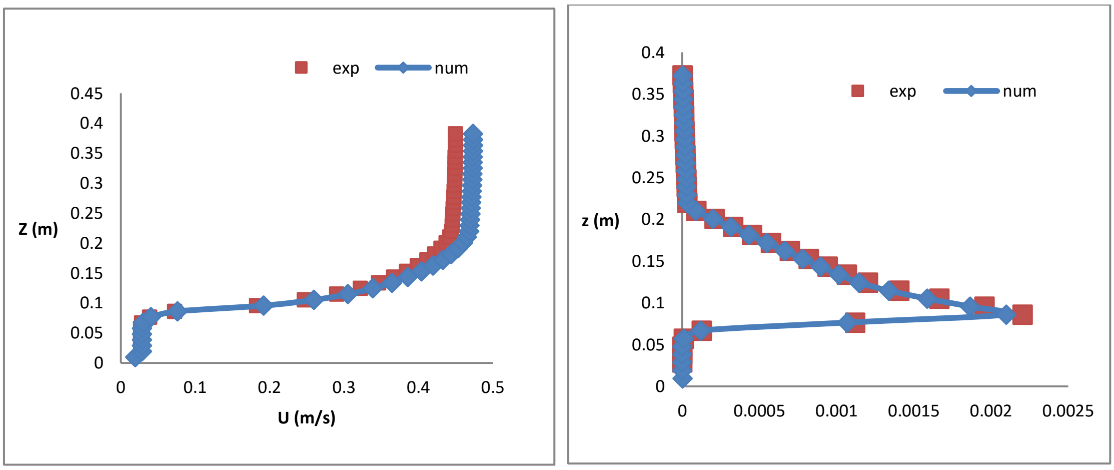

Figure 4.4c: Plot of mean vertical streamwise velocity profile and Reynolds shear stresses (RUN3)

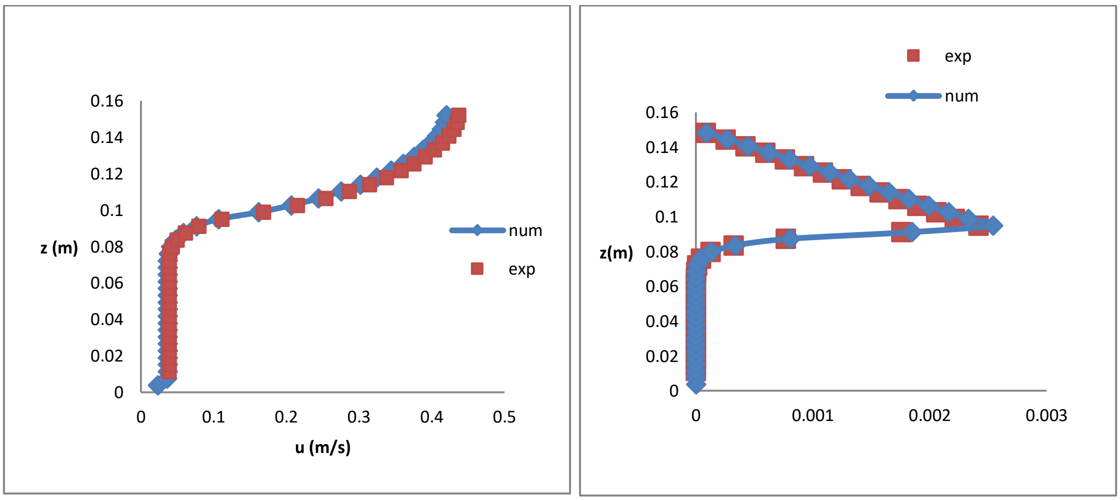

Figure 4.4d: Plot of mean vertical streamwise velocity profile and Reynolds shear stresses (RUN4)

