OPTIMAL AQUIFER WITHDRAWAL MODEL FOR IRRIGATION SCHEDULE IN BIDA BASIN, NIGER STATE, NIGERIA

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ABSTRACT

This research work stemed from the challenges of crop water need within Bida basin. It is focused on three identified problems of irrigation farming. These are the quantity of groundwater available for irrigation; optimum quantity of water to be drawn from well during the dry season as well as crop water need. The mass balance equation was solved and use to determine the groundwater level. The Lagrange multiplier method was used in estimating the optimum quantity of water to be drawn from the well during the dry season while computational method was used to solve energy balance equation and Pen – Monteith equation to determine estimated crop water needs using rice and soya beans as samples. The results obtained for the estimation of the quantity of the groundwater show that there is considerable amount of water ($27.19\times10^{6}\,ft^{3}$, $21.81\times10^{6}\,ft^{3}$ and $5.94\times10^{6}\,ft^{3}$) available for irrigation. For groundwater withdrawal the results show that moderate amount of water is drawn from the wells in each of the study area while the results obtained for estimated crop water needs conformed to crop water requirement estimated under standard conditions these are $5960.5mm$ in Mokwa; $2326mm$ in Bida and $5584.22mm$ in Lapai. This research project provides useful information to farmers on how to make proper planning for irrigation farming in order to achieve optimal crop yield.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

Irrigation is the science of artificial application of controlled amount of water for plant at needed intervals. Irrigation helps to grow agricultural crops, maintain landscape, and re-vegetate disturbed soil in dry area. Nigeria has not been left out among the committee of African nations which are battling with food crisis and food security. For any nation to stand fit for economic development such nation must take her Agricultural sector serious in which irrigation practices are paramount not only in solving hunger crisis but also economic recovery and employment creation (Geir, 1997).

Our famers or irrigators need to be educated, orientated and sensitized on how to carried out proper planning for irrigation farming in order to achieve optimal crop yield. Most sources of water for irrigation practices in developed nations are canal and surface water therefore, there is need for irrigator farmers to be sensitised on the quantity of the groundwater available during the dry season before commencing irrigation farming. Allocation of water supply for agricultural purposes has been a major constraint of irrigation farming. The irrigated agricultural areas has to be survey and monitor if the available groundwater is enough to carry out the irrigation without causing lowering of the water table.Typical issues such as environmental degradation, decline ground water tables and deterioration of the ecological state of wetlands are mostly common in the areas where extensive groundwater withdrawal occured (Molle *et al.*, 2010). An increase in Agricultural water productivity is an important component to sustain and improve food production towards ensuring sustainable food security in the food-insecure region. Recently, emphasis has been placed on the concept of water productivity, and it is defined as a measure of the economic or biophysical gain from the use of a unit of water consumed in crop production. (FAO, 2010).

The irrigation development continues to increase the demand for water abstraction and the irrigation water demand superse industrial and domestical water demand therefore there is need for communities to increase the protection of the natural environment and maintanace of biodiversity. (Dandy *et al.*, 1993).

Irrigation planners need to analyse complex climate-soil relationship and apply mathematical optimization techniques to determine the optimally beneficial crop pattern and water allocation. In arid and semi-arid wetland, salinity and water scarcity are two serious and chronic environmental problems threatening the ecosystem. Wetlands are now often experiencing extended periods of high salinization level and associated water availability problems due to impacts of high evaporative conditions, poor surface drainage, human population pressure, and the associated change in land use. Irrigation scheduling enables farmers to schedule watering to minimize crop water stress and maximize yield, the farmer's cost of water and labour reduced through less irrigation, thereby making maximum use of soil moisture storage. Irrigation scheduling increases net returns by increasing crop yield and crop quality (Goldberg, 1989).

The fastest growing branch of mathematics with appalling history is mathematical programming (optimisation) which essentially deals with maximisation or minimisation of some functions subject to one or more constraints (Bixby, 2016).

Mathematical programming (MP) is the use of mathematical models particularly optimising models, to assist in taking decisions. In computer science, the Large Scale

Integration (LSI) area gives rise to many optimisation problems such as scheduling of organisation activities, layout of microchips, routing, via minimizatio. In telecommunications, the scheduling design of network leads to many different optimisation problems, instance of this is that of scheduling network design or expansion costs subject to constraints, reflecting that network can support the described traffic (Krivulin, 2017).

Optimisation problems are common in many disciplines. In optimization problems, we have to find solutions which are optimal or near optimal with respect to some goals. In optimisation, problems are not usually solved using one step, but many steps through problem solving. Often, the solution process is broken into different steps which are executed one after the other. Commonly used steps are; recognising and defining problems, constructing and solving models, evaluating and implementing solutions (Rothlauf, 2011).

In economics (econometrics) optimization models are used for describing money transfer between sectors in society or describing the efficiency of production units. The large amount of application, combined with the development of fast computer has led to massive innovation in optimisation problems and with the standard linear programming problem, the assumption that choice variables are infinitely divisible (can be any real number) is unrealistic in many setting, integer programming problems are typically much harder to solve than linear programming problems and there are no fundamental theoretical results like duality or computational algorithm like simplex method to help one to understand and solve the problems. Resource allocation is the process of assigning and managing assets in a manner that supports an organization's strategic goals. Resource allocation includes managing tangible assets such as hardware to make the best use of softer assets such as human capital and scheduling problem is optimization problem that involves assigning resources to perform a set of tasks at specific time (Teodor, 2017).

