COMPARATIVE DECOMPOSITION APPROACH OF MAGNETOHYDRODYNAMICS ROTATIONAL STAGNATION ON POINT FLOW BETWEEN A STRETCHING AND SHRINKING SHEET WITH HEAT GENERATION

ABSTRACT

The problem of Magnetohydrodynamics rotational stagnation point flow over a shrinking and stretching sheet with heat generation and absorption was considered . The Partial differential equations were transformed using similarity variables to ordinary nonlinear coupled differential equations. The analytical solutions were presented using the Adomian decomposition method. The results presented were validated with the literature and a good agreement was observed. The effects of various dimensionless parameters like Rotational parameter , Thermal Grashof number , Concentration Grashof number , Magnetic parameter , Prandtl number , Heat generation , Schimidt number , shrinking/stretching parameter and Suction/injection parameter that appeared were graphically presented and the magnetic parameter was found to enhanced the fluid temperature.

CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

Magnetohydrodynamic flows with or without the movement of heat in an electrically conducting fluids have attracted a large interest in the context of metallurgical fluid dynamics, aerothermodynamics, astronautics, geophysics, nuclear engineering and applied mathematics. Carrier and Greenspan (1960) considered unsteady hydromagnetic flows past a semi-infinite flat plate moving impulsively in its own plane. Gupta (1960) considered unsteady magneto-convection under buoyancy forces. Singer (1965) carried out further study rtyon unsteady free convection heat transfer with magnetohydrodynamic effects in a channel regime. Pop (1969) works on transient buoyancy-driven convective hydromagnetics from a vertical surface. Tokis (1986) implored the Laplace transforms to analyze the three dimensional free-convection hydromagnetic flows near an infinite vertical plate moving in a rotating fluid when the plate temperature undergoes a thermal transient. The influence of oscillatory pressure gradient on transiently rotating hydromagnetic flow was reported by Ghosh (1993).

Abd-El Aziz (2006) carried out a study on the thermal radiation flux effects on unsteady Magnetohydrodynamics micropolar fluid convection. Ogulu and Prakash (2006) in their work, presented an analytical solutions for variable suction and radiation effects on dissipative-free convective, optically-thin, Magnetohydrodynamic flow using a differential approximation to describe the radiative flux. Recent studies involving thermal radiation and transient hydromagnetic convection with specie transfer and viscous heating can be found in the analyses of Prasad et al. (2006) and Zueco (2007). In many geophysical and metallurgical flows, porous medium can arise also. Classically, the Darcian model is used to showcase the bulk effects of porous materials on flow dynamics and is valid for Reynolds numbers based on the pore radius. Chamkha (1996) works on the transient-free convection Magnetohydrodynamic boundary layer flow in a fluid-saturated porous medium channel, and later Chamkha (2001) extended the study to consider the influence of temperature-dependent properties and inertial effects on the convection regime. B´eg et al. (2005) presented perturbation solutions for the transient oscillatory hydromagnetic convection in a Darcian porous media with present of heat source. Chaudhary and Jain (2008) carried out the influence of oscillating temperature on Magnetohydrodynamic convection heat transfer past a vertical plane in a Darcian porous medium. Lately, Variational iteration method was applied for squeezing MHD Nano fluid flow in a rotating channel with the lower stretching porous surface, see Shahmohamadi and Rashidi (2016) for example. More extensive works as contained in the works of Mishra and Bhatti (2017), Rashidi et al. (2014), Sheikholeslami and Bhatti (2017), Abbas et al. (2017) and Bhatti and Rashidi (2016).

1.2 Statement of the Problem

Das and Ahmed (1992) employed the perturbation technique to analyze the buoyancy-driven magnetohydrodynamic (MHD) flow and heat transfer for a viscous incompressible fluid confined between a long vertical undulated plate and a parallel flat plate. They considered the effects of relative temperature of the channel walls on the velocity and temperature profiles without considering the wavy wall amplitude and inclination angle effects.

Recently, Bhatti *et al.* (2018) consider the problem of magnetohydrodynamic boundary layer flow with suction on a stretching and shrinking sheet. In their work, they used numerical approach to obtain the solution to the problem formulated. Natural convection, fluid rotation, fluid temperature and concentration were neglected. There by, no temperature and concentration dependent variables were considered.

1.3 Justification of the Study

A wide variety of industrial processes involve the transfer of heat energy. Throughout any industrial facility, heat must be added, removed, or moved from one process stream to another and it has become a major task for industrial necessity. These processes provide a source for energy recovery and process fluid heating/cooling.

Several works have been cited in this research work and we have decided to consider the areas that have been left out by the previous researchers, thus making the present study justifiable.

1.4 Aim and Objectives of the Study

1.4.1 Aim of the study

The aim of this study is to carry out a comparative analysis of magnetohydrodynamics boundary layer flow between stretching and shrinking sheet with heat generation and fluid rotation.

1.4.2 Objectives of the study

The objectives of this present study are :-

i. To transform the Partial differential equation (PDE) formulated to ordinary differential equations (ODE) using the similarity equations.

ii. To solve the set of transformed non linear, coupled, ordinary differential equations (ODE) using the Adomian Decomposition Method (ADM).

iii. To validate the results obtain with the work of Wang (2008) and Bhatti *et al.*(2018).

iv. To present and analyse the solutions with the help of the graphical representations.

v. To verify the effects of the various dimensionless parameters that appears in the solutions on both stretching and shrinking sheet.

1.5 Scope and Limitation

The Partial Differential Equation (PDE) formulated from the problem is presented in its rectangular coordinate system. The appropriate similarity transformations and stream functions are used to transform the partial differential equations to ordinary differential equations. Non linear coupled ordinary differential equations are derived, corresponding to momentum, energy equation and concentration equations. These equations are solved using improved Adomian Decomposition Method. The effect of various parameters that appears are analysed with the help of graphs. This work is limited to incompressible fluid dynamics.

1.6 Definition of Terms

Fluid: A substance which deforms continuously when shear stress is applied to it no matter how small, such as liquid or gas which can flow, has no fixed shape and offers little resistance to an external stress.

Grashof number: is a dimensionless number in fluid dynamics and heat transfer which approximates the ratio of the buoyancy to viscous force acting on a fluid. It frequently arises in the study of situations involving natural convection. It is named after the German engineer Franz Grashof.

Prandtl number: the relationship between the thickness of two boundary layers at a given point along the plate depend on the dimensionless prandtl number which is the ratio of the momentum diffusivity ν or $\frac{\mu}{\rho}$ to the thermal diffusivity $\propto$ or $\frac{k}{\rho Cp}$.

Boundary Layers:- boundary layer is defined as that part of moving fluid in which the fluid motion is influenced by the presence of a solid boundary. As a specific example of boundary layer formation, consider the flow of fluid parallel with a thin plate, when a fluid flows at htigh Reynolds number past a body, the viscous effects may be neglected everywhere except in a thin region in the vicinity of the walls . This region is termed as the boundary layer.

Magnetohydrodynamics: is the study of magnetic properties and behavior of electrically conducting fluids.

Convection: is the conveying of heat from part of a liquid or gas to another by the movement of heated substances.

Stagnation Point: A point in the flow where the local velocity is zero.

Compressible fluid: change in density of fluid with time

$$\frac{\Delta\rho}{\Delta t} \neq 0 \tag{1.1}$$

Incompressible flow: fluid motion with negligible changes in density ρ

$$\frac{\Delta\rho}{\Delta t} = 0 \tag{1.2}$$

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Reviews on Fluid Dynamics

Stagnation-point flow generally describes the fluid motion near the stagnation region of a solid surface, which exists in the case of fixed as well as a moving body in a fluid. Stagnation-point flow with various physical effects has greater physical importance, including the prediction of the skin friction as well as the heat/mass transfer near the stagnation regions of bodies in high-speed flows; design of thrust bearings and radial diffusers; and drag reduction, transpiration cooling, and thermal oil recovery, among others. The flow in the neighbourhood of a stagnation line has attracted many investigations during the past several decades. Ramachandra *et al.* (1988) have investigated the mixed convection flow in the stagnation flow region of a vertical plate. The steady stagnation-point flow towards a permeable vertical surface was investigated by Ishak *et al.* (2008). Li et al. (2011) introduced an analysis of the steady mixed convection flow of a viscoelastic fluid stagnating orthogonally on a heated or cooled vertical flat plate. Makinde (2012) examined the hydromagnetic mixed convection stagnation-point flow towards a vertical plate embedded in a highly porous medium with radiation and internal heat generation. Mabood and Khan (2014) introduced an accurate analytic solution (series solution) for MHD stagnation-point flow in a porous medium for different values of the Prandtl number and the suction/injection parameter An unsteady boundary layer plays important roles in many engineering problems like a start-up process and a periodic fluid motion. An unsteady boundary layer has different behaviors due to extra time-dependent terms, which will influence the fluid motion pattern and the boundary-layer separation. Kumari *et al.* (1992) have studied the unsteady mixed convection flow of an electrically conducting fluid at the stagnation point of a two-dimensional body and an axisymmetric body in the presence of an applied magnetic field. Seshadri *et al.* (2002) studied the unsteady mixed convection in the stagnation-point flow on a heated vertical plate where the unsteadiness is caused by the impulsive motion of the free stream velocity and by sudden increase in the surface temperature (heat flux). Hassanien *et al.* (2004) analyzed the problem of unsteady free convection flow in the stagnation-point region of a rotating sphere embedded in a porous medium. The unsteady flow and heat transfer of a viscous fluid in the stagnation region of a three-dimensional body embedded in a porous medium was investigated by Hassanien et al. (2006). Hassanien and Al-Arabi (2008) studied the problem of thermal radiation and variable viscosity effects on unsteady mixed convection flow in the stagnation region on a vertical surface embedded in a porous medium with surface heat flux. Fang *et al.* (2011) investigated the boundary layers of an unsteady incompressible stagnation-point flow with mass transfer. Shateyi and Marewo (2014) have numerically investigated the problem of unsteady MHD flow near a stagnation point of a two-dimensional porous body with heat and mass transfer in the presence of thermal radiation and chemical reaction. Rosali *et al.* (2014) discussed the effect of unsteadiness on mixed convection boundary-layer stagnation-point flow over a vertical flat surface embedded in a porous medium.

During the past decade, the study of nanofluids has attracted enormous interest from researchers due to their exceptional applications to electronics, automotive, communication, computing technologies, optical devices, lasers, high-power X-rays, scientific measurement, material processing, medicine, and material synthesis, where efficient heat dissipation is necessary. Nanobiotechnology is also a fast-developing field of research and application in many domains, such as in medicine, pharmacy, cosmetics and agro-industry. Nanofluids are prepared by dispersing solid nanoparticles in base fluids such as water, oil, ethylene glycol, or others. According to Yacob et al. (2011), nanofluids are produced by dispersing the nanometer-scale solid particles into base liquids with low thermal conductivity such as water and ethylene glycol. Nanoparticles are usually made of metal, metal oxide, carbide, nitride, and even immiscible nanoscale liquid droplets. Congedo *et al.* (2009) compared different models of nanofluids (regarded as a single phase) to investigate the density, specific heat, viscosity, and thermal conductivity, and discussed the water–Al2O3 nanofluid in detail by using CFD. Hamad *et al.* (2011) introduced a one-parameter group to represent similarity reductions for the problem of magnetic field effects on free-convective nanofluid flow past a semi-infinite vertical flat plate following a nanofluid model proposed by Buongiorno (2006). Hady *et al.* (2012a) studied the radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet with variable wall temperature. Also, Hady et al. (2012b) studied the problem of natural convection boundary-layer flow past a porous plate embedded in a porous medium saturated with a nanofluid using Buongiorno's model. Further, Abu-Nada and Chamkha (2010) presented the natural convection heat transfer characteristics in a differentially heated enclosure filled with CuO–ethylene glycol (EG)–water nanofluids for different variable thermal conductivity and variable viscosity models. Rudraswamy and Gireesha (2014) studied the problem of flow and heat transfer of a nanofluid over an exponentially stretching sheet by considering the effect of chemical reaction and thermal radiation. Besthapu and Bandari (2015) presented a study on the mixed convection MHD flow of a Casson nanofluid over a nonlinear permeable stretching sheet with viscous dissipation. Numerical solutions of the natural convection flow of a two-phase dusty nanofluid along a vertical wavy frustum of a cone is discussed by Siddiqa *et al.* (2016a). The bioconvection flow with heat and mass transfer of a water-based nanofluid containing gyrotactic microorganisms over a vertical wavy surface is studied by Siddiqaet al. (2016b). Kameswaran et al. (2016) studied convective heat transfer in the influence of nonlinear Boussinesq approximation, thermal stratification, and convective boundary conditions on non-Darcy nanofluid flow over a vertical wavy surface.

Vasu and Manish (2015) studied the problem of two-dimensional transient hydrodynamic boundary-layer flow of an incompressible Newtonian nanofluid past a cone and plate with constant boundary conditions. Gireesha *et al.* (2015) introduced a numerical solution for hydromagnetic boundary-layer flow and heat transfer past a stretching surface embedded in a non-Darcy porous medium with fluid-particle suspension. The unsteady forced convective boundary-layer flow of an incompressible non-Newtonian nanofluid over a stretching sheet when the sheet is stretched in its own plane is investigated by Gorla and Vasu (2016). Gorla *et al.* (2016) investigated the transient mixed convective boundary-layer flow of an incompressible non-Newtonian quiescent nanofluid adjacent to a vertical stretching surface. The unsteady flow and heat transfer of a nanofluid over a contracting cylinder was studied by Zaimi *et al.* (2014). Srinivasacharya and Surender (2014) studied the effects of thermal and mass stratification on natural convection boundary-layer flow over a vertical plate embedded in a porous medium saturated by a nanofluid.

Abdullah *et al.* (2018) studies the effects of Brownian motion and thermophoresis on unsteady mixed convection flow near the stagnation-point region of a heated vertical plate embedded in a porous medium saturated by a nanofluid. The plate is maintained at a variable wall temperature and nanoparticle volume fraction. The presence of a solid matrix, which exerts first and second resistance parameters, is considered in the study. A suitable coordinate transformation is introduced and the resulting governing equations are transformed and then solved numerically using the local nonsimilarity method and the Runge-Kutta shooting quadrature. The effects of various governing parameters on the flow and heat and mass transfer on the dimensionless velocity, temperature, and nanoparticle volume fraction profiles as well as the skin-friction coefficient, Nusselt number, and the Sherwood number are displayed graphically and discussed to illustrate interesting features of the solutions. The results indicate that as the values of the thermophoresis and Brownian motion parameters increase, the local skin-friction coefficient increases whereas the Nusselt number decreases.