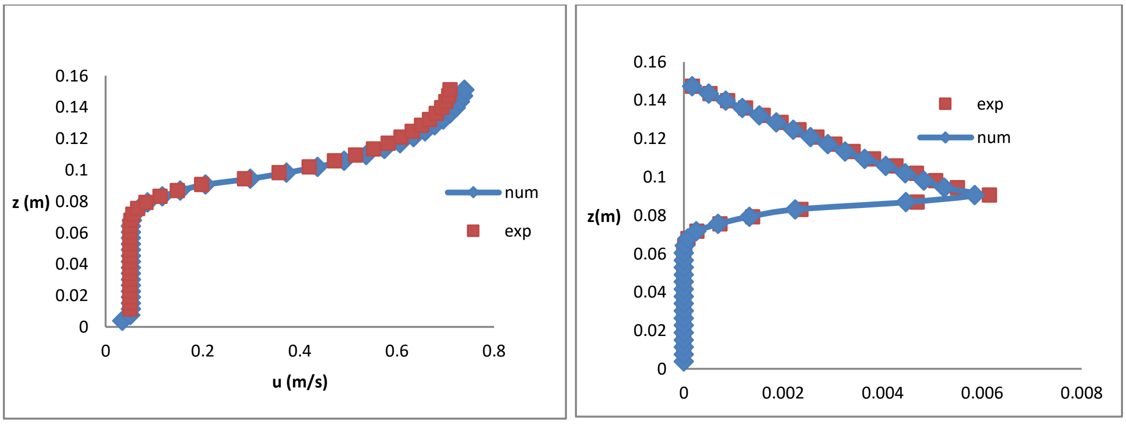

Figure 4.4e: Plot of mean vertical streamwise velocity profile and Reynolds shear stresses (RUN5)

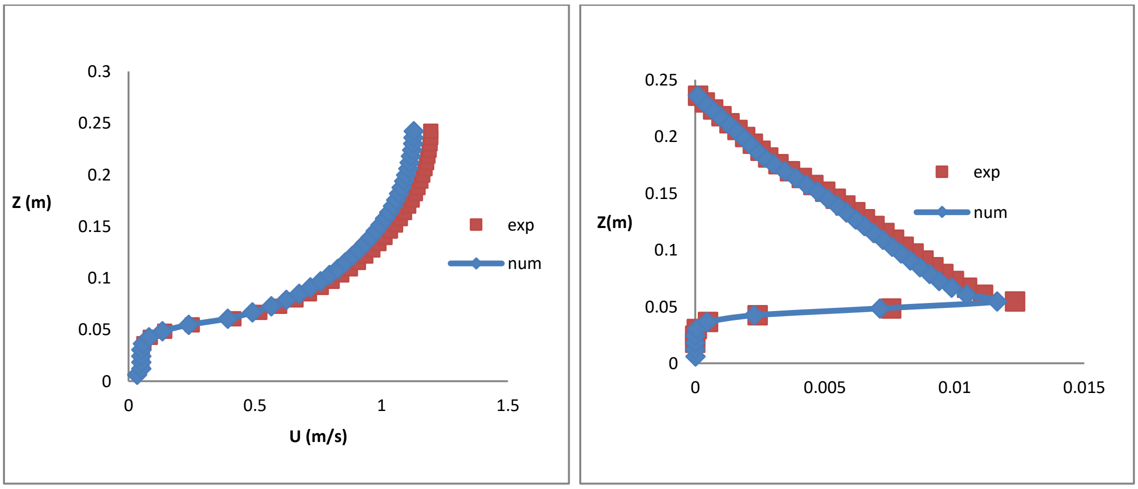

Figure 4.4f: Plot of mean vertical streamwise velocity profile and Reynolds shear stresses (RUN6)

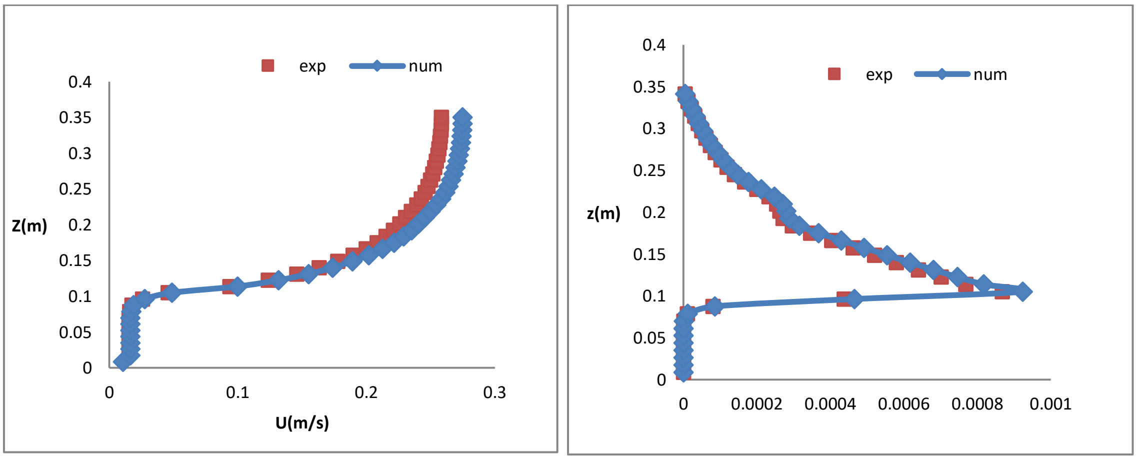

Figure 4.4g: Plot of mean vertical streamwise velocity profile and Reynolds shear stresses (RUN7)

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The study formulate the required theory of vegetated flow based on vertical mean streamwise velocity and Reynolds shear stresses.