The scheduling problem (SP) has many practical applications such as transportation, agriculture, networking. Irrigation scheduling is a decision-making process used by farmers to decide when to irrigate and how much water to apply. The question of when to irrigate is approached in several ways. However, this method can be broadly classified under the following headings (Phene *et al.*, 1990):

(i) Plant indicators

(ii) Soil indicators and

(iii) Water balance approach

The plant indicator methods include appearance and growth (Haise and Hagen, 1967), leaf temperature, leaf water potential and stomata resistance. However, these methods are either too crude or subjective, they demand for the use of specialized instrumentation (Singh *et al.*, 1994). The major setback in these methods is that the plant has suffered some amount of moisture stress, which may sometimes irreversibly damage the plant and affect the crop yield, before deciding to irrigate.

Irrigation scheduling requires the estimation of crop evapo-transpiration and expected rainfall during the irrigation interval. As we stated earlier, the irrigation scheduling programs required an accurate estimate of water used by the crop. A common procedure for estimating crop water use is to first determine the daily reference crop evapotranspiration (ET_0) and then multiply it by a specific crop coefficient (K_c), as given by (Doorenbos and Pruth, 1977).

Rainfall season start ealier in south west, south east, and south- south of the country and the rainfall season in the middle belt is between May and October while the rainy season in the northern part of Nigeria lasts for only three to four months (June – September). The rest of the year is hot and dry with temperatures climbing as high as $40^{o}C$ $(104.0^{0}\,\mathrm{F})$. Agriculture in Nigeria is a significant sector of the economy, providing employment for about 70% of the population. Most of the farms are focused on production of cocoa, beans, rice, rubber, cotton, yam, corn and sweet potato. The county's livestock sector includes the breeding of pigs, cattle, donkeys, camels and poultry products. The dry season is usually a challenging period for farmers, with an average precipitation that is below 60 millimetres and lack of watering holes, farmers face many challenges in planting (Yahya, 2015).

1.2 Statement of the Research Problem

Irrigation practice in Nigeria is the highest user of total consumptive water and it is profit returns have not been impressive. (Nkondo *et al.*, 2012). The sustainable management of irrigation water resources is therefore a necessity. Crop development and food security are supposed to be dependent on irrigation due to low annual average rainfall experienced in Bida Basin and its environs. It is obvious that several methods like simulation and optimization techniques have been developed and applied to manage irrigation water allocation around the world, yet there exists some uncertainties about finding a generally trustworthy method that can consistently find real time solutions which are really close to the global optimum of the problems in all circumstances, therefore mathematician are forced to use Meta – heuristic algorithm to solve irrigation scheduling problems. With these, there is need to develop a mathematical model for solving irrigational scheduling problems that causes low irrigation practices in Bida Basin in Niger state Nigeria.

1.3 Aim and Objectives of the Study

The aim of this research is to estimate quantity of groundwater available, optimum groundwater withdrawal and determine the exact amount of crop water need for irrigation farming in Bida Basin, Niger State Nigeria.

The Objectives of the Study are to:

(i) estimate the quantity of groundwater available for irrigation farming during the dry season (November to April) by solving mass balance model equation and its solution are trained and tested by the cimatic data obtained from Nigeria Metrological (NIMET) using artificial neural network.

(ii) determine the optimum quantity of water to be drawn from each well in the selected irrigation fields at a given time using Lagrange Multiplier method and genetic algorithm.

(iii) estimate the crop water needs for Rice and Soya periodically

(iv) estimate the crop yield for a given area of the land.

1.4 Justification of the Study

The study will help the farmers to know when to irrigate and to determine irrigation intervals.

Farmers will benefit from the study, if they have proper planning orientation on how to engage in moderate groundwater withdrawal and avoiding over withdrawal of groundwater and under withdrawal of ground water for irrigation farming.

Farmers will have an improved knowledge on crop water requirement and expected crop yields.

Farmers will benefit from the study by having orientation during and after implementation.

Federal, State and Local Governments will benefit from the study when planning for irrigation to boost food productivity.

A large number of irrigation/ irrigation scheduling systems fail each year in Nigeria and in other sub – Sahara African countries (Yahya, 2015). The failure of irrigation systems is attributed not only to water shortages but also drainage problems associated with excessive water application.

A very useful procedure for the optimization technique that provides reliable solution to mathematical modelling for optimal irrigation scheduling incorporating aquifer withdrawal system is adaptation techniques. Development of artificial neural network will be used for Groundwater level estimation in irrigation catchment areas and Genetic algorithm methods for solving irrigation scheduling problem will go a long way in solving the physical problems associated with it.

1.5 Definition of the Basic Terms

(i) **Aquifer**: A saturated geologic formation (rock or sediment) capable of storing, transmitting and yielding usable amount of groundwater to wells and springs (Ferrie and Ferguson 2007).

(ii) **Genetic Algorithm** (GA) is a search-based optimization technique based on the principles of Genetics and Natural Selection. It is frequently used to find optimal or near-optimal solutions to difficult problems which otherwise would take a lifetime to solve.

(iii) **Storability (S)**: is the volume of water per unit aquifer surface area taken into or released from storage per unit increase or decrease in head respectively and is given as: $S = S_s b$

where,

S_s =specific storage

b= saturated thickness of the aquifer

S = storativity

(iv) **water table**: - is the depth at which soil pore spaces or fractures and voids in rock become completely saturated with water.

(v) **Artificial Neural Network**: is a mathematical model that tries to simulate the structure and functionalities of biological neural networks. (Andrej, 2018).

(vi) **Irrigation**: is a science of artificial application of water to land for the purpose of agriculture production.