Moreover, the Sherwood number increases as the thermophoresis parameter increases, and decreases as the Brownian motion parameter increases. On the other hand, the unsteadiness parameter and the resistance parameters enhance the local skin-friction coefficient, local Nusselt number, and the local Sherwood number.

The effects of radiation on unsteady free convection flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for design of reliable equipments, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. Based on these applications, Cogley *et al.* (1968) showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Hossain and Takhar (1996) have considered the radiation effects on mixed convection boundary layer flow of an optically dense viscous incompressible fluid along a vertical plate with uniform surface temperature. Makinde (2005) examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate. Satter and Hamid (1996) investigated the unsteady free convection interaction with thermal radiation of an absorbing emitting plate. Rahman and Satter (2007) studied transient convective flow of micropolar fluid past a continuous moving porous plate in the presence of radiation. Heat and mass transfer effects on unsteady magneto hydrodynamics free convection flow near a moving vertical plate embedded in a porous medium was presented by Das and Jana (2010). Olajuwon (2008) examine convection heat and mass transfer in a hydromagnetic flow of a second grade fluid past a semi-infinite stretching sheet in the presence of thermal radiation and thermal diffusion. Haque *et al.* (2011) studied micropolar fluid behavior on steady magneto hydrodynamics free convection flow and mass transfer through a porous medium with heat and mass fluxes. Soret and Dufour effects on mixed convection in a non- Darcy porous medium saturated with micropolar fluid was studied by Srinivasacharya (2011). Rebhi (2007) studied unsteady natural convection heat and mass transfer of micropolar fluid over a vertical surface with constant heat ux. The governing equations were solved numerically using McCormack's technique and the effects of various parameters were investigated on the flow. Eldabe and Ouaf (2006) solved the problem of heat and mass transfer in a hydromagnetic flow of a micropolar uid past a stretching surface with ohmic heating and viscous dissipation using the Chebyshev finite difference method. Keelson and Desseaux (2001) studied the effect of surface conditions on the flow of a micropolar fluid driven by a porous stretching surface. The governing equations were solved numerically. Sunil *et al.* (2006) studied the effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium. The resulting non-linear coupled differential equations from the transformation were solved using finite-difference method. Mahmoud (2007) investigated thermal radiation effect on magneto hydrodynamic flow of a micropolar fluid over a stretching surface with variable thermal conductivity. The solution was obtained numerically by iterative, Runge-Kuta order-four method. Magdy (2005) studied unsteady free convection flow of an incompressible electrically conducting micropolar fluid, bounded by an infinite vertical plane surface of constant temperature with thermal relaxation including heat sources. The governing equations were solved using Laplace transformation. The inversion of the Laplace transforms was carried out with a numerical method. The obtained self-similar equation was solved numerically by an efficient implicit, iterative, infinite-difference method. Reena and Rana (2009) investigated double diffusive convection in a micropolar fluid layer heated and saluted from below saturating a porous medium. A linear stability analysis theory and normal mode analysis method was used. Kandasamy *et al.* (2005) studied the nonlinear MHD flow, with heat and mass transfer characteristics, of an incompressible, viscous, electrically conducting, Boussinesq uid on a vertical stretching surface with chemical reaction and thermal stratification effects. Modather *et al.* (2009) studied MHD heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate in a porous medium. Seddek (2005) studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a at plate in the presence of heat generation/absorption. Patil and Kulkarni (2008) studied the effects of chemical reaction flow of a polar fluid through porous medium in the presence of internal heat generation. Double-diffusive convection radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects was studied by Mohamed (2009).

Chaudhary (2008) studied the effect of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip flow regime. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driven potential are important. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradient as well. The energy caused by a composition gradient is called the Dufour or the diffusion-thermo effect, also the mass fluxes can also be caused by the temperature gradient and this is called the Soret or thermal diffusion effect, the diffusion-thermo effect is neglected in this study because it is of a smaller order of magnitude than the magnitude of thermal radiation which exerts a stronger effect on the energy flux. Thermal diffusion effect or Soret effect has been utilized for isotope separation in mixtures between gases with very light molecular weight and medium molecular weight and it was found to be of a magnitude that it cannot be neglected due to its practical applications in engineering and sciences. For some industrial applications such as glass production and furnace design, and in space technology applications such as cosmic alight aerodynamics rocket and spacecraft re-entry aerothermodynamics which operate at higher temperature, thermal radiation effect can be significant.

Flow of a nanofluid in a boundary layer in an inclined moving sheet at angle Θ is considered analytically by Yusuf *et al.* (2019), the Mathematical formulation consists of the Magnetic parameter, thermophoresis, and Brownian motion. Solutions to momentum, temperature and concentration distribution depends on some parameters. The non linear coupled Differential equations were solved using the improved Adomian decomposition method and a good agreement was established with the numerical method (Shooting technique).

Bhatti *et al.* (2018) Considered a steady three-dimensional incompressible, non rotational flow of a Magnetohydrodynamics fluid, near a stagnation point over a permeable shrinking/stretching sheet coinciding with the plane at z=0, the governing equations for continuity and momentum equation were given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2}-\sigma B_0^2\left(u-u_0\right) \tag{2.2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\frac{\partial^2 v}{\partial z^2}-\sigma B_0^2\left(v-v_0\right) \tag{2.3}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\frac{\partial^2 w}{\partial z^2} \tag{2.4}$$

Subject to the boundary condition:

$$\left.\begin{array}{l} u=v=w=0,\ z \text{ p } 0 \\ u=bx,\ \ v=cx,\ \ w=-S,\ \ z=0 \\ u_0=ax,\ \ v=0,\ \ w=-az,\ \ z\to\infty \end{array}\right\} \tag{2.5}$$

$P=P_0-\frac{1}{2}a^2\rho x^2-\frac{1}{2}\rho w^2+\rho\upsilon\frac{\partial w}{\partial x}$, where P_0 is the stagnation pressure.

The present work extends Bhatti *et al.* (2018) by introducing the rotational parameter, natural convection, heat generation parameter, temperature and concentration equations. Considering the literatures available to us, this innovation is new in the literature.

CHAPTER THREE

3.0 MATERIALS AND METHODS

3.1 Problem Formulation

Considering a steady three-dimensional incompressible rotational flow of a Magnetohydrodynamics fluid, near a stagnation point over a permeable shrinking/stretching sheet coinciding with the plane at z=0. Following the work of Bhatti *et al.* (2018) with T_w and C_w as temperature and concentration at wall and T_∞ and C_∞ far away from the wall respectively, with heat generation Q_o and buoyancy effects. The governing equations for continuity, momentum, temperature and concentration equations can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{3.1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+2\Omega w\right)=-\frac{\partial P}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2}-\sigma B_0^2\left(u-u_0\right)+g\beta\left(T-T_\infty\right) \\ +g\beta\left(C-C\infty\right) \tag{3.2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+2\Omega w\right)=-\frac{\partial P}{\partial y}+\mu\frac{\partial^2 v}{\partial z^2}-\sigma B_0^2\left(v-v_0\right) \tag{3.3}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\frac{\partial^2 w}{\partial z^2} \tag{3.4}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+Q_0\left(T-T_\infty\right) \tag{3.5}$$

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}\right)=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right) \tag{3.6}$$

Subject to the boundary condition:

$$\left.\begin{array}{l} u=v=w=T=C=0,\ z\ \text{p}0 \\ u=bx,\ \ v=cx,\ \ w=-S,\ \ T=T_w,\ \text{C}=\text{C}_w,\ z=0 \\ u_0=ax,\ \ v=0,\ \ w=-az,\ \ T=T_\infty,\text{C}=\text{C}_\infty\quad z\rightarrow\infty \end{array}\right\} \tag{3.7}$$

$P=P_0-\frac{1}{2}a^2\rho x^2-\frac{1}{2}\rho w^2+\rho\upsilon\frac{\partial w}{\partial x}$, where P_0 is the stagnation pressure.

The similarity variables are defined as follows:

$$\eta=\sqrt{\frac{a}{\upsilon}}z,\ \ u=axf^{/}\left(\eta\right),\ \ v=cxh\left(\eta\right),\ \ w=-\sqrt{a\upsilon}\,f\left(\eta\right),\ \ \theta=\frac{T-T_\infty}{T_w-T_\infty},\phi=\frac{C-C_\infty}{C_w-C_\infty}$$
(3.8)

$$
\left.
\begin{array}{l}
\frac{\partial P}{\partial x} = -a^2 \rho x \ , \quad \frac{\partial u}{\partial x} = af^{/}, \ \frac{\partial u}{\partial y} = 0, \quad \frac{\partial w}{\partial z} = \frac{\partial w}{\partial f}\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial z} = -\sqrt{a\upsilon}\sqrt{\frac{a}{\upsilon}} f^{/} = -af^{/} \\
\frac{\partial u}{\partial y} = 0, \quad \frac{\partial u}{\partial z} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial z} = axf^{//}\sqrt{\frac{a}{\upsilon}}, \quad \frac{\partial v}{\partial x} = ch(\eta), \quad \frac{\partial v}{\partial y} = 0, \frac{\partial v}{\partial z} = \frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial z} = cx\sqrt{\frac{a}{\upsilon}} h^{/} \\
\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(axf^{//}\sqrt{\frac{a}{\upsilon}} \right) = \frac{\partial \eta}{\partial z}\frac{\partial}{\partial \eta}\left(axf^{//}\sqrt{\frac{a}{\upsilon}} \right) = \frac{a^2 x}{\upsilon} f^{///}, \\
\frac{\partial^2 v}{\partial z^2} = \frac{\partial}{\partial z}\left(cx\sqrt{\frac{a}{\upsilon}} h^{/} \right) = \frac{\partial \eta}{\partial z}\frac{\partial}{\partial \eta}\left(cx\sqrt{\frac{a}{\upsilon}} h^{/} \right) = \sqrt{\frac{a}{\upsilon}} cx\sqrt{\frac{a}{\upsilon}} h^{//}, \ u\frac{\partial u}{\partial x} = a^2 xf^{/2} \\
w\frac{\partial u}{\partial z} = -a^2 xff^{//}, \ u\frac{\partial v}{\partial x} = acxf^{/} h(\eta), \ w\frac{\partial v}{\partial z} = -acxfh^{/}, \ T = T_\infty + (T_W - T_\infty)\theta(\eta) \\
\frac{\partial T}{\partial x} = 0, \quad \frac{\partial T}{\partial y} = 0, \ \frac{\partial T}{\partial z} = (T_W - T_\infty)\frac{\partial \eta}{\partial z}\frac{\partial}{\partial \eta}\theta(\eta) = (T_W - T_\infty)\sqrt{\frac{a}{\upsilon}}\theta^{/} \\
\frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z}\left((T_W - T_\infty)\sqrt{\frac{a}{\upsilon}}\theta^{/} \right) = (T_W - T_\infty)\frac{a}{\upsilon}\theta^{//}, \mathrm{C} = C_\infty + (C_W - C_\infty)\phi(\eta) \\
\frac{\partial C}{\partial x} = 0, \quad \frac{\partial C}{\partial y} = 0, \ \frac{\partial C}{\partial z} = (C_W - C_\infty)\frac{\partial \eta}{\partial z}\frac{\partial}{\partial \eta}\phi(\eta) = (C_W - C_\infty)\sqrt{\frac{a}{\upsilon}}\phi^{/} \\
\frac{\partial^2 C}{\partial z^2} = \frac{\partial}{\partial z}\left((C_W - C_\infty)\sqrt{\frac{a}{\upsilon}}\phi^{/} \right) = (C_W - C_\infty)\frac{a}{\upsilon}\phi^{//},
\end{array}
\right\} \quad (3.9)
$$

Introducing equation (3.9) into (3.1) to (3.7), the equations reduces to

$$
\left.
\begin{array}{l}
f^{///} + ff^{//} - f^{/2} + 2K_r f + 1 - M\left(f^{/} - 1 \right) + G_{rT}\theta + G_{rC}\phi = 0 \\
h^{//} - f^{/} h + fh^{/} - Mh + 2K_r f = 0 \\
\theta^{//} + P_r f\theta^{/} + P_r Q_0 \theta = 0 \\
\phi^{//} + Scf\phi^{/} = 0
\end{array}
\right\} \quad (3.10)
$$

With the corresponding boundary condition:

$$\left.\begin{array}{l} f(0)=k,\ f'(0)=\alpha,\ f'(\infty)=1 \\ h(0)=1,\ h(\infty)=0 \\ \theta(0)=1,\ \theta(\infty)=0 \\ \phi(0)=1,\ \phi(\infty)=0 \end{array}\right\} \tag{3.11}$$

in which : $K_r=\dfrac{\Omega\sqrt{a\upsilon}}{a^2x}$, $G_{rT}=\dfrac{g\beta(T_w-T_\infty)}{a^2x\rho}$, $G_{rC}=\dfrac{g\beta(C_w-C_\infty)}{a^2x\rho}$ $M=\dfrac{\sigma B_0^2}{\rho ax}$, $P_r=\dfrac{\upsilon}{\alpha}$, $Q_0=aQ$, $S_c=\dfrac{\upsilon}{D_B}$, $\alpha=\dfrac{b}{a}$ $k=\dfrac{S}{\sqrt{a\nu}}$ are the Rotational parameter, Thermal Grashof number, Concentration Grashof number, Magnetic parameter, Prandtl number, Heat generation, Schimidt number, shrinking/stretching parameter, and Suction/injection parameter respectively.