The most significant hydraulic parameters have been evaluated using Buckingham pi (π) method of dimensional analysis. The variation of the parameters yield a hydraulic roughness equation relating the flow and the plants flexibility. More so, a clear explanation of increasing Reynolds number with increasing submergence ratio has been provided.

A Mathematical model has been developed for aquatic canopies. The model predicts well the velocity within and above the vegetation zone as well as the Shear stresses.

Finally, the model outputs were compared with the experimental datasets. The predictions of mean velocity profile and Reynolds stress for different hydraulic conditions shows the capability of the model in modeling of vegetated open channel flows.

5.2 Recommendations

i. This thesis aimed at reinforcing sustainable development and Engineering growth through innovations, hence, it is advised to start at lower Engineering Researchers from as low as undergraduate levels. However Vegetation and other linings maybe effective in checking erosion and similar environmental conditions in place of the conventional use of concrete and other rigid structures, where the channel longitudinal slope is less than (<) 10%.

5.3 Contribution to Knowledge

The study applies a modified one-dimensional (1-D) hydraulic model to replicate the vegetative velocity profile and Reynolds stresses from laboratory experimental and field datasets found in laboratory investigation from Literature. The model is as shown

$$\frac{\boldsymbol{R}}{\boldsymbol{\mu U k_d}^3} = f\left\{\boldsymbol{Re_{veg}}, \boldsymbol{Fr},\ \frac{\boldsymbol{h}}{\boldsymbol{k_d}}, \frac{\boldsymbol{D}}{\boldsymbol{k h_v}},\ \boldsymbol{N b_v h_v}\right\}$$ flow parameters used from dimensional analysis

$$\frac{\partial}{\partial z}\left(z\frac{\partial u}{\partial z}\right)^2 + \frac{gS_o}{(\eta k)^2} = 0$$

$$\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{gS_o}{{l_0}^2} - \frac{C_D N b_v u^2}{2{l_0}^2} = 0$$

$$2\frac{u_1 - u_{i-1}}{\Delta z}\ \frac{u_{i+1} - 2u_i + u_{i-1}}{{\Delta_z}^2} + \frac{gS_o}{{l_0}^2} - \frac{C_D N b_v u^2}{2{l_0}^2} = 0$$