(vii) **Irrigation Scheduling**: is attempting to apply water to crops at the appropriate amount and time for a specific stage in the plants development and growth. (Massie and Curwen, 1994)

(viii) **Porosity**: is a dimensionless value that expresses the ratio of the volume of pores to the total volume of porous material and usually expressed as:

$$P = \frac{V_p}{V} \tag{1.1}$$

$P =$ porosity

$V_p =$ volume of pore space L^3

$V =$ volume of materials L^3

(ix) **Algorithm**: is a step by step procedure or method for solving a problem by a compute r in an infinite number of steps (Kumar, 2013).

(x) **Aquifer withdrawal**: is when water is withdrawn from an aquifer more rapidly than it is replenished.

(xi) **Mean Square error**: is a single value that provides information about the goodness of fit of the regression line, (Chai, 2014) and is given as:

$$\frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y}_i)^2 \tag{1.2}$$

$n =$ number of data poits

$Y_i =$ observed values.

$\overline{Y}_i =$ predicted values

(xii) The **correlation coefficient** (r) is used to estimate the correlation between model and observations.

$$r = \frac{Cov(X,Y)}{\sqrt{S_x S_y}} \quad (1.3)$$

where,

$r =$ correlation coefficient

Cov (X, Y) = the coverance.

$S_x =$ sample variance for X.

$S_y =$ sample variance for Y

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Artificial Neural Network

Harsh *et al.* (2016) made it known that, Artificial Neural Network (ANN) is gaining prominence in various applications like pattern recognition, weather estimation handwriting recognition, face recognition autopilot and robotics.

An artificial neural network (ANN) is a computational mode based on the structure and function of biological neural networks. Information that flows through the network affects the structure of the ANN.

Andrej *et al.* (2018) defined Artificial Neural Network (ANN) as a mathematical model that tries to simulate the structure and functionalities of biological neural networks. The mathematical description of the artificial neuron is:

$$y(k) = F = \left(\sum_{i=0}^{m} w_i(k).x_i(k) + b\right) \quad (2.1)$$

Where,

$x_i(k)$ = input value in discrete time k where i goes from 0 to m;

$w_i(k)$ = weight value in discrete time k where i goes from 0 to m;

$b =$ is bias;

$F =$ is a transfer function and;

$y(k) =$ is the output value in discrete time k.

They concluded that the algorithm is based on the ability of specialized ANNs to resolve the overlapping values of the intensive optical parameter calculated for each layer in the multi-wave length Raman Lidar profile.

2.1.1 Feed-forward artificial neural networks

Artificial neural network with feed-forward topology is called feed-forward artificial neural network and as such has only one condition: information must flow from input to output in only one direction with no back-loops. Simple multi-layer feed –forward artificial neural network (Andrej *et al*, 2018) is as follows:

$$n_1 = F_1(w_1 x_1 + b_1)$$

(2.2)

$$n_2 = F_2(w_2 x_2 + b_2)$$

(2.3)

$$n_3 = F_1(w_2 x_2 + b_2)$$

(2.4)

$$n_4 = F_3(w_3 x_3 + b_3)$$

(2.5)

$$m_1 = F_4(\mathrm{q}_1 n_1 + \mathrm{q}_2 n_2 + b_4)$$

(2.6)

$$m_2 = F_5(\mathrm{q}_3 n_3 + \mathrm{q}_4 n_4 + b_5)$$

(2.7)

$$y = F_6(\mathrm{r}_1 m_1 + r_2 m_2 + b_6)$$

(2.8)

$$y = F_6[r_1(\mathrm{F}_4[q_1F_1(w_1x_1 + b_1) + q_2F_2(w_2x_2 + b_2)] + \mathrm{b}_4) + ...$$
$$.. + r_2(\mathrm{F}_5[q_3F_2(w_2x_2 + b_2) + q_4F_3(w_3x_3 + b_3)] + \mathrm{b}_5] + b_6]$$

(2.9)

$r =$ threds hoid unit.

$q =$ weight function.

There is always changing in the pattern of Mult – layer feed forward artificial neural network. They made it known, that in the recent years, feed forward neural network has been used in chemical plant, oil refineries, and power system and power electronics.

2.1.2 The input layer

The input layer communicates with the external environments which present a pattern to the neural network. The pattern may be single layer or Multi-layer neural network. The input layer is responsible for all the inputs only. The input gets transferred to hidden layer where every input neuron represents some independent variables that have an influence over the output of the neural network.

2.1.3 The hidden layer

Hidden Layer is the collection of neurons which has activation function applied on it and it is intermediate layer found between the input layer and output layer. It is responsible for the processing of the input obtained by its previous layer, thus, it is the layer which is responsible for extracting the required feature from the input data.

2.1.4 The output layer

Output layer of the neural network collects and transmits the information accordingly in a way it has been designed to give the pattern presented by the output. The number of neurons in output layer should be directly related to the type of work that the neural network was performing.