3.2 Implementation of Adomian Decomposition Method

Begin with an equation $Fu(t)=g(t)$, where F represents a general nonlinear ordinary differential operator involving both linear and nonlinear terms. The linear term is decomposed into $L+R$, where L is easily invertible and R is the remainder of the linear operator. For convenience, L may be taken as the highest order derivative which avoids difficult integrations which result when complicated Green's functions are involved (Adomian, 1994). Thus the equation may be written

$$Lu+Ru+Nu=g \tag{3.12}$$

where Nu represents the nonlinear terms. Solving for Lu,

$$Lu=g-Ru-Nu \tag{3.13}$$

Because L is invertible, an equivalent expression is

$$L^{-1}Lu = L^{-1}g + L^{-1}Ru - L^{-1}Nu \tag{3.14}$$

If this corresponds to an initial-value problem, the integral operator L^{-1} may be regarded as definite integrals from t_0 to t. If L is a second-order operator, L^{-1} is a twofold integration operator and $L^{-1}Lu = u - u(t_0) - (t - t_0)u'(t_0)$. For boundary value problems (and, if desired, for initial-value problems as well), indefinite integrations are used and the constants are evaluated from the given conditions. Solving (3.73) for u yields

$$u = A + Bt + L^{-1}g + L^{-1}Ru - L^{-1}Nu \tag{3.15}$$

The nonlinear term Nu will be equated to $\sum_{n=0}^{\infty} A_n$, where the A_n, are special polynomials to be discussed, and u will be decomposed into $\sum_{n=0}^{\infty} u_n$, with u_0 identified as $A + Bt + L^{-1}g$

$$\sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \tag{3.16}$$

consequently, we can write

$$\left.\begin{aligned} u_1 &= -L^{-1}Ru_0 - L^{-1}A_0 \\ u_2 &= -L^{-1}Ru_1 - L^{-1}A_0 \\ &. \\ &. \\ &. \\ u_{n+1} &= -L^{-1}Ru_n - L^{-1}A_n \end{aligned}\right\} \tag{3.17}$$

The polynomials A_n, are generated for each nonlinearity so that A_0, depends only on u_0, A_1, depends only on u_0 ,and u_1, A_2, depends on u_0, u_1, u_2, etc. All of the u_n, components are calculable, and $u=\sum_{n=0}^{\infty}u_n$. If the series converges, the n-term partial sum $\phi_n=\sum_{i=0}^{n-1}u_i$ will be the approximate solution since $\lim_{n\to\infty}\phi_n=\sum_{i=0}^{\infty}u_i=u$ by definition. It is important to emphasize that the A_n can be calculated for complicated nonlinearities of the form $f\left(u,\ u',...\right)$ or $f\left(g\left(u\right)\right)$.

We start by introducing the improved Adomian decomposition method to get the solution by letting $\frac{d^3}{d\eta^3}=L_1$ and $\frac{d^2}{d\eta^2}=L_2$ and from equation (3.10) we have

$$\left.\begin{aligned} f''' &= -ff''+f'^2-2K_r f-1+M\left(f'-1\right)-G_{rT}\theta-G_{rC}\phi \\ h'' &= f'h-fh'+Mh-2K_r f=0 \\ \theta'' &= -P_r f\theta'-P_rQ_0\theta \\ \phi'' &= -Scf\phi' \end{aligned}\right\} \tag{3.18}$$

Introducing the operators into equations (3.10) we have

Type equation here.

$$\left.\begin{aligned} L_1L_1^{-1}\left[f\left(\eta\right)\right] &= L_1^{-1}\left[-ff''+f'^2-1-2K_r+M\left(f'-1\right)-G_{rT}\theta-G_{rC}\phi\right] \\ L_2L_2^{-1}\left[h\left(\eta\right)\right] &= L_2^{-1}\left[f'h-fh'+Mh-2K_r\right] \\ L_2L_2^{-1}\left[\theta\left(\eta\right)\right] &= L_2^{-1}\left[-P_r f\theta'-P_rQ_0\theta\right] \\ L_2L_2^{-1}\left[\phi\left(\eta\right)\right] &= L_2^{-1}\left[-S_c f\phi'\right] \end{aligned}\right\} \tag{3.19}$$

Where $L_1^{-1}=\int\int\int\left(\bullet\right)d\eta d\eta d\eta$ and $L_2^{-1}=\int\int\left(\bullet\right)d\eta d\eta$

(3.20)

Introducing the Adomian polynomials into (3.19) we have

$$\left.\begin{aligned}&\sum_{n=0}^{\infty} f_n = -L_1^{-1}\sum_{n=0}^{\infty} A_n + L_1^{-1}\sum_{n=0}^{\infty} B_n - 2K_r L_1^{-1} f_n + ML_1^{-1}\left(f_n^{/} - 1\right)\\&-L_1^{-1}G_{rT}\theta_n - L_1^{-1}G_{rC}\phi_n\\&\sum_{n=0}^{\infty} h_n = L_2^{-1}\sum_{n=0}^{\infty} C_n - L_2^{-1}\sum_{n=0}^{\infty} D_n - 2K_r L_2^{-1} f_n - ML_2^{-1} f_n\\&\sum_{n=0}^{\infty}\theta_n = -P_r L_2^{-1}\sum_{n=0}^{\infty} E_n - P_r Q_0 L_2^{-1}\theta_n\\&\sum_{n=0}^{\infty}\phi_n = -S_c L_2^{-1}\sum_{n=0}^{\infty} F_n\end{aligned}\right\}$$

(3.21)

Where $A_n = f_n f_{n-k}^{//},\ \mathrm{B}_n = f_n^{/} f_{n-k}^{/},\ \mathrm{C}_n = f_n^{/} h_{n-k},\ \mathrm{D}_n = f_n h_{n-k}^{/},$
$E_n = f_n \theta_{n-k}^{/},\ \mathrm{F}_n = f_n \phi_{n-k}^{/}$

$$\left.\begin{aligned}&f_{n+1} = -L_1^{-1}\sum_{k=0}^{n} f_k f_{n-k}^{//} + L_1^{-1}\sum_{k=0}^{n} f_k^{/} f_{n-k}^{/} - 2K_r L_1^{-1} f_n + ML_1^{-1}\left(f_n^{/} - 1\right)\\&-L_1^{-1}G_{rT}\theta_n - L_1^{-1}G_{rC}\phi_n\\&h_{n+1} = L_2^{-1}\sum_{k=0}^{n} f_k^{/} h_{n-k} - L_2^{-1}\sum_{k=0}^{n} f_k h_{n-k}^{/} - 2K_r L_2^{-1} f_n - ML_2^{-1} f_n\\&\theta_{n+1} = -P_r L_2^{-1}\sum_{k=0}^{n} f_k \theta_{n-k}^{/} - P_r Q_0 L_2^{-1}\theta_n\\&\phi_n = -S_c L_2^{-1}\sum_{n=0}^{k} f_k \phi_{n-k}^{/}\end{aligned}\right\} \quad (3.22)$$

Where
$$\left.\begin{aligned}&f_0(\eta) = k + \frac{\eta^2\alpha_1}{2} + \alpha\eta\\&h_0(\eta) = 1 + \eta\alpha_2\\&\theta_0(\eta) = 1 + \eta\alpha_3\\&\phi_0(\eta) = 1 + \eta\alpha_4\end{aligned}\right\} \quad (3.23)$$

are the starting points.

Using maple18 to evaluate the integrals we have the final solutions as:

$$\left.\begin{aligned} f(\eta) &= \sum_{n=1}^{4} f_n(\eta) \\ h(\eta) &= \sum_{n=1}^{4} h_n(\eta) \\ \theta(\eta) &= \sum_{n=1}^{4} \theta_n(\eta) \\ \phi(\eta) &= \sum_{n=1}^{4} \phi_n(\eta) \end{aligned}\right\} \tag{3.24}$$

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Results

In this chapter, The table, showing the comparison of Adomian Decomposition Method and the results obtained by Bhatti et al. (2018) are presented and the variation of each dimensionless property that appear such as thermal Grashof number,concentration Grashof number,Schimidt number, Prandtl number and Magnetic parameter (*M*) are also presented graphically with the aid of Maple 18.

4.2 Presentation of Tables of Comparison Between the Present Work and the Literature

Table 4.1: Comparison of Results between Numerical and Analytical for $\alpha \geq 0$ with $M = S = G_{rC} = G_{rT} = 0$

$\alpha \geq 0$	$f''(0)$ Wang (2008)	$f''(0)$ Bhatti *et al.* (2018)	$f''(0)$ Present Results	$-h'(0)$ Wang (2008)	$-h'(0)$ Bhatti *et al.* (2018)	$-h'(0)$ Present Results
0	1.232588	1.23258777	1.23276	0.8113	0.8113013	0.812251
0.1	1.14656	1.146561	1.14678	0.86345	0.8634517	0.863281
0.2	1.05113	1.05112999	1.05162	0.9133	0.9133028	0.910313
0.3	-	0.94681611	0.94786	-	0.9611159	0.952974
0.5	0.7133	0.71329495	0.71666	1.05239	1.0514584	1.025327
1	0	0	0	1.25331	1.2533141	1.202451
2	-1.88731	-1.8873067	-1.9861	1.58957	1.5895668	1.07536
5	-10.2647	-10.264749	-9.3979	2.3381	2.338099	1.965189

Table 4.2: Comparison of results between Numerical and Analytical for α p 0 with $M = S = G_{rC} = G_{rT} = 0$

α f 0	$f^{//}(0)$ Wang (2008)	$f^{//}(0)$ Bhatti et al (2018)	$f^{//}(0)$ Present Results	$-h^{/}(0)$ Wang (2008)	$-h^{/}(0)$ Bhatti et al (2018)	$-h^{/}(0)$ Present Results
-0.25	1.40224	1.402241	1.40278091	0.66857	0.66857	0.6697137
-0.5	1.49567	1.495668	1.49643034	0.50145	0.50145	0.5044947
-0.75	1.4893	1.489298	1.49156502	0.29376	0.29376	0.2977485
-1	1.32882	1.328817	1.33300894	0	0	0
-1.15	1.08223	1.082231	1.07144745	-0.298	-0.298	-0.258003
-1.2465	0.5543	0.584282	0.69677895	-0.999	-0.9478	-0.558264

Tables 4.1 and 4.2 shows present the comparison of the present work and previous works published in the literature shrinking and stretching sheet respectively. A good agreement is observe between the present method and the previous ones as seen on the tables.

4.3 Presentation of Results on the Shrinking Sheet

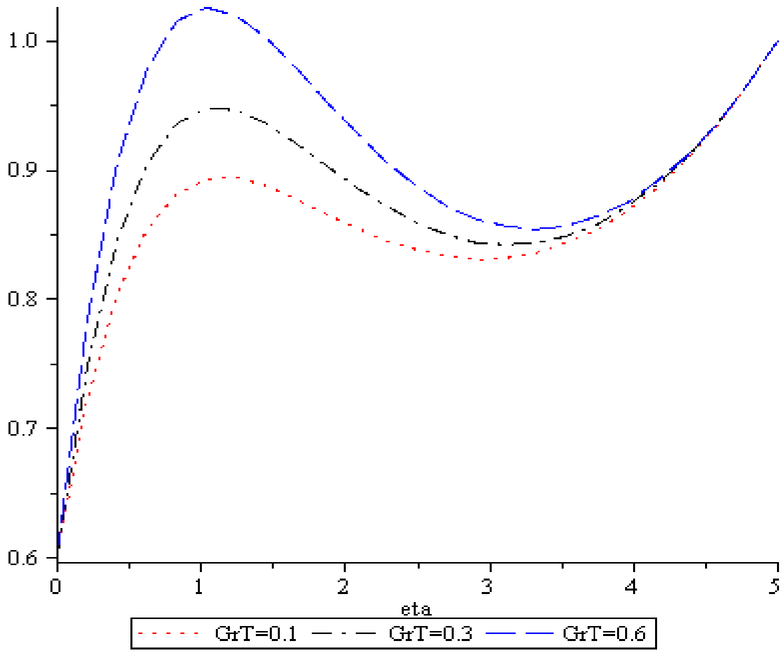

Figure 4.1: Variation of thermal Grashof number on velocity f on a stretching sheet

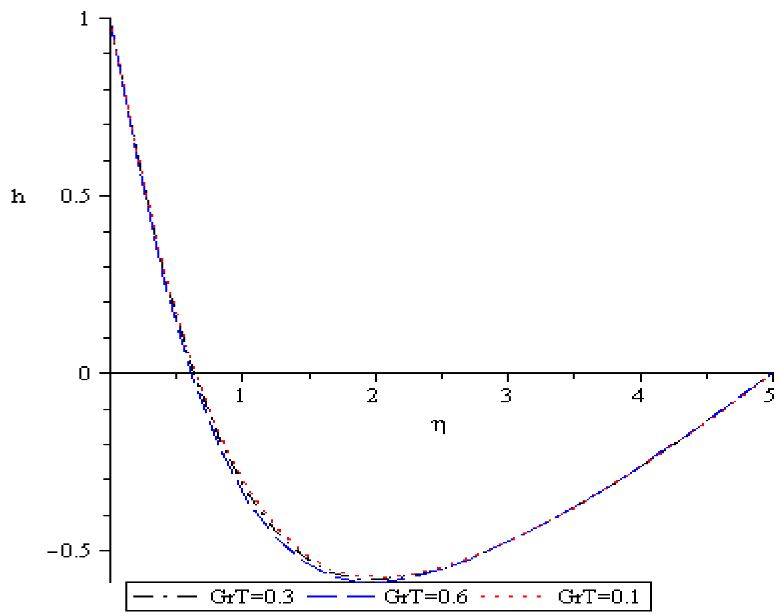

Figure 4.2: Variation of thermal Grashof number on velocity h on a stretching sheet

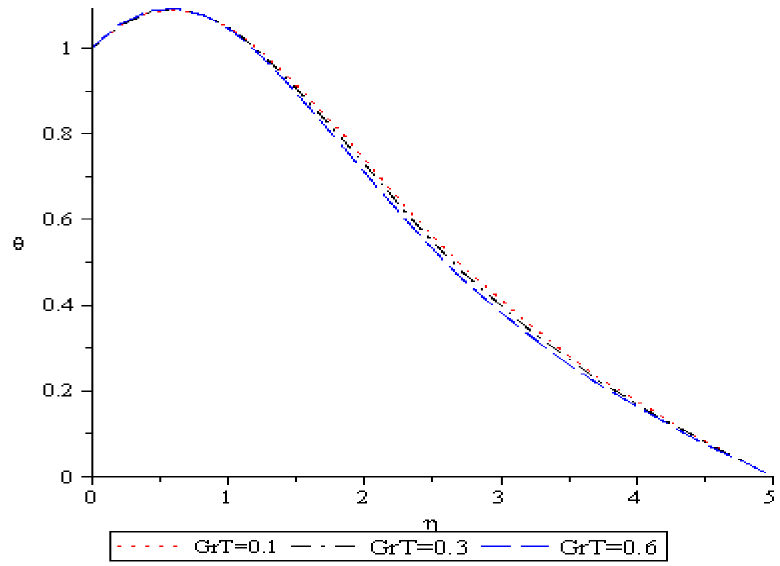

Figure 4.3: Variation of thermal Grashof number on temperature on a stretching sheet