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APPENDIX

Script Code

```
c......velocity profile and Reynolds shear stresses for flexible vegetation

      real*4 u1(2000),xn1(2000),xn2(2000),xn3(2000),u2(2000)

      real*4 rke(2000),rys(2000),dist(2000),z(2000),zp(2000)

      rk=0.41

      sigm=0.6667

      cb1=0.1355

      cb2=0.622

      cv1=7.1

      cw1=cb1/rk**2+(1+cb2)/sigm

      cw2=0.3

      cw3=2.

      pi=3.14159265

      xnu=1.e-6

      cmu=0.09

      rho=1000.

      open(1,file='1dfl.dat')

      open(7,file='vel.out')
```

```
      open(8,file='rke.out')
      open(9,file='rys.out')
      open(10,file='xnu.out')
      open(13,file='defl.out')
      read (1,*) yh,aa,kmax,uav,dt,nt,iout,gs0,yhv,frk,rz0
      read (1,*) cd,width,EI
      read (1,*) alpha,beta

c      stokes layer
c       stok=sqrt(2.e-6/omega)
c       ustar=u0/25.
      ustar=sqrt(yh*gs0)
      ustar1=ustar
c      grid layout
      if (aa.eq.1.) then
      dz0=yh/(kmax-1)
      else
      dz0=yh*(aa-1)/(aa**(kmax-1)-1)
      endif
```

```
c     write (*,*) 'dz0 ', dz0
      zp(1)=0.
      dist(1)=0.
      do k=1,kmax-1
      z(k)=dz0*aa**(k-1)
      zp(k+1)=zp(k)+z(k)
      dist(k+1)=max(zp(k+1)-rz0*yhv,zp(k+1)*(1-rz0))
      enddo
      write (7,'(1000f12.5)') 0.,(zp(k)/yh,k=1,kmax)
      write (7,'(1000f12.5)') 0.,(zp(k),k=1,kmax)
      write (10,'(1000f12.5)') 0.,(zp(k)/yh,k=1,kmax)
      write (8,'(1000f12.5)') 0.,(zp(k)/yh,k=2,kmax-1)
      write (9,'(1000f12.5)') 0.,(zp(k)/yh,k=2,kmax-1)
      write (7,'(1000f12.5)') 0.,(zp(k)*ustar/xnu,k=1,kmax)
      write (7,'(1000f12.5)') 0.,0.,(1./rk*log(zp(k)*ustar/xnu)+8.5,    &   k=2,kmax)
c     write (10,'(1000f12.5)') 0.,(zp(k)*ustar/xnu,k=1,kmax)
c     write (8,'(1000f12.5)') 0.,(zp(k)*ustar/xnu,k=2,kmax-1)
c     write (9,'(1000f12.5)') 0.,(zp(k)*ustar/xnu,k=2,kmax-1)
      write (10,'(1000f12.5)') 0.,(zp(k),k=1,kmax)
```

```
      write (8,'(1000f12.5)') 0.,(zp(k),k=2,kmax-1)
      write (9,'(1000f12.5)') 0.,(zp(k),k=2,kmax-1)
c       write (*,*) (z(k),k=1,kmax)
c       write (*,*) yh
c       pause
c      intitial condition
c      power law
      al=0.1
      u0=uav*(1+al)
      do k=1,kmax
      if (k.gt.1) then
      u1(k)=u0*(zp(k)/yh)**al
      else
      u1(1)=0.
      endif
      xn1(k)=ustar*rk*zp(k)
      xn2(k)=3.*xn1(k)
      xn3(k)=xn2(k)
      enddo
```

```
c       write (*,*) (xn1(k),k=1,200)
c       pause
        hv=yhv
        do it=1,nt
        t=it*dt
c
c      boundary conidtion
        u1(kmax)=u1(kmax-1)
c       u1(1)=0.
c       u2(1)=0.
        u2(kmax)=u1(kmax)
        xn1(1)=0.
        xn2(1)=0.
        xn3(1)=0.
        ustar1=u1(2)/(1./rk*log(zp(2)*ustar1/xnu)+8.5)
c       ustar2=u1(3)/(1./rk*log(zp(3)*ustar/xnu)+8.5)
c       ustar=0.5*(ustar1+ustar2)
        xn1(2)=ustar1*rk*zp(2)
        xn2(2)=xn1(2)
```

```
      xn3(2)=xn2(2)ro766i6
      u1(1)=u1(2)-ustar1**2/xn1(2)*(zp(2)-zp(1))*2
c     u1(2)=2*ustar/rk+u1(1)
c      u1(2)=ustar*(1./rk*log(zp(2)*ustar/xnu)+5.0)
      u2(1)=u1(1)
c      u2(2)=u1(2)


c      xn1(1)=xn1(2)
c      xn2(1)=xn2(2)
c      xn3(1)=xn3(2)
      xn1(kmax)=xn1(kmax-1)
      xn2(kmax)=xn2(kmax-1)
      xn3(kmax)=xn3(kmax-1)


c
c     determine deflected height
c
      if (mod(it,iout).eq.0) then
      Q=0.
```

```
      do k=2,kmax-1
c       if (zp(k).lt. 2./3*hv+1./3*yhv) then