2.1.5 Neuron with vector input

A neuron with a single R- element input vector shown below:

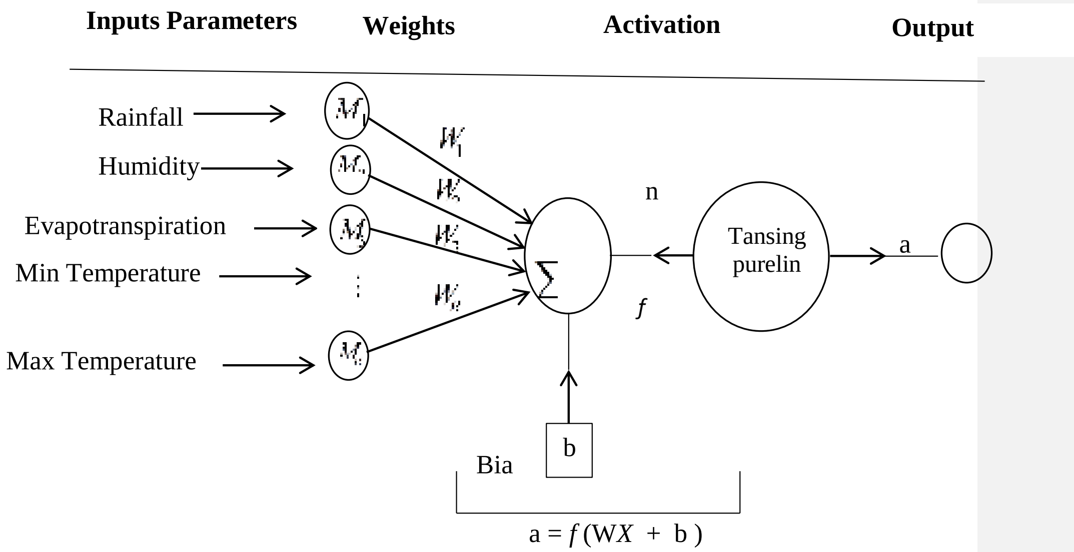

Figure 2.1 Shows a neuron with a single R- element input vector

The individual element inputs $M_1, M_2, M_3 ... M_R$ are multiplied by the weight input $W_I, W_2, W_3, ..., W_R$ and the weight values are fed to summation junction, The sum is simply $W \times M$, the dot product of the single row matrix Wand the vector M. The neuron a bias b which summed with the weighted input to form the net input n. this sum n is the argument of the transfer function f.

$n = W_{I1,} M_1 + W_{I,2} M_2 + ..., + W_{I,R} M_R + b$.

2.2 Genetic Algorithm (GA)

The Genetic Algorithm begins, like any other optimization algorithm, by defining the optimization variables. GA also ends like other optimization algorithms too, by testing for convergence. The algorithm was developed by Holland (1975) and there was sequence of development in 1960s and 1970s and finally applied by one of his students, David Goldberg, who was able to solve a difficult problem involving the control of gas-pipeline transmission for his dissertation (Goldberg, 1989).

A path through the components of the GA is shown as a flowchart in Figure 2.2. Each block in the flow chart is discussed in detail in this chapter.

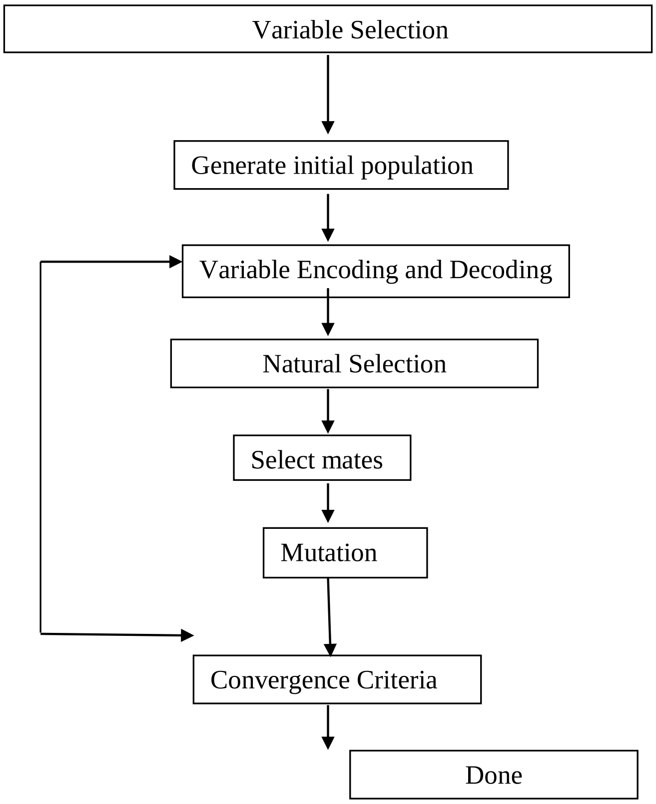

Figure 2.2 Flow chart of genetic algorithm

2.2.1 Variable selection

The Genetic Algorithm begins by defining a chromosome or an array of variable values to be optimized. If the chromosome has N_{var} variables (an N_{var}-dimensional optimization problem) given by p_1, p_2, . . . , , then the chromosome is written as an N_{var} element row vector, following the idea of Goldberg and Deb (1991).

$$Chromosome = [p_1, p_2, p_3, \ldots p_{Nvar}] \quad (2.10)$$

In a chromosome, the genes are represented as in table 2.1

Table 2.1 Representation of genes in a chromosome

1010	1110	1111	0101
gene 1	gene2	gene3	gene4

2.2.2 Variable encoding and decoding

Since the variable values are represented in binary, there must be a way of converting continuous values into binary, and vice versa as outlined by Back (1994). The mathematical formulas for the binary encoding and decoding of the n^{th} variable, p_n, are given as follows:

For encoding,s

$$P_{norm} = \frac{P_n - P_{l0}}{P_{hi} - P_{L0}}$$

(2.11)

$$\text{gene}[m] = \text{round}[m]\left\{ p_{norm} - 2^{-m} - \sum_{p=1}^{m-1} gene[p]2^{-p} \right. \text{ for decoding} \quad (2.12)$$

$$P_{quant} = \sum_{m=1}^{Ngene} gene[m]2^{-m} + 2^{-1(m+1)} \quad (2.13)$$

$$q_n = P_{quant}(P_{hi} - P_{lo}) + P_{lo} \quad (2.14)$$

In each case,

P_{norm} = normalized variable, $0 \leq P_{norm} \leq 1$

P_{lo} = lowest variable value

P_{hi} = highest variable value

$gene[m]$ = binary version of P_n

round{·} = round to nearest integer

P_{quant} = quantized version of P_{norm}

q_n = quantized version of P_n

P_n = *nth* variables

The binary GA works with bits. The variable x has a value represented by a string of bits that is N_{gene} long. If $N_{gene} = 2$ and x has limits defined by $1 \leq x \leq 4$, then a gene with 2 bits has = 4 possible values.