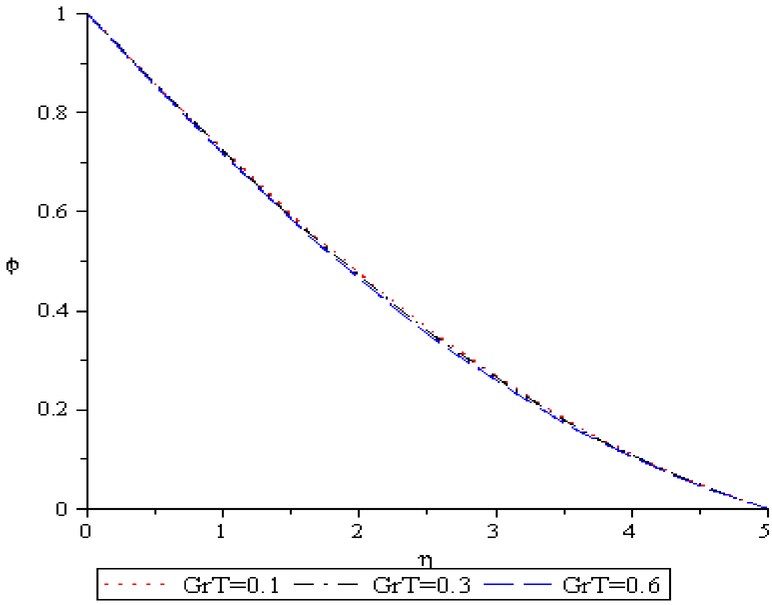

Figure 4.4: Variation of thermal Grashof number on concentration on a stretching sheet

Figures 4.1 to 4.4 display the variation of thermal Grashof number on velocities, temperature, and concentration. It is observe that the thermal Grashof number enhances the velocity *f* due to the presence of buoyancy while the velocity *h*, temperature and concentration dropped.

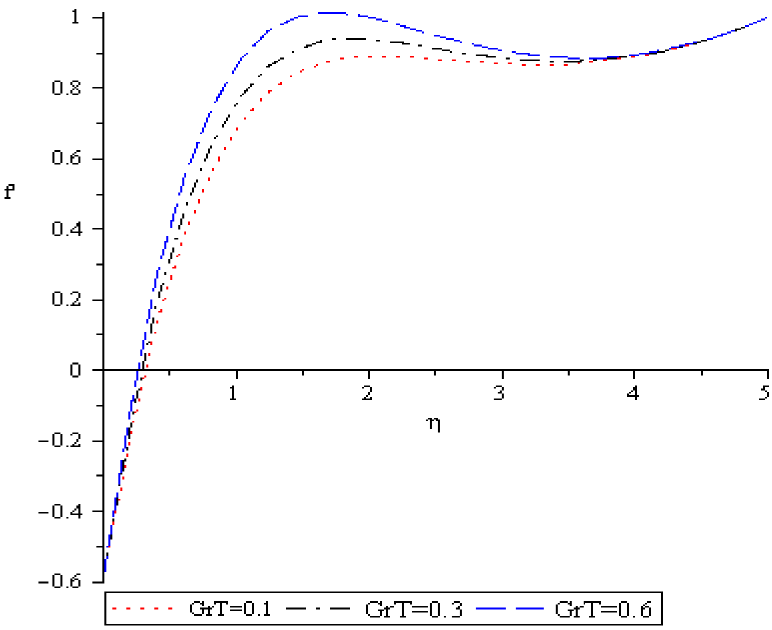

Figure 4.5: Variation of thermal Grashof number on velocity *f* on a shrinking sheet

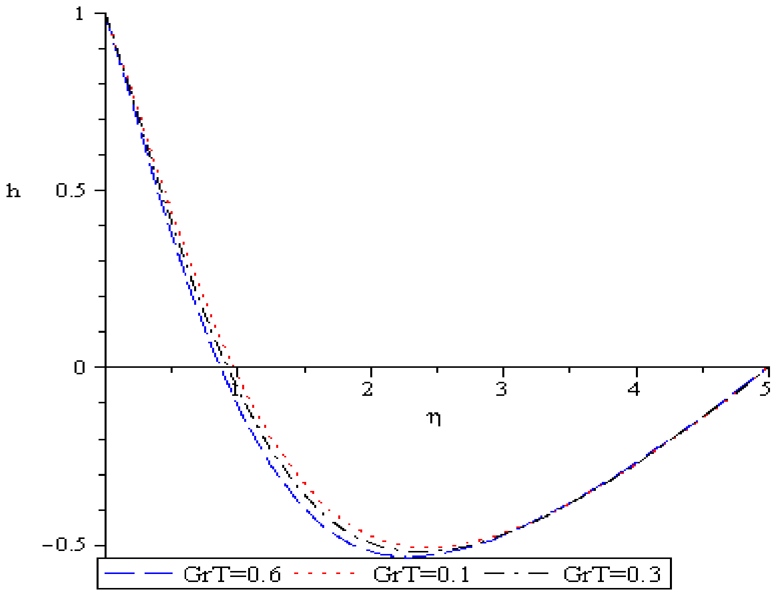

Figure 4.6: Variation of thermal Grashof number on velocity *h* on a shrinking sheet

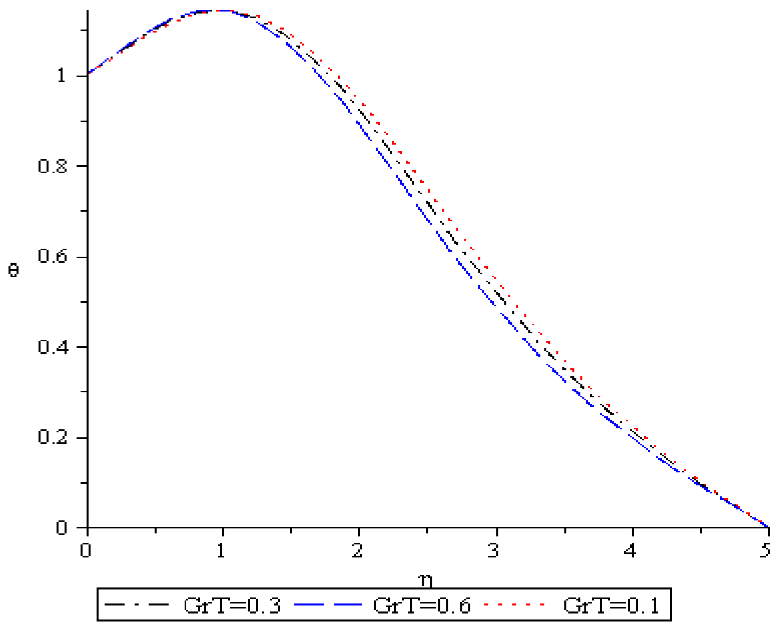

Figure 4.7: Variation of thermal Grashof number on temperature on a shrinking sheet

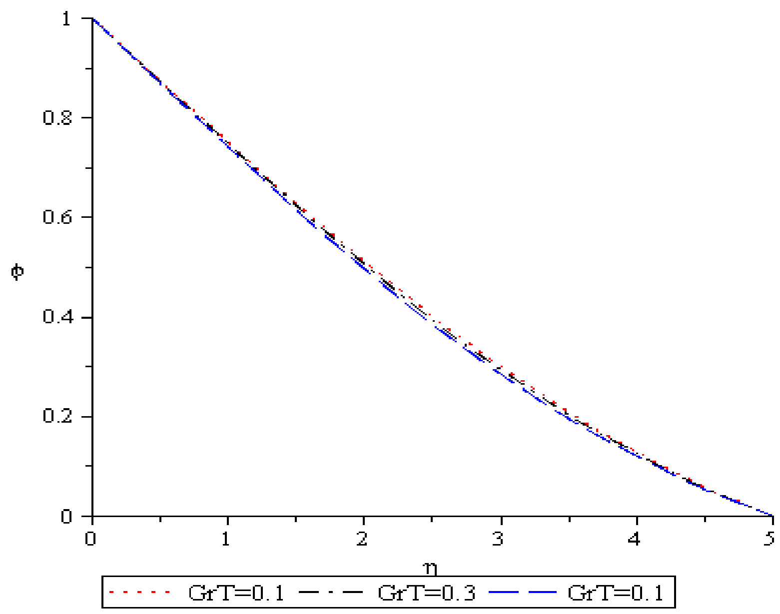

Figure 4.8: Variation of thermal Grashof number on concentration on a shrinking sheet

Figures 4.5 to 4.8 shows the effects of thermal Grashof number on velocities, temperature, and concentration on the shrinking sheet. It is observe that as the the thermal Grashof number increases the velocity profile *f* thickens due to the presence of buoyancy while the velocity *h*, temperature and concentration dropped on the shrinking sheet.

Generally, it is observe that velocity profile *f* on the stretching sheet is thicker than on the shrinking sheet while the velocity profile *h*, temperature and concentration profiles are thicker on the shrinking sheet.

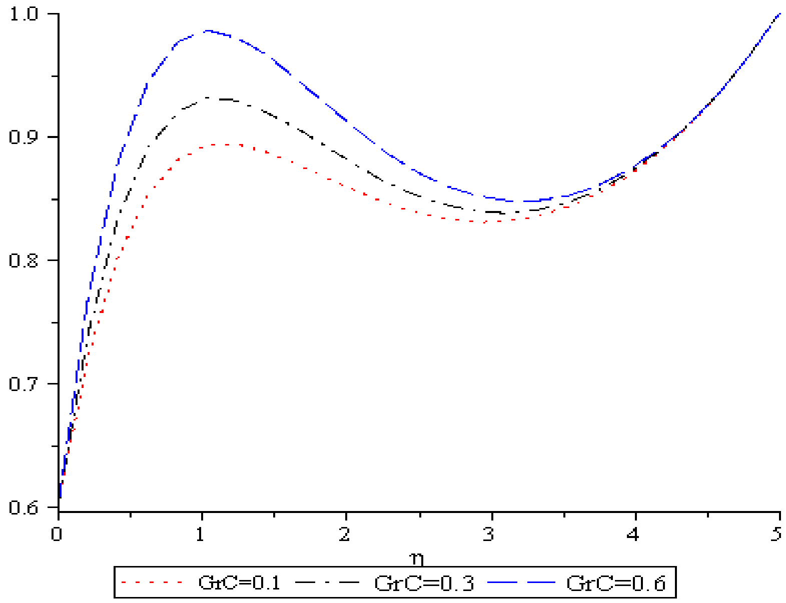

Figure 4.9: Variation of concentration Grashof number on velocity *f* on a stretching sheet

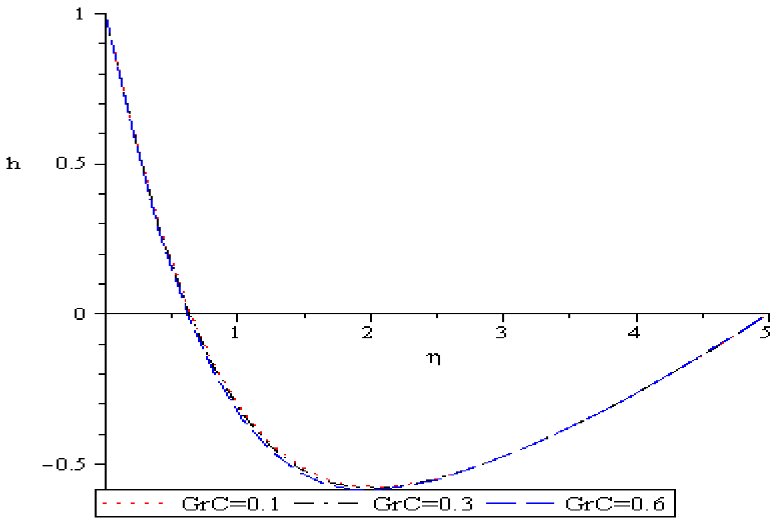

Figure 4.10: Variation of concentration Grashof number on velocity *h* on a stretching sheet

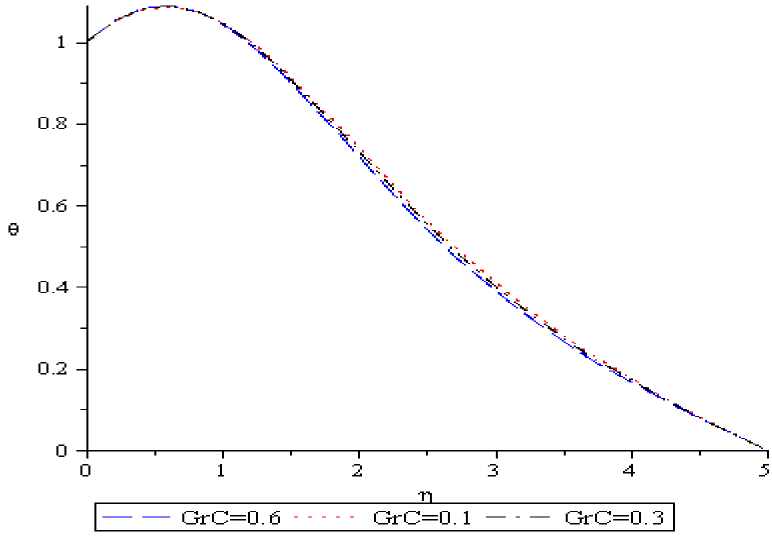

Figure 4.11: Variation of concentration Grashof number on temperature on a stretching sheet

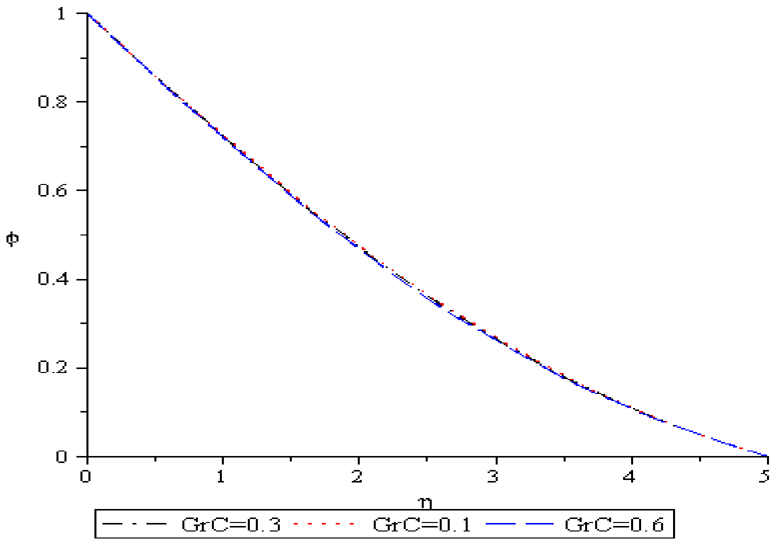

Figure 4.12: Variation of concentration Grashof number on concentration on a stretching sheet

Figures 4.9 to 4.12 present the variation of concentration Grashof number on velocities, temperature, and concentration on stretching sheet. It is observe that the concentration Grashof number increases the velocity *f* due to the presence of buoyancy while the velocity *h*, temperature and concentration dropped on the stretching sheet.