c       if (zp(k).lt.yhv) then
      if (zp(k).lt.hv) then
c      single stem
      Q=Q+0.5*rho*u1(k)*abs(u1(k))*z(k-1)*width*cd
      endif
      enddo
c       Q=Q/(2./3*hv+1./3*yhv)
c       Q=Q/yhv
      Q=Q/hv
      qyn=abs(Q*yhv**3/EI)
c       qyn=abs(Q*yhv**2/EI)
c       qyn=abs(Q*hv*yhv**2/EI)
      qyn = min (1000., qyn)
      if (qyn.gt. 100.) then
     rkend=yhv*(5.4583e-18*qyn**6-1.9598e-14*qyn**5+2.8577e-11*qyn**4
    &  -2.1897e-8*qyn**3+9.5802e-6*qyn**2-2.5010e-3*qyn+0.55942)
      else
```

```
      rkend=yhv*(-1.1093e-12*qyn**6-7.9352e-12*qyn**5+7.3301e-8*qyn**4
     &  -1.2141e-5*qyn**3+8.5414e-4*qyn**2-3.171e-2*qyn+1.0143)
      endif
      hv=rkend
      endif


      do k=2,kmax-1
         difz1=(xnu+(xn1(k)+xn1(k+1))*0.5)
     &       *(u1(k+1)-u1(k))/z(k)
         difz2=(xnu+(xn1(k)+xn1(k-1))*0.5)
     &       *(u1(k)-u1(k-1))/z(k-1)
         diffz=(difz1-difz2)/(z(k)+z(k-1))*2.
      u2(k)=u1(k)+dt*(gs0+diffz)


      if (zp(k).le.hv) u2(k)=u2(k)-dt*frk*u1(k)*abs(u1(k))*0.5
c       secondary current
      if (zp(k).gt.hv) u2(k)=u2(k)-dt*0.5*(1+tanh(beta*(zp(k)/yh-
     &   alpha)))*gs0
```

```
c        write (*,*) u1(k),diffz,xn1(k)
      enddo
c        pause
      do k=2,kmax-1
      sv=abs((u1(k+1)-u1(k-1))/(z(k)+z(k-1)))
        difz1=(xnu+(xn2(k)+xn2(k+1))*0.5)
     &       *(xn2(k+1)-xn2(k))/z(k)
        difz2=(xnu+(xn2(k)+xn2(k-1))*0.5)
     &       *(xn2(k)-xn2(k-1))/z(k-1)
        diffz=(difz1-difz2)/(z(k)+z(k-1))*2.
        xnuxyz=((xn2(k+1)-xn2(k-1))/(z(k)+z(k-1)))**2
      phi=xn2(k)/xnu
      fv1=phi**3/(phi**3+cv1**3)
      fv2=1-phi/(1+phi*fv1)
      svb=sv+xn2(k)*fv2/(rk*dist(k))**2
c....to avoid potential stability problem
      svb=max(svb,0.3*sv)
      rrr=xn2(k)/svb/(rk*dist(k))**2
      rrr=min(rrr,10.)
```

```
   ggg=rrr+cw2*(rrr**6-rrr)
   fw=ggg*((1+cw3**6)/(ggg**6+cw3**6))**(1./6)
     xn3(k)=xn2(k)+dt*(cb1*svb*xn2(k)+
 &      (diffx+diffy+diffz+cb2*xnuxyz)/sigm
 &      -cw1*fw*xn2(k)**2/dist(k)**2)
   enddo
   do k=2,kmax-1
   xn2(k)=xn3(k)
   phi=xn2(k)/xnu
   fv1=phi**3/(phi**3+cv1**3)
   xn1(k)=xn2(k)*fv1
   u1(k)=u2(k)
   enddo

   if (mod(it,iout).eq.0) then
   do k=2,kmax-1
   sv=abs((u1(k+1)-u1(k-1))/(z(k)+z(k-1)))
   rke(k)=xn1(k)*sv/sqrt(cmu)*1.3
   rys(k)=xn1(k)*(u1(k+1)-u1(k-1))/(z(k)+z(k-1))
```

```
      enddo
      uav1=0.
      do k=1,kmax-1
      uav1=0.5*(u1(k)+u1(k+1))*z(k)
      enddo
      uav1=uav1/yh
      time=t
c       write (7,'(1000e12.4)') time,(u1(k)/ustar,k=1,kmax)
      write (7,'(1000e12.4)') time,(u1(k),k=1,kmax),uav1
      write (8,'(1000e12.4)') time,(rke(k),k=2,kmax-1)
      write (9,'(1000e12.4)') time,(rys(k),k=2,kmax-1)
      write (10,'(1000e12.4)') time,(xn1(k),k=1,kmax)
c       write (8,'(1000e12.4)') time,(rke(k)/ustar**2,k=2,kmax-1)
c       write (9,'(1000e12.4)') time,(rys(k)/ustar**2,k=2,kmax-1)
c       write (10,'(1000e12.4)') time,(xn1(k)/xnu,k=1,kmax)
      write (13,'(1000e12.4)') time,hv,hv/yh,qyn
      endif
      if (mod(it,iout*10).eq.0) then
      write (*,*) 'time step = ',it
```

```
c       residual stress
        ustar2=ustar**2
        do k=kmax,2,-1
        if (zp(k).lt.hv) ustar2=ustar2-
     &   frk*u1(k)*abs(u1(k))*0.5*z(k-1)
        enddo
        write (*,*) 'ustar2', ustar2,ustar**2,ustar1**2
        endif
        enddo
        stop
        end
```