The quantized value of the gene or variable is mathematically found by multiplying the vector containing the bits by a vector containing the quantization levels:

$$q_n = gene \times Q^T \quad (2.15)$$

where

$gene = [b_1\ b_2 \ldots b_{Ngene}]$

N_{gene} = number bits in a gene

b_n = binary bit = 1 or 0

Q = quantization vector = $[2^{-1}\ 2^{-2} \ldots 2^{Ngene}]$

Q^T = transpose of Q

2.2.3 Generating initial population

The GA starts with a group of chromosomes known as the population (Goldberg and Deb, 1991). The population has N_{pop} chromosomes and is an $N_{pop} \times N_{bits}$ matrix filled with random ones and zeros generated using equation (2.16)

$$pop=round(rand(N_{pop}, N_{bits})) \quad (2.16)$$

where the function (N_{pop}, N_{bits}) generates a $N_{pop} \times N_{bits}$ matrix of uniform random numbers between zero and one. The function *round* rounds the numbers to the nearest integer which in this case is either 0 or 1. Each row in the pop matrix is a chromosome. Population being combination of various chromosomes is represented as in Table 2.2 below. This population consists of four chromosomes.

Table 2.2 Population

Population	Chromosome 1	1 1 1 0 0 0 1 0
	Chromosome 2	0 1 1 1 1 0 1 1
	Chromosome 3	1 0 1 0 1 0 1 0
	Chromosome 4	1 1 0 0 1 1 0 0

The best chromosomes from the population will be rank and computed using the equation below using genetic algorithm approach (Goldberg, 1989):

$$rsum_i = \sum_{i=1}^{ngene} r_{i,j}$$

(2.17)

$$PRANK_i = \frac{r_{i,j}}{rsum_i} \qquad (2.18)$$

where i varies from 1 to $ngen$,

$rsum_i$ = sum of ranks in the generation

$r_{i,j}$ = rank of jth individual in ith generation for rank selection

$ngen$ = number of generation

2.2.4 Natural selection

In GA survival of the fittest means translates into retaining the chromosomes with the highest fitness. First, the N_{pop} fitness and associated chromosomes are ranked from lowest fitness to highest fitness. Then, only the best are selected to proceed, while the rest are deleted. The selection rate, X_{rate}, is the fraction of N_{pop} that survives for the next step of mating (Goldberg, 1993). The number of chromosomes that are kept each generation is given as:

$$N_{keep} = X_{rate} N_{pop} \qquad (2.19)$$

Natural selection occurs in each generation (iteration) of the algorithm. We allow, only the top N_{keep} survive for mating, and the bottom N_{pop} - N_{keep} are discarded to make room for the new offspring. Choosing what number of chromosomes to keep is somewhat arbitrary. Allowing only a few chromosomes to survive to the next generation restricts the available genes in the offspring. Keeping too many chromosomes allows bad performers a chance to contribute their traits to the next generation. We often keep 50% (X_{rate} = 0.5) in the natural selection process.

2.2.5 Selection of mates

Two chromosomes will be selected from the mating (ma) pool of N_{keep} chromosomes to produce two new offspring. Pairing (pa) takes place in the mating population until N_{pop} - N_{keep} offspring are born to replace the discarded chromosomes. Pairing chromosomes in a GA can be as interesting and varied as pairing in an animal species. In this study we shall be using random pairing as explained by Russel (1998).

This approach uses a uniform random number generator to select chromosomes. The row numbers of the parents are found using

$$ma=ceil(N_{keep}*rand(1, N_{keep})) \quad (2.20)$$

$$pa=ceil(N_{keep}*rand(1, N_{keep})) \quad (2.21)$$

where ceil rounds the value to the next highest integer.

2.2.6 Crossover process

In GA Crossover operators is used to divide a pair of selected chromosomes into two or more parts. It consists of combining the chromosomes of two parents to produce a new offspring (child). The reason behind using crossover is that the new chromosomes will be formed (child) may be better than both of the parents, if it takes the best chromosomes from both parents. For the purpose of this work, the following Crossover will be used:

(i) **Single point crossover (SPC)**

A single point crossover involves the two mating chromosomes (parents) are cut once at corresponding points and the selection exchanged after the cuts (Reeves *et al*, 2003).

Figure 2.3 below shows the single point crossover (SP). The shaded area is the

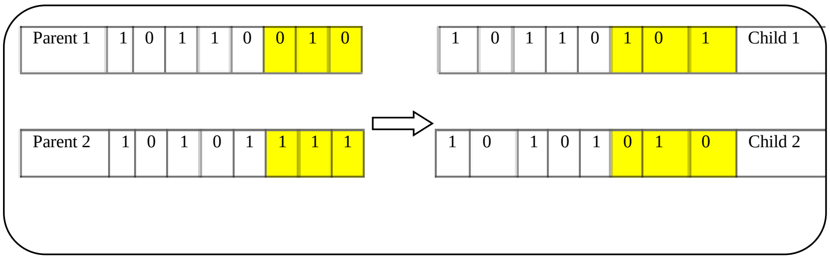

Figure 2.3 Single point crossover

crossover point.