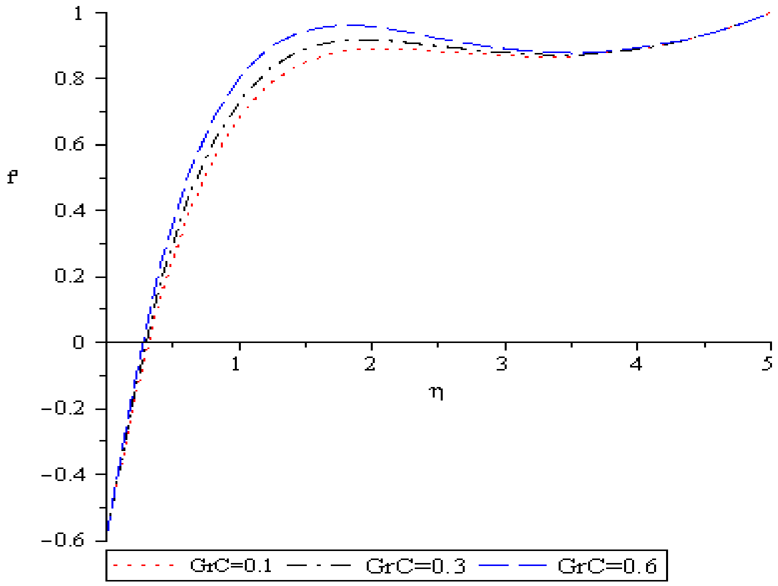

Figure 4.13: Variation of concentration Grashof number on velocity *f* on a shrinking sheet

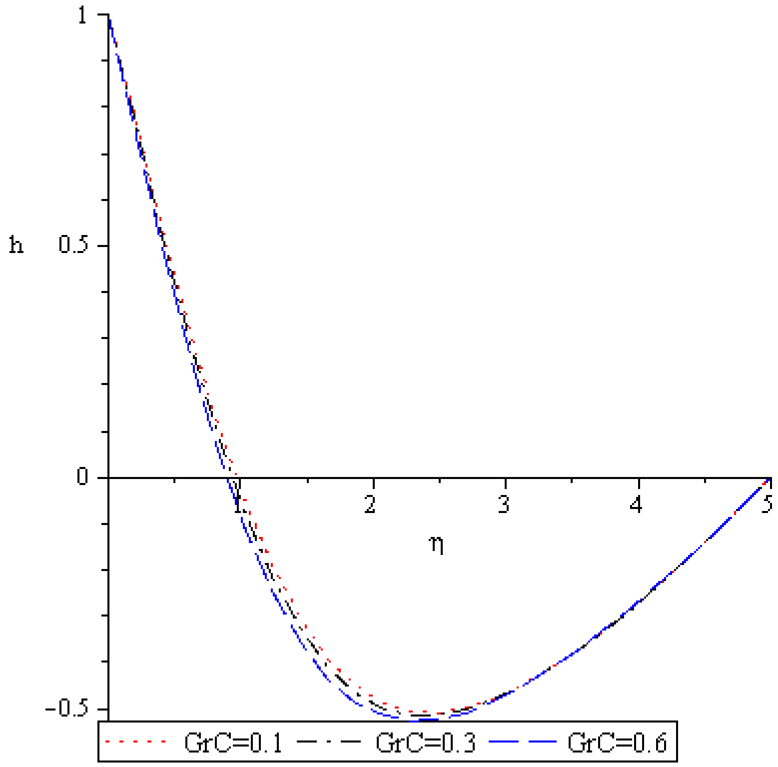

Figure 4.14: Variation of concentration Grashof number on velocity *h* on a shrinking sheet

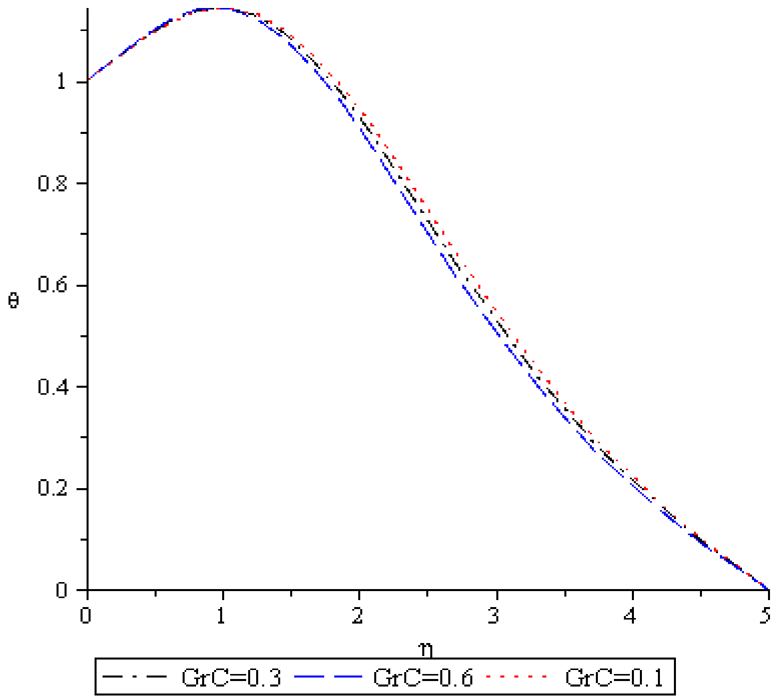

Figure 4.15: Variation of concentration Grashof number on temperature on a shrinking sheet

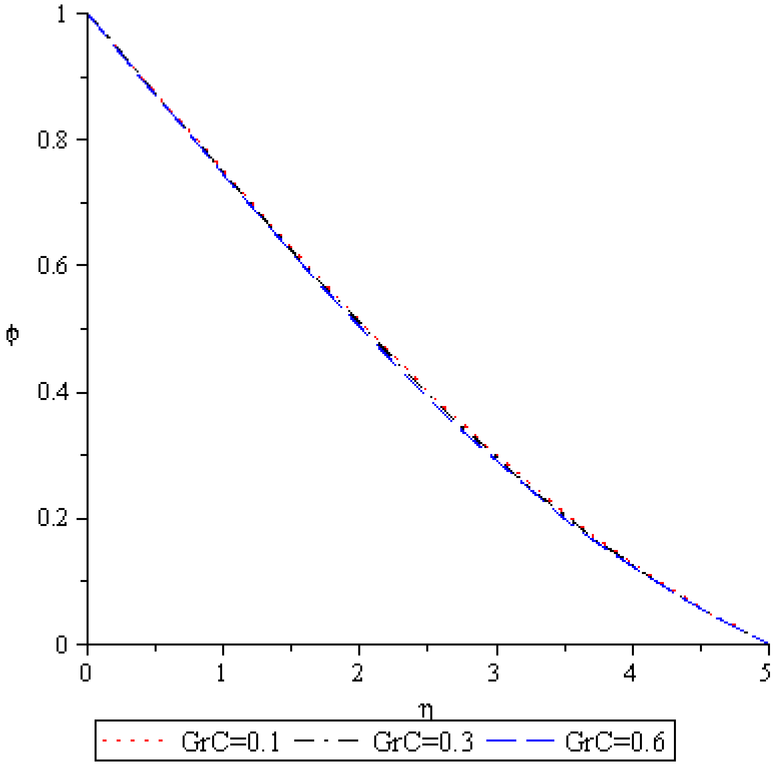

Figure 4.16: Variation of concentration Grashof number on concentration on a shrinking sheet

Figures 4.13 to 4.8 displays the effects of concentration Grashof number on velocities, temperature, and concentration on the shrinking sheet. It is observe that as the concentration Grashof number increases the velocity profile *f* thickens due to the presence of buoyancy while the velocity *h*, temperature and concentration dropped on the shrinking sheet.

Generally, it is observe that velocity profile *f* on the stretching sheet is thicker than on the shrinking sheet while the velocity profile *h*, temperature and concentration profiles are thicker on the shrinking sheet than the stretching sheet.

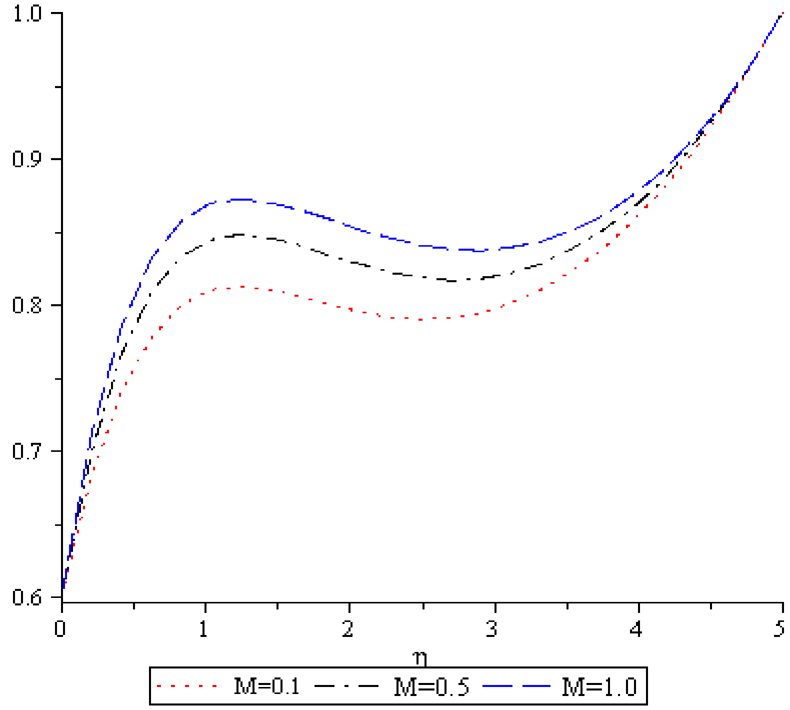

Figure 4.17: Variation of magnetic parameter on velocity *f* on a stretching sheet

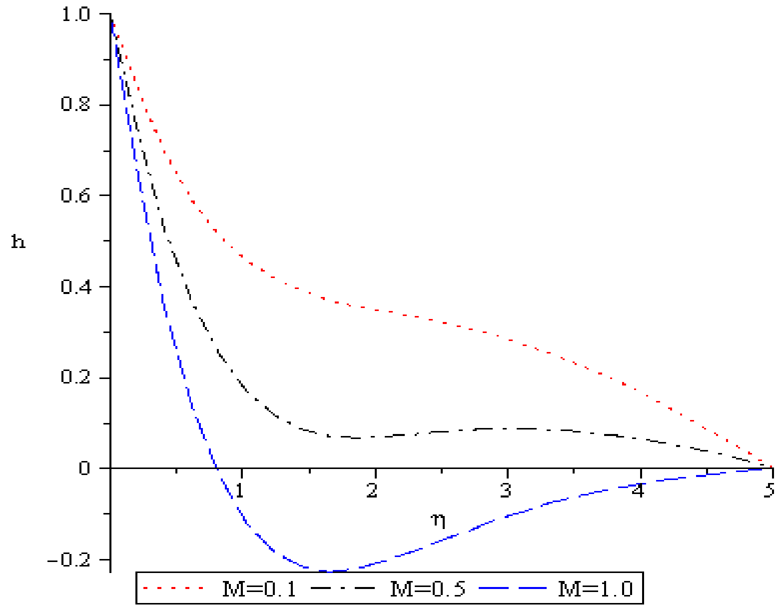

Figure 4.18: Variation of magnetic parameter on velocity *h* on a stretching sheet

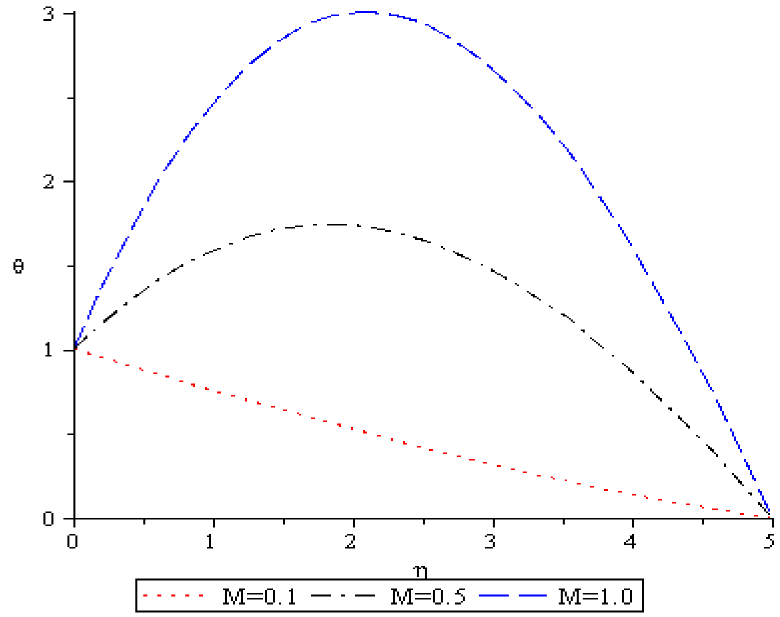

Figure 4.19: Variation of magnetic parameter on temperature on a stretching sheet

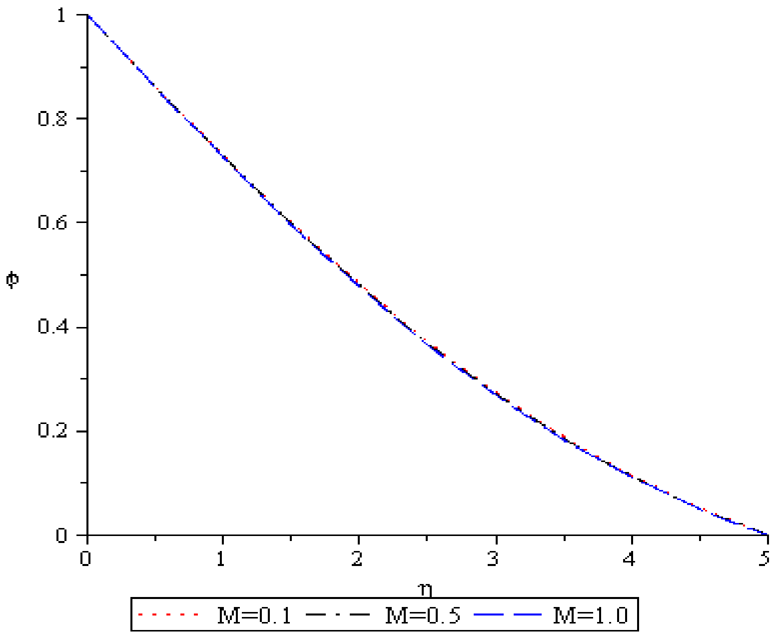

Figure 4.20: Variation of magnetic parameter on concentration on a stretching sheet

Figures 4.17 to 4.20 present the variation of magnetic parameter on velocities, temperature, and concentration on stretching sheet. It is observe that the magnetic parameter increases the velocity *f* and temperature profile while the velocity *h* reduces due to drag like force and concentration dropped on the stretching sheet.

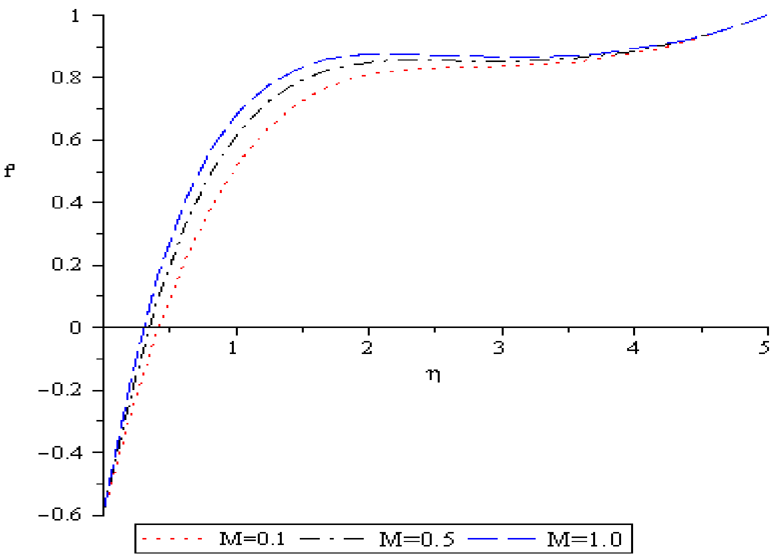

Figure 4.21: Variation of magnetic parameter on velocity *f* on a shrinking sheet

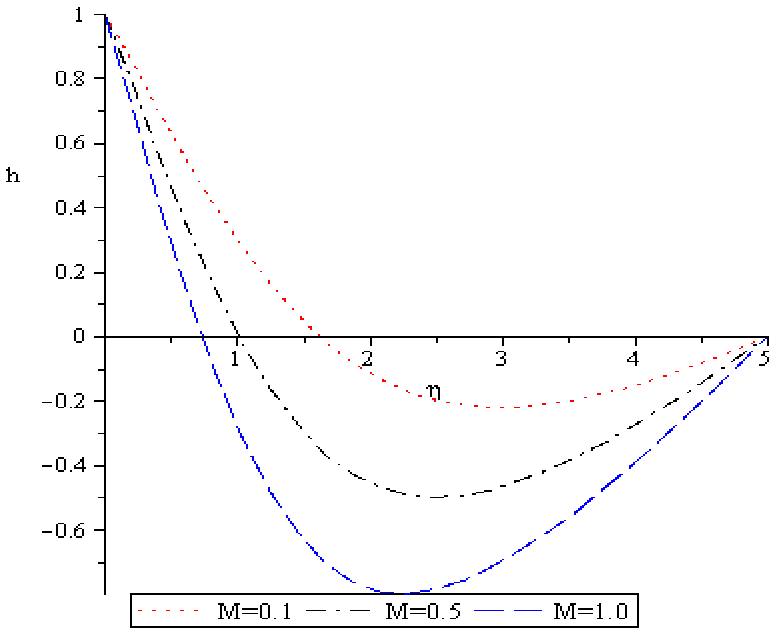

Figure 4.22: Variation of magnetic parameter on velocity *h* on a shrinking sheet

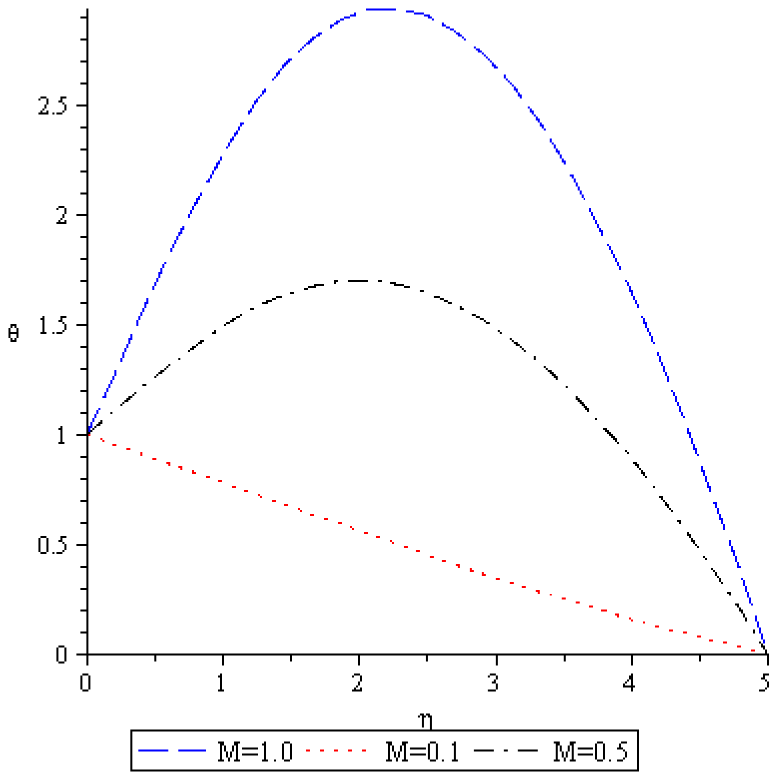

Figure 4.23: Variation of magnetic parameter on temperature on a shrinking sheet

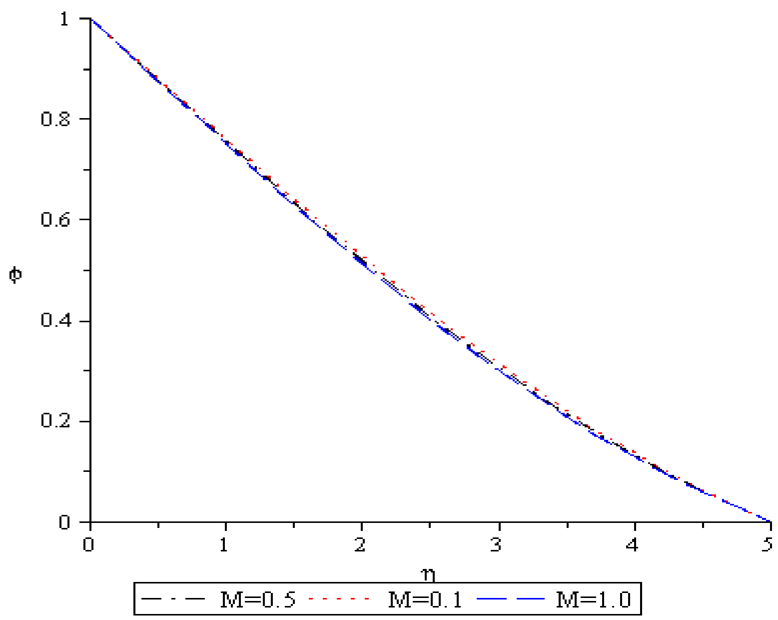

Figure 4.24: Variation of magnetic parameter on concentration on a shrinking sheet

Figures 4.21 to 4.24 display the effects of magnetic parameter on velocities, temperature, and concentration on the shrinking sheet. It is observe that as the magnetic parameter increases the velocity profile *f* and temperature thickens while the velocity *h* and concentration dropped on the shrinking sheet.

Generally, it is observe that velocity profiles and temperature profile on the stretching sheet has a thicker boundary layer than on the shrinking sheet and concentration boundary layer are thicker on the shrinking sheet than the stretching sheet.

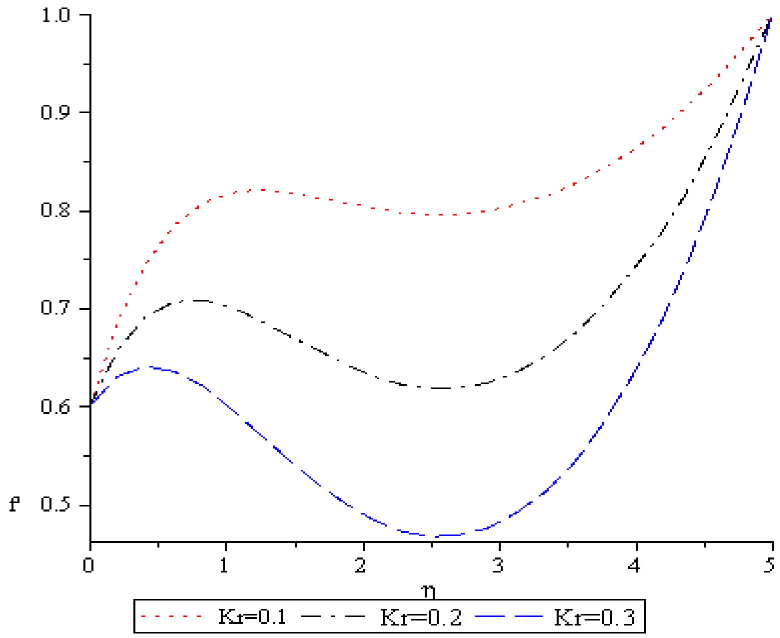

Figure 4.25: Variation of rotational parameter on velocity *f* on a stretching sheet

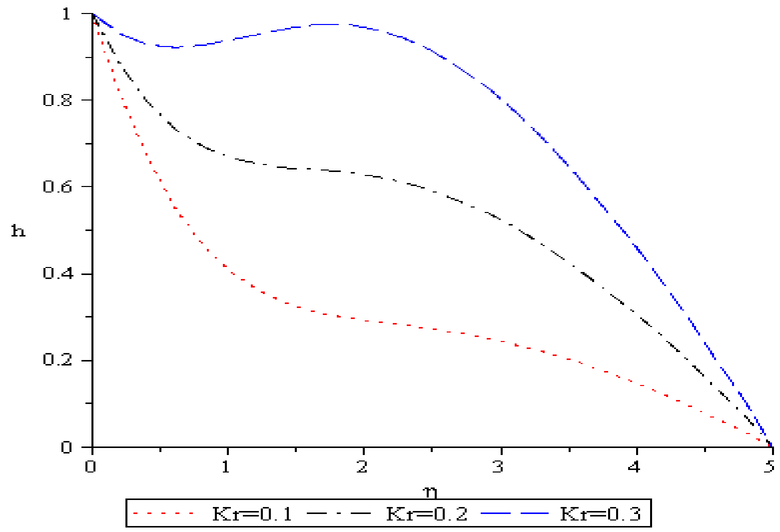

Figure 4.26: Variation of rotational parameter on velocity *h* on a stretching sheet

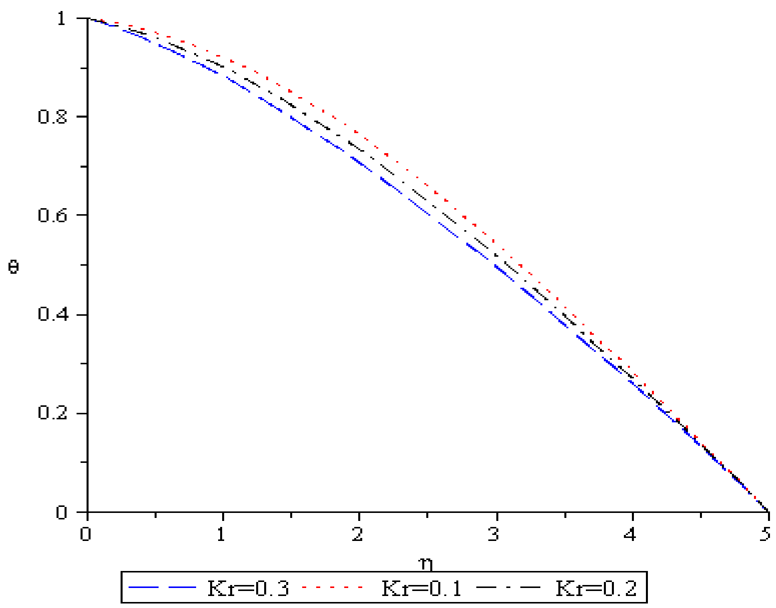

Figure 4.27: Variation of rotational parameter on temperature on a stretching sheet

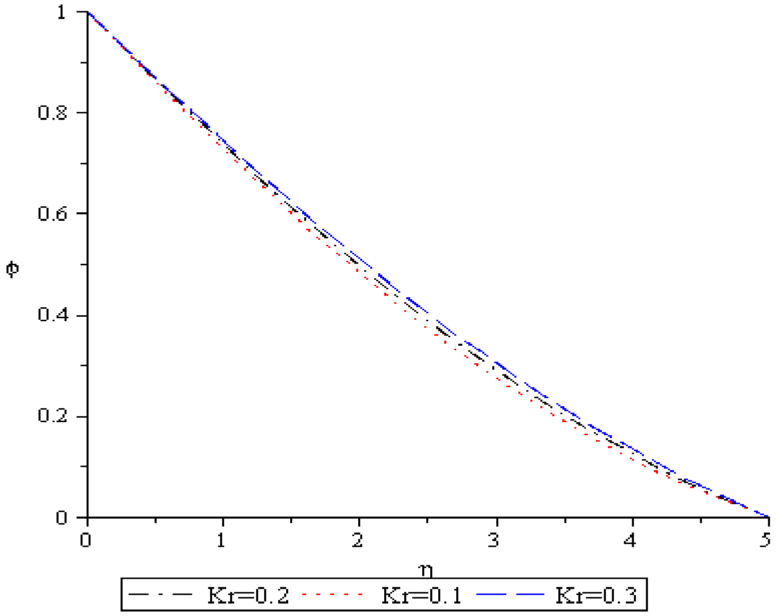

Figure 4.28: Variation of rotational parameter on concentration on a stretching sheet

Figures 4.25 to 4.28 present the variation of rotational parameter on velocities, temperature, and concentration on stretching sheet. It is observe that the rotational parameter increases the velocity *h*, and concentration profiles while the velocity *f* and temperature reduces as the fluid rotation increases stretching sheet.