(ii) Two-point crossover (TPC)

Two-point crossover often involving more than one cut point (Kaya, 2011). The two mating chromosomes (parents) may cut in more than point end and the selection after the cut may be exchanged.

Figure 2.4 below show the two-point crossover, the shaded area indicated the crossover point.

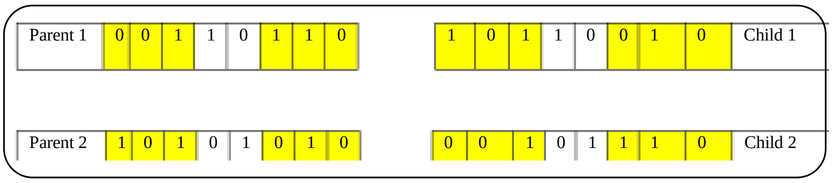

Figure 2.4 Two-point crossover

iii) Intermediate crossover (ITC)

Intermediate crossover creates offspring (child) by a weighted average of the two mating parents (Mathworks, 2015). If parent 1 and parent 2 are the two mating chromosomes and Ratio is in the range [0, 1], then the returns the child (*offspring*). The equation is given as:

$$offspring(child) = Parent\ 1 + rand \times Ratio \times (Parent2 - Parent\ 1) \qquad (2.22)$$

(iv)) Heuristic crossover (HEC)

Heuristic crossover (HE), produces an offspring of the two parents which lies a small distance away from the parents with better fitness value in the direction away from the parent with the worse fitness value (Mathworks, 2015).

$$offspring(child) = \ Parent2 + Ratio \ \times (Parent\ 1 - Parent\ 2) \qquad (2.23)$$

where default value of $Ratio$ is 1.2

(v) Arithmetic crossover (AC)

Arithmetic crossover (AM), it produces an offspring (child) that is a weighted arithmetic mean of two parents, α is random value between [0,1] (Mathwork, 2015). If parent 1 and parent 2 are the Parents, and parent 1 has the better fitness value, the function returns a child (offspring)

$$offspring = \ \alpha \times Parent\ 1 + (1 - \alpha) \times Parent2 \qquad (2.24)$$

(vi) Scattered crossover (SC)

Scattered Crossover (SC) creates a random binary chromosomes and selects the genes where the chromosome is 1 from the first parent, and the genes where the chromosome is 0 from the second parent and later combines the genes to form a child. Figure 2.4 below shows the scattered crossover (Stepaj, 2010).

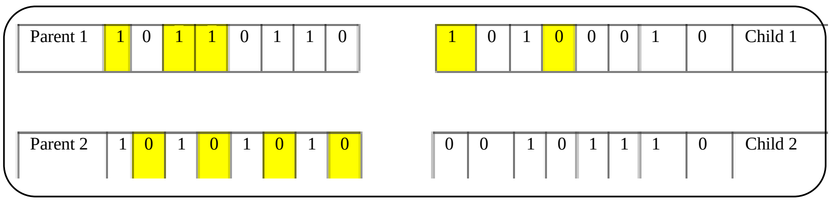

Figure 2.5 Scattered Crossover

2.2.7 Mutations

Random mutations alter a certain percentage of the bits in the list of chromosomes.Mutation is the second way a GA explores a fitness surface (Goldberg, 1989). It introduces traits not in the original population and keeps the GA from convergingtoo fast before sampling the entire cost surface. A single point mutationchanges 1 to 0, and vice versa. Mutation points are randomly selectedfrom the $N_{pop} \times N_{bits}$ total number of bits in the population matrix. Increasingthe number of mutations increases the algorithm’s freedom to search outsidethe current region of variable space. It has a tendency to distract the algorithm from converging on a popular solution. Mutations do not occur on the final iteration.

2.2.8 Mutation process

Immediately after crossover, the fitness functions are subjected to mutation. Mutation prevents the algorithm from being caught in a local minimum. Mutation plays the role of recuperating from the lost genetic materials as well as for randomly disturbing genetic information. If crossover is supposed to exploit the current solution to find better ones, mutation is supposed to help in for the exploration of the whole search space. Mutation presents new genetic structures in the population by randomly altering some of its building blocks. The building block is being exceedingly fit, low order short defining length schemes, and encoding schemes. Mutation helps to escape from local minima’s trap and keeps up diversity in the population (Goldberg, 1989). In this study, we shall consider the following mutation: Single Point Mutation and Adaptive feasible mutation.

(i) **Single point mutation**

A commonly used method for mutation is called single point mutation (Goldberg, 1989). Single gene (chromosome or even individual) is randomly selected to be mutated and its value is changed depending on the encoding type used. The shaded area in Figure 2.5 is the mutation point.

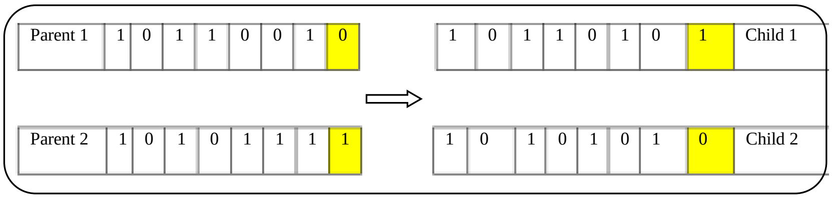

Figure 2.6 Single point mutation

(ii) Adaptive feasible mutation

Adaptive Feasible Mutation used in this research work randomly generates directions that are adaptive with respect to the last successful or unsuccessful generation (Angeline, 1995). The feasible region is bounded by the constraints. A gene locus is called non-inclined if there is no trend of increasing or decreasing of 1's in the gene locus. The probability of mutation for each locus *i* at the generation *t* is adjusted by using the equation:

$$Probm(i,t) = Pmax - 2 \times |f_1(i,t) - 0.5| \times (P_{max} - P_{min}) \tag{2.25}$$

where,

$f_1(i,t)$ = frequency of 1's in the locus *i* over the population at generation *t*

P_{max} = maximum value of the mutation probability for the locus *i*

P_{min} = minimum value of the mutation probability for the locus *i*

In mutation adaptive feasible, each gene has two different mutation probabilities which are $Probm^1$ and $Probm^0$

$Probm^1$ = for loci that have the value of 1

$Probm^0$ = for loci that have the value of 0

The new mutation probability for each locus *i* at generation *t* + 1 is given as:

$$Probm^0(i, t+1) = \begin{cases} Probm^0(i,t) + \gamma, if\ G_{avg}^1(i,t) > P_{avg}(t) \\ Probm^0(i,t) - \gamma, otherwise \end{cases} \quad (2.26)$$

$$Probm^1(i, t+1) = \begin{cases} Probm^1(i,t) - \gamma, if\ G_{avg}^1(i,t) > P_{avg}(t) \\ Probm^1(i,t) + \gamma, otherwise \end{cases} \quad (2.27)$$

where

γ = updated value for the mutation rate

$G_{avg}^1(i,t)$ = average fitness of the individual with allele "1" for locus *i* at generation *t*

$P_{avg}(t)$ = average fitness of the population at generation *t*

2.2.9 The next generation

After the mutations take place, the fitness associated with the offspring and mutated chromosomes are calculated. The process described is iterated.

The bottom chromosomes are discarded and replaced by offspring from the top parents. Another random bits are selected for mutation from the bottom chromosomes and then generate the population (Piszcz, 2006).

2.2.10 Convergence criteria

The number of generations that evolve depends on whether an acceptable solution is reached or a set number of iterations is exceeded. After a while all the chromosomes and associated costs would become the same if it were not for mutations. At this point the algorithm should be stopped. The Genetic Algorithms is a stochastic search method; it is difficult to formally specify convergence criteria. As the fitness of a population may remain static for a number of generations before a predominant individual is found, the application of conventional termination criteria becomes problematic. A common practice is to terminate the Genetic Algorithm after a pre-specified number of generations and then test the quality of the best members of the population against the problem definition. If no acceptable solutions are found, the Genetic Algorithms may be restarted or a fresh search initiated (Stepaj, 2010).

The following stopping conditions were used in their study:

(i) Maximum generations: - the genetic algorithm stops when the specific numbers of generations have evolved.

(ii) Elapsed time: - The genetic process will end when a specific time has elapsed.

Note: If the maximum number of generation has been reached before the specific time elapsed, the process will end.

(iii) Best individual:- A best individual convergence criterion stops the search once the minimum fitness in the population drops below the convergence value. This brings the search to a faster conclusion guaranteeing at least one good solution.

(iv) Sum of fitness: - In this termination scheme, the search is considered to have Satisfaction converged when the sum of the fitness in the entire population is less than or equal to the convergence value in the population record. This guarantees that virtually all individuals in the population will be within particular fitness range, although it is better to pair this convergence criterion with weakest gene replacement, otherwise a few unfit individuals in the population will blow out the fitness sum.

2.3 Irrigation Scheduling

Neda *et al.* (2015) used deterministic optimization methods to solve the problem of irrigation planning and cropping pattern. They observed that there is water resource limitation and demand for water is increasing and there is occurrence of intermittent draughts. Their objective was to maximize annual benefit of system by considering most economical values of water use efficiency (WUE) and relative yield.

The climate conditions for seven years and the meteorological required for estimating evapotranspiration is considered. These include minimum and maximum temperature, wind speed, average relative humidity, solar radiation and rainfall were gathered as data and used. Their mathematical model for cropping pattern optimization was:

$$z = \sum_{i=1}^{n\ canal} \sum_{j=1}^{n\ crop} \left(P_{ci,j} Y_{i,j} - C_{i,j} \right) A_{i,j} - P_w \sum_{i=1}^{n\ canal} \sum_{j=1}^{n\ crop} IR_{ij} \qquad (2.28)$$

Where

$z = net\ profit$
$P_{ci,j} = \text{Pr}\,ice\ for\ crop$
$Y_{i,j} = actual\ crop\ yield\ per\ hectare\ for\ crop$
$A_{i,j} = cropping\ area\ for\ crop$
$c = \cos t\ of\ crop$
$P_w = irrigation\ water\ price$
$IR = value\ of\ irrigation\ water\ for\ crop$

They proposed the maximum water required of crop in different growth stages and that was obtained from the following equation:

$$w_{ak} = (1 - x)\, w_{pk} \qquad (2.29)$$

where

x *represent* irrigation water reduction fraction

w_p *represent* is calculated according to the following (Ku *et al.*, 2000)

$$w_{p_{i,j}} = \tfrac{IN}{Ea} \times A_{i,j} \times 10 \qquad (2.30)$$

where,

IN_i represent actual water required for crop i

$E_a\ (mm/10\,days)$ *represent* efficiency of water application in the field (decimal),

IN_j was obtained according to the following equation

$$IN_j = ET_{crop\,(i)} - P_e \qquad (2.31)$$

where

P_e *represent* effective rainfall in month (i).

The value which was calculated aiding cropwat software through USDA method for the months therein rainfall occurred.

ET *represent* evapotranspiration of crop (i). This was obtained according to the following equation.

$$ET_{crop\,j} = K_c \cdot ET_o \tag{2.32}$$

There result revealed that relative crop yield and water use efficiency for all crops in all deficit scenarios were determined. Deficit irrigation had the high values of relative (WUE). They estimated the amount of water for deficit irrigation to about 57cm. They found out that it is possible to irrigate the total irrigation areas of studied lands. After determining total cropping area with an optima deficit irrigation which was applied for the entire cropping pattern at all their growth stages. The cropping pattern was optimized using genetic algorithm.

Memon *et al.* (2018) estimated crop water requirement and irrigation scheduling of Soya Bean and Tomatoe using crop WAT 8.0. They carry out irrigation scheduling for various scenarios through which it was observed that the critical depletion scenario is the best one as the yield reduction is minimum the model equation is given as :

$$ET_c = ET_0 \times K_c \tag{2.32a}$$

$$ET_0 = \left(\frac{0.408\Delta(R_n - G) + \gamma \dfrac{900}{Ta + 273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \right) \tag{2.32b}$$

where;

$ET_0 =$ crop water need

$ET_0 =$ reference crop evapotranspiration

$K_c =$ crop water coefficient

The results show that the gross water requirement for Soya Bean and Tomato are obtain 637.2mm and 1458.5 respectively.

Sumayah *et al.* (2014) developed a non – linear programming optimization model with an integrated soil – water balance to determine the optimal reservoir release policies and the optimal cropping pattern. They identified the combining probability levels of rainfall, evapotranspiration and inflow. They modelled two irrigation strategies, full irrigation and deficit irrigation under each weather condition. The objective function of the model maximizes the total farm income which is based on the crop – water production function developed by Jasen. In their study, rainfall, evapotranspiration and inflow were considered as being stochastic.

The objective function, which maximizes the total farm income, is considered for the optimal operation of the reservoir and the irrigation on n crops at any time interval j during the irrigation seasons are:

$$z^* = \max A_i R_{i,j} \sum_{i=1}^{n} \left[P_i \left(y_a \right)_i - c_i \right] A_i = \max A_i R_{i,j} \sum_{i=1}^{n} \left\{ P_i \left(Y_m \right)_i \prod_{j=1}^{k} \left[\frac{\left(ET_a \right)_{i,j}}{\left(ET_m \right)_{i,j}} \right]^{\lambda_{i,j}} - c_i \right\} A_i$$

(2.33)

where;

n represent the number of crops

K represent number of time intervals

i represent the crop index

J represent the time interval

z^* represent total farm income

P_i *represent* price of crop i

$R_{i,j}$ represent reservoir release for crop i during time interval j

c_i represent the production cost of crop i

A_i represent the cultivated area of crop i

Y_a represent the actual yield

Y_m represent maximum crop yield under the given management conditions with limited water supply

ET_a represent the actual evapotranspiration

ET_m represent maximum evapotranspiration (mm) which is the product of a crop factor K_c and reference evapotranspiration.

K_c represent three growing stages (initial, middle, end)

$\lambda_{i,j}$ = the sensitivity index of crop i to water stress during time interval j.

The equation (2.6) was later reduced to

$$z^* = \max A_i \sum_{i=1}^{n} \left[P_i \left(y_a \right)_i - c_i \right] A_i = \max \sum_{i=1}^{n} \left\{ P_i \left(Y_m \right)_i - c_i \right\} A_i \tag{2.34}$$

with the following constraint soil moisture constraint, crop irrigation requirement and reservoir release constraint, actual evapotranspiration constraint. The optimization model is solved using genetic algorithm and the constrained problem is converted into an unconstrained problem in a genetic algorithm by introducing a penalty function as

$$F_i = F(x) + \sum_{j=1}^{k} \delta_i (\phi_j)^2 \tag{2.35}$$

where;

F_i represent fitness value

$F(x)$ represent Objective function value

K *represent* number of constraints

$E =$ - 1 for maximization and +1 for minimization

δ_j *represent* the penalty coefficient

ϕ *represent* amount of violation

By applying the models which were described in the stochastic generation section to the historical data, their result shows the comparison between the monthly means and the monthly standard deviation of the aforementioned data. They observed that the monthly means and monthly standard deviations of the generated synthetic rainfall, evapotranspiration and inflow are closed to those of the measure rainfall, evapotranspiration and inflow.

The result from genetic algorithm shows that the models indicates appropriate performance as the model tends to converge to a maximum after 200 iterations. For wet and normal weather conditions similar results were obtained. The model could not find a feasible solution under hot and dry weather conditions. The reason seems to be that, the initial reservoir storage and the inflow during the irrigation season were insufficient to irrigate crops given the imposed constraints of the minimum desire area (mm) and the minimum desired relative yield $\left({}^{Y_a}\!/_{Y_m} \right)$.

Bernardo *et al.* (1987), developed an irrigation model for management of limited water supply. The presented two-stage simulation / mathematical programming model to determining the optimal intra-seasonal allocation of irrigation water under conditions of scarce water supply. Their model is applied to a series of water shortage scenarios under surface and pivot irrigation. They presented a methodology for developing economically efficient seasonal irrigation plans and they apply the model to limited water supply settings characterized by alternative irrigation scheduling. They represented water supply limits in the mathematical programming model as

$$\sum_j \sum_k w_{ijk} X_{jk} \le b_i \cdot \frac{E}{L} \qquad (i = 1, 2, ..., 50) \tag{2.36}$$

$$\sum_j \sum_k w_{jk} X_{jk} \le b_i \cdot E(L) \tag{