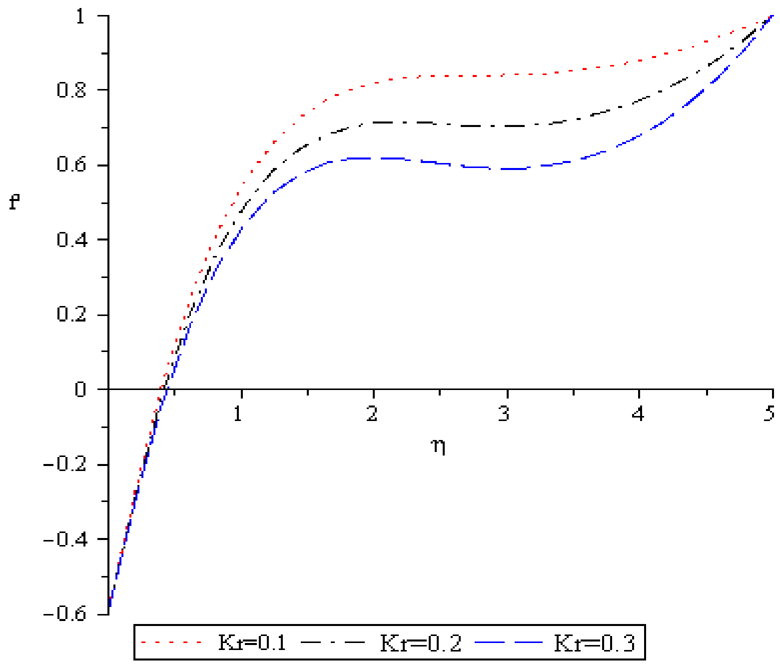

Figure 4.29: Variation of rotational parameter on velocity *f* on a shrinking sheet

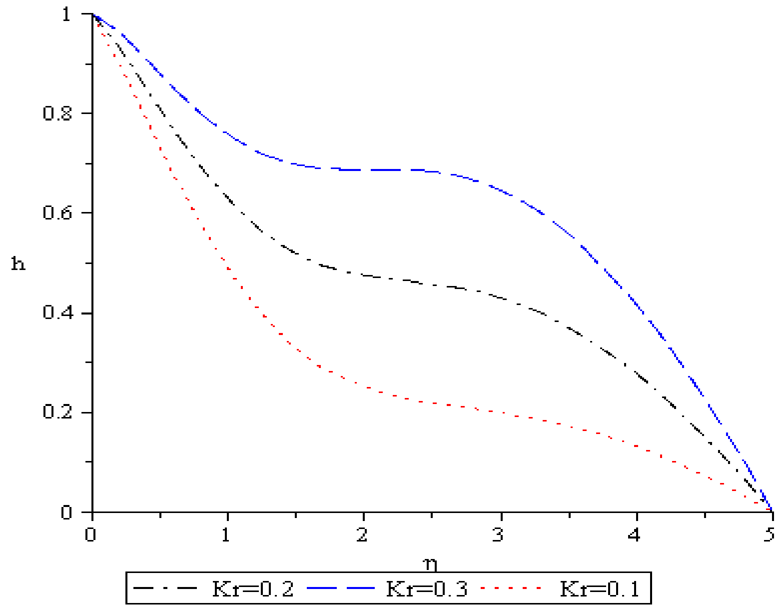

Figure 4.30: Variation of rotational parameter on velocity *h* on a shrinking sheet

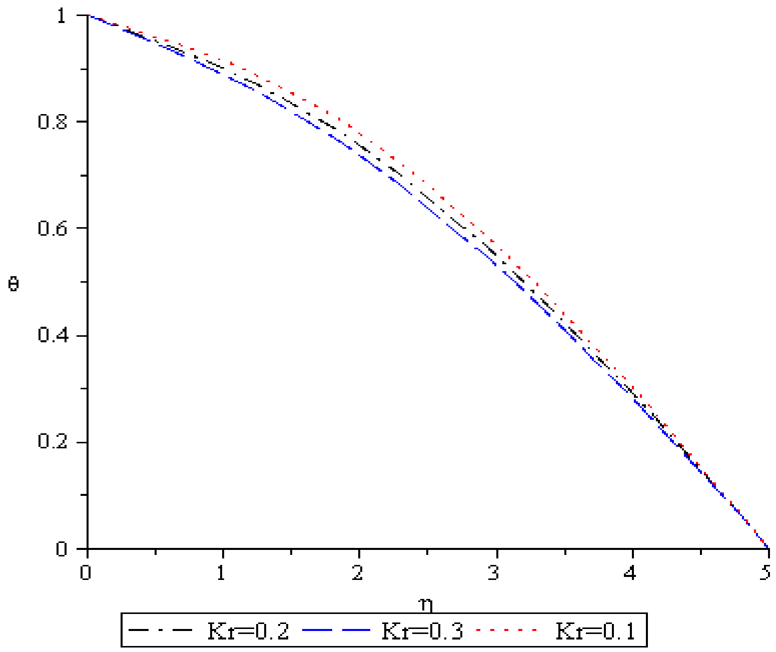

Figure 4.31: VariatioJHn of rotational parameter on temperature on a shrinking sheet

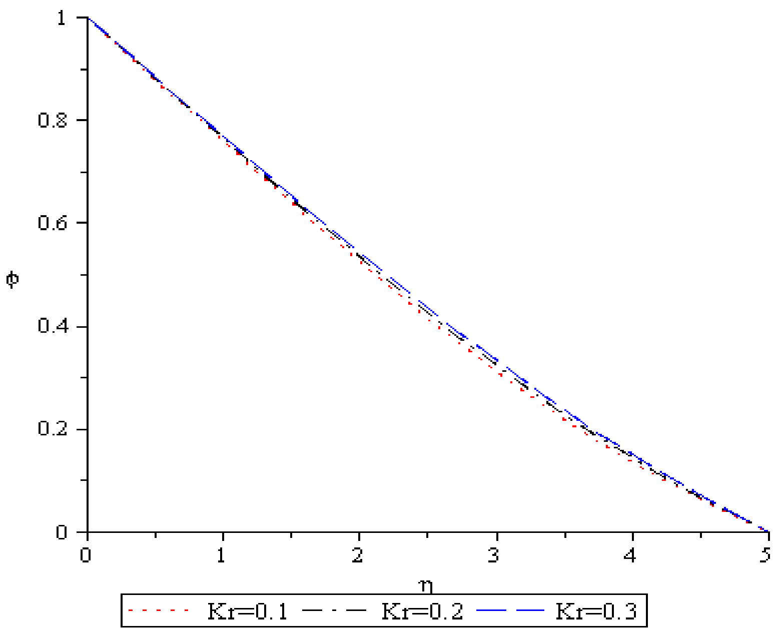

Figure 4.32: Variation of rotational parameter on concentration on a shrinking sheet

Figures 4.29 to 4.32 present the variation of rotational parameter on velocities, temperature, and concentration on shrinking sheet. It is observe that the rotational parameter increases the velocity *h*, and concentration profiles while the velocity *f* and temperature reduces as the fluid rotation increases shrinking sheet. The boundary layers on the shrinking sheet are found to be thicker than the stretching sheet.

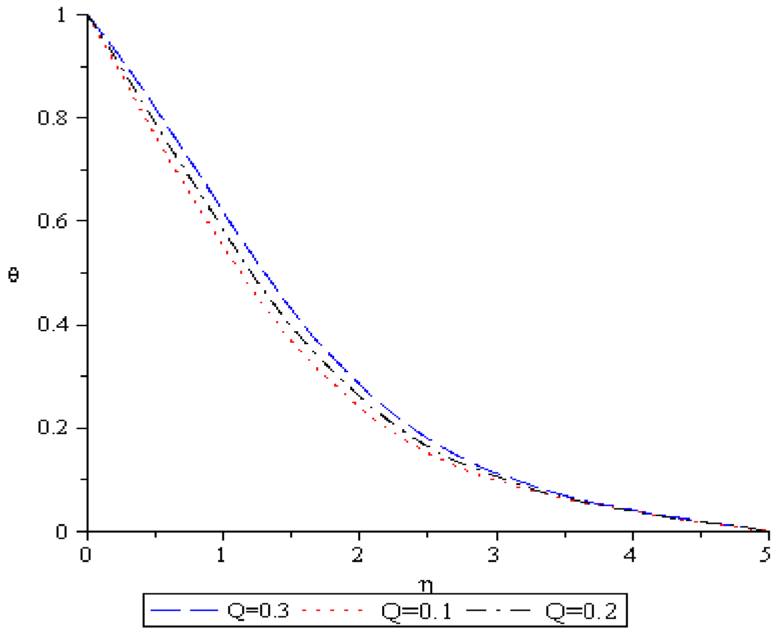

Figure 4.33: Variation of heat generation parameter on temperature on a stretching sheet

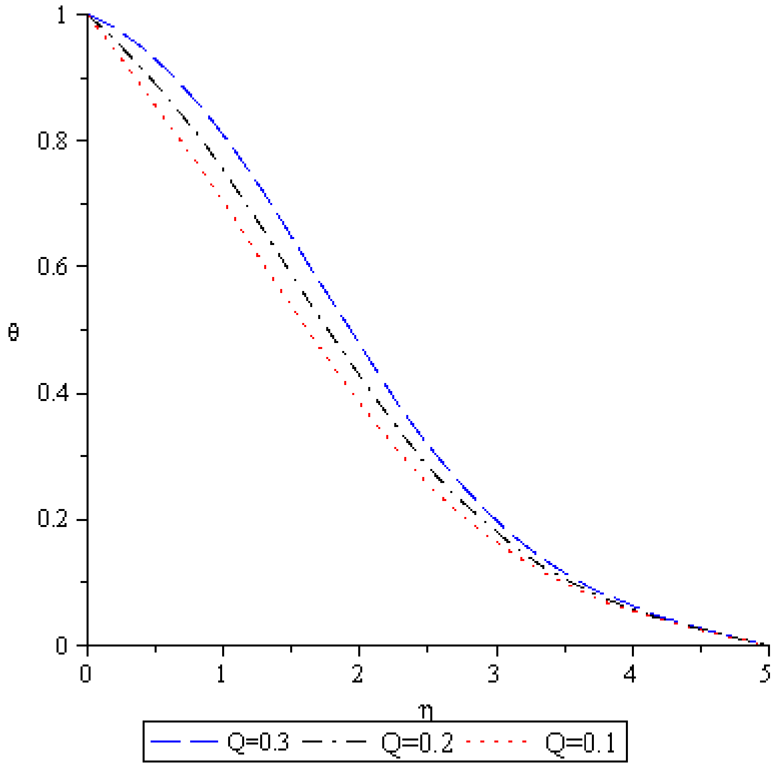

Figure 4.34: Variation of heat generation parameter on temperature on a shrinking sheet

Figures 4.33 to 4.34 are the variation of heat generation parameter on the stretching sheet and shrinking sheet respectively. As the heat generation parameter increases, temperature also increases on both sheets. But the temperature boundary layer on the shrinking sheet is thicker than on the stretching sheet.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This present work considered the work of Bhatti et al. (2018) with natural convection, heat generation. The PDE formulated in rectangular system was reduced to ODE via some similarity variables. The non linear coupled ODE depends on some physical parameters such as magnetic parameter, shrinking/ stretching parameter, and Grashof number. The following observations were made:-

1. The graphs presented in this work clearly satisfy the boundary conditions, which imply that the problem is well posed.

2. The larger values of the dimensionless distance is choosing to be at $\eta = 5$.

3. The results presented in this work were compared with the results of the existing literatures as seen in Table 4.1 and 4.2 for stretching sheet and shrinking sheet and a good agreement was established.

4. The values of the stretching sheet $\alpha = 0.6$ and for shrinking sheet $\alpha = -0.6$ as seen on the graphs.

5. All the parameters varied on velocities, temperature and concentration have same effects on both stretching and shrinking sheet, but the boundary layer thickness varies.

5.2 Recommendations

It is recommended that researchers:

- Who wish to extend this problem by introducing more parameters are advice to do so.

- Should employ another method to obtained better results than what we have here.

REFERENCES

Abbas, T., Bhatti, M. M. & Ayub, M. (2017). Aiding and opposing of mixed convection Casson nanofluid flow with chemical reactions through a porous Riga plate, *Journal of Process Mechanical Engineering,* 19, doi: 10.1177/0954408917719791.

Abd-El Aziz, M. (2006). Thermal radiation effects on magnetohydrodynamic mixed convection flow of a micropolar fluid past a continuously moving semi-infinite plate for high temperature differences. *Acta Mechanica*, 187(2006), 113–127.

Abu-Nada, E. & Chamkha, A (2010). Effect of nanofluid variable properties on natural convection in enclosures filled with a CUO–water nanofluid, *Internal Journal of Thermal. Science*, 49, 2339–2352.

Abdallah, A., Ibrahim, F., & Chamkha, A. (2018). Nonsimilar solution of unsteady mixed convection flow near the stagnation point of a heated vertical plate in a porous medium saturated with a nanofluid. *Journal of Porous Media*, 21(4), 363–388

Adomian, G. (1994). Solving frontier problem of Physics: The Decomposition method. *Springer-Science +Business, 6.*

Bhatti, M. M., Ali Abbas, M., Rashidi, M. M. (2018). A robust method for solving stagnation point flow over a permeable shrinking sheet under the influence of MHD. *Journal of Applied Mathematics and Computation*, 316, 381-389.

Bhatti, M. M., & Rashidi, M.M. (2016). Effects of thermo-diffusion and thermal radiation on Williamson nanofluid over a porous shrinking/stretching sheet. *Journal of Molecular Liquids,* 221, 567–573.

B´eg, O. A., Takhar, H. S. & Singh, A. K. (2005). Multiparameter perturbation analysis of unsteady oscillatory magnetoconvection in porous media with heat source effects. *International Journal of Fluid Mechanics Research*, 32(6), 635–661.

Besthapu, P. and Bandari, S (2015). Mixed convection MHD Flow of a Casson nanofluid over a nonlines'ar permeable stretching sheet with viscous dissipation. *Journal of Applied. Mathematical Physics*, 3, 1580–1593.

Buongiorno, J. (2009). Convective transport in nanofluids. *Journal of Heat Transfer,* 128, 240-250.

Congedo, P., Collura, S., & Congedo, P (2009). Modeling and analysis of natural convection of heat transfer in nanofluids. *Heat Transfer Conference*, Jacksonville, FL, 3, 569–579.

Carrier, G. F. & Greenspan, H. P. (1960). The time-dependent magnetohydrodynamic flow past a flat plate. *Journal of Fluid Mechanics*, 7, 22–32.

Chaudhary, R.C. & Abhay, K. J (2008). Effect of chemical reaction on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Applied Mathematics and Mechanics, 29(9), 117-1194, 2008

Chamkha, A. J. (2001). Unsteady laminar hydromagnetic flow and heat transfer in porous channels with temperature-dependent properties. *International Journal of Numerical Methods for Heat and Fluid Flow*, vol. 11(5-6), 430–448.

Chamkha, A. J. (1996). Unsteady hydromagnetic natural convection in a fluid-saturated porous medium channel. *Advanced Filtration and Separation Technology*, 10, 369–375.

Chaudhary, R. C. & Jain, A. (2008). Magnetohydrodynamic transient convection flow past a vertical surface embedded in a porous medium with oscillating temperature. *Turkish Journal of Engineering and Environmental Sciences*, 32(1), 13–22.

Cogley A.C., Vincent W.E., Gilles S.E. (1968). Differential approximation for radiation in a non-gray gas near equilibrium. *Non-analysis modelling and control*, 6, 551-553.

Das, U. N., & Ahmed, N. (1992). Free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. *Indian Journal of Pure and Applied Mathematics*, *23*, 295–204.

Das, S., Choi, S., Yu, W. & Pradeep, T (2008). Nanofluids: *Science and Technology*, Hoboken, NJ: John Wiley & Sons, 2008.

Das, K., Jana, S., (2010). Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in a porous medium. *International communication on Heat and Mass Transfer*, 17, 15-32.

Eldabe, N. T. & Ouat, M. E. (2006). Chebyshev finite difference method for heat and mass transfer in hydromagnetic flow of a micropolar fluid past a stretching surface with Ohmic heating and viscous dissipation. *Applied Mathematical Computation*, 177, 561-571.

Fang, T., Lee, C., & Zhang, J (2011). The boundary layers of an unsteady incompressible stagnation-point flow with mass transfer. *International Journal of Non-Linear Mechanics*, 46, 942–948, 2011.

Ghosh, S. K. (1993). Unsteady hydromagnetic flow in a rotating channel with oscillating pressure gradient. *Journal of the Physical Society of Japan*, 62(11), 3893–3903.

Gireesha, B.J., Mahanthesh, B., Manjunatha, P.T., & Gorla, R .S. R (2015). Numerical solution for hydromagnetic boundary layer flow and heat transfer past a stretching surface embedded in non-darcy porous medium with fluid-particle suspension. *Journal of Nigeria Mathematical Society*, 34, 267–285, 2015.

Gorla, R.S.R. & Vasu, B.(2016). Unsteady convective heat transfer to a stretching surface in a non-newtonian nanofluid. *Journal of Nanofluids*, 5, 581–594, 2016.

Gorla, R.S.R., Vasu, B., & Siddiqa, S.(2016). Transient combined convective heat transfer over a stretching surface in a non-newtonian nanofluid using Buongiorno's model. *Journal of Applied. Mathematical Physics,* 4, 443–460.

Gupta, A. S. (1960). Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of a magnetic field. *Applied Scientific Research,* 9(1), 319–333.

Hady, F., Ibrahim, F., Abdel-Gaid, S., & Eid, M.(2012a). Radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet. *Nanoscale Residual Letters*, **7**, 229–241.

Hady, F., Ibrahim, F., El-Hawary, H., & Abdelhady, A. (2012b). Effect of suction/injection on natural convective boundary-layer flow of a nanofluid past a vertical porous plate through a porous medium. *Journal of Modern Methods in Numerical Mathematics*, 3, 53–63.

Haque, M. Z., Alam, M. M., Ferdows, M., Postelnicu, A. (2011). Micropolar fluid behaviors on steady MHD free convection flow and mass transfer with constant heat and mass fluxes, joule heating and viscous dissipation. *Journal of King Saud University Engineering and Science,* 23-29, doi:10.1016/j.jksues.

Hamad, M., Pop, I. & Ismail, A. (2011). Magnetic field effects on free convection flow of a nanofluid past a semi-infinite vertical flat plate. *Nonlinear Analysis: Real World Application*, 12, 1338–1346.

Hassanien, I. & Al-Arabi, T. (2008). Thermal radiation and variable viscosity effects on unsteady mixed convection flow in the stagnation region on a vertical surface embedded in a porous medium with surface heat flux, *Far East Journal of Mathematical Science*, 28, 187–207.

Hossain, M. A. Takhar, H. S. (1996). Radiation effect on mixed convection along a vertical plate with uniform surface temperature. *Heat mass transfer*, 31,243-248.

Hassanien, I., Ibrahim, F., & Omer, G. (2004). Unsteady free convection flow in the stagnation-point region of a rotating sphere embedded in a porous medium. *Journal of Mechanical Engineering*, 7, 89–98.

Hassanien, I., Ibrahim F. & Omer, G. (2006). Unsteady flow and heat transfer of a viscous fluid in the stagnation region of a three-dimensional body embedded in a porous medium. *Journal of Porous Media, 9*(4), 357–372.

Hossain, M. A. Takhar, H. S. (1996). Radiation effect on mixed convection along a vertical plate with uniform surface temperature. *Heat mass transfer*, 31,243-248.

Ishak, A., Nazar, R., & Pop, I.(2008). Dual solutions in mixed convection flow near a stagnation point on a vertical surface in a porous medium. *International Journal of Heat Mass Transfer,* 51, 1150–1155.

Kakac¸, S. & Pramuanjaroenkij, A. (2009). Review of convective heat transfer enhancement with nanofluids. *International Journal of Heat Mass Transf*, 52, 3187–3196.

Kameswaran, P.K., Vasu, B., Murthy, P.V.S.N., & Gorla, R.S.R. (2016). Mixed convection from a wavy surface embedded in a thermally stratified nanofluid saturated porous medium with non-linear boussinesq approximation. *International Communication on Heat Mass Transfer*, 77, 78–86.

Kandasamy R., Periasamy K. & Sivagnana, K (2005). Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and stratification effects. *International Journal of Heat and Mass Transfer*, 4550, 4557.

Keelson, N.A. & Desseaux, A. (2001). Effects of surface condition on flow of a micropolar fluid driven by a porous stretching sheet. *International Journal of Engineering and Science,* 39, 1881-1897.

Kumari, M., Takhar, H.S., & Nath, G. (1992) Unsteady mixed convection flow at the stagnation point. *International Journal of Engineering and Sciences*, 30, 1789–1800, 1992.

Mabood, F. & Khan, W.(2014). Approximate analytic solutions for influence of heat transfer on MHD stagnation point flow in porous medium. *Computational Fluids,* 100, 72–78.

Mahmoud, M.A.A. (2007). Thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity, *Physical A.* 375, 401-410.

Makinde, O.D. (2012). Heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation. *Meccanica,* 47, 1173–1184, 2012.

Makinde, O. D. (2005). Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. *International communication on Heat and Mass Transfer,* 25, 289-295.

Magdy, A.C.(2005). Free convection flow of conducting micropolar fluid with thermal relaxation including heat sources. *Journal of Applied Mathematics*, 2(4), 271-292.

Mohamad, A. (2009). Double diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effect, Applied mathematical sciences 13, 629-651.

Mishra, S. R., & Bhatti, M. M. (2017). Simultaneous effects of chemical reaction and Ohmic heating with heat and mass transfer over a stretching surface: a numerical study, Chin. *Journal of Chemical Engineering*, doi: 10.1016/j.cjche.2019.05.016.

Modather, M., Rashad, A. M., Chamkha, A. J. (2009). Study of MHD heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate in a porous medium. *Turkish Journal of engineering Environmental Sciences,* 33, 245-257.

Ogulu, A., and Prakash, J. (2006). Heat transfer to unsteady magneto-hydrodynamic flow past an infinite moving vertical plate with variable suction. *Physica Scripta*, 74(2), 232–239.

Olajuwon, B.I. (2008). Convection heat and mass transfer in a hydromagnetic flow of a second grade uid in the presence of thermal radiation and thermal diffusion. *International communication on Heat and mass*, 38, 377-382.

Pop, I. (1969). Unsteady hydromagnetic free convection flow from a vertical infinite flat plate. *Zeitschrift f ̈ur Angewandte Mathematik und Mechanik*, 49(12), 756–757.

Prasad, V. R., Reddy, N. B. and Muthucumaraswamy. (2006). Transient radiative hydromagnetic free convection flow past an impulsively started vertical plate with uniform heat and mass flux. *Theoretical Applied Mechanics*, 33(1), 31–63.

Patil, P.M. & Kulkarni P.S. (2008). Effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation. *International Journal of Thermal Science,* 4, 1043-1054.

Rashidi, M. M., Rostami, B., Freidoonimehr, N. & Abbasbandy, S. (2014). Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects. *Ain Shams Engineering Journal,* 5, 901–912.

Rahman, M.A. & Sattar, M.A. (2007). Transient convective flow of micropolar fluid past a continuously moving vertical porous plate in the presence of radiation. *International Journal of Applied Mechanics and Engineering, 12*(2), 497- 513.

Reena- I. & Rana U.S. (2009). Linear stability of thermo solutal convection in a micropolar fluid saturating a porous medium. *International Journal of Application and Applied mathematics,* 4(1), 62-87.

Srinivasacharya, D., Ramreddy. C. (2011). Soret and Dufour effect on mixed convection in a non-Darcy porous medium saturated with micropolar fluid. *Non-analysis modelling and control*, 16(1), 100-115.

Shahmohamadi, H., & Rashidi, M. M. (2016). VIM solution of squeezing MHD nanofluid flow in a rotating channel with lower stretching porous surface. *Advance Powder Technology*, 27 (1), 171–178.

Sheikholeslami, M., & Bhatti, M. M. (2017). Active method for nanofluid heat transfer enhancement by means of EHD. *International Journal of Heat Mass Transfer,* 109, 115–122.

Shateyi, S. & Marewo, G. T. (2014). Numerical analysis of unsteady MHD flow near a stagnation point of a two-dimensional porous body with heat and mass transfer, Thermal Radiation, and Chemical Reaction. *Boundary Value Problem*, 218,

Siddiqa, S., Begum, N., Hossain, M.A., & Gorla, R.S.R (2016a) Numerical solutions of natural convection flow of a dusty nanofluid about a vertical wavy truncated cone. *Journal of Heat Transfer,* 139, 022-503.

Siddiqa, S., Sulaiman, M., Hossain, M.A., Islam, S., & Gorla, R.S.R. (2016b). Gyrotactic bioconvection flow of a nanofluid past a vertical wavy surface. *International Journal of Thermal Science*, 108, 244–250.

Srinivasacharya, D. & Surender, O. (2014). Non-similar solution for natural convective boundary layer flow of a nanofluid past a vertical plate embedded in a doubly stratified porous medium. *Journal of Heat Mass Transfer*, 71, 431–438.

Singer, R. M. (1965). Transient magnetohydrodynamic flow and heat transfer. *Zeitschrift f ¨ur Angewandte Mathematik und Mechanik*, 16(4), 483–494.

Sunil, A. Sharma, A. Bharti, P.K. & Shandi, R. G. (2006). Effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium. *International Journal of Engineering science,* 44 (11-12), 683-698.

Seddek, M. A (2005). Finite-element method for effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydromagnetic flow with heat and mass transfer over a heat surface. Acta mechanics, 177, 1-18.

Satter M.D. A., Hamid, M. D. K (1996). Unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. *Journal of Mathematical Physics Science.* 30, 25-37.

Tokis, J. N. (1986). Unsteady magnetohydrodynamic free-convection flows in a rotating fluid. *Astrophysics and Space Science*, 119(2), 305–313.

Vasu, B. & Manish, K. (2015). Transient boundary layer laminar free convective flow of a nanofluid over a vertical cone/plate. *International Journal of Applied Computational Mathematics*, 1, 427–448.

Wang, C. Y. (2008). Stagnation flow towards a shrinking sheet. *International Journal of Non-Linear Mechanics,* 43, 377-382.

Yacob, N., Ishak, A., & Pop, I (2011). Falkner–Skan problem for a static or moving wedge in nanofluids. *International Journal of Thermal. Science,* 50, 133–139, 2011.

Yusuf, A., Bolarin, G. & Adekunle, S. T (2019). Analytical Solution of unsteady Boundary Layer Flow of a nanofluidpast a Stretching inclined sheet with effect of magnetic Field. *FUOYE Journal of Engineering and Technology (FUOYEJET), 4(1),* 97-101, engineering.fuoye.edu.ng/journal.

Zaimi, K., Ishak, A., & Pop, I.(2014). Unsteady flow due to a contracting cylinder in a nanofluid using Buongiorno’s model. *International Journal of Heat and Mass Transfer*, 68, 509–513.

Zueco, J. (2007). Network simulation method applied to radiation and viscous dissipation effects on MHD unsteady free convection over vertical porous plate. *Applied Mathematical Modelling*, 31(9), 2019–2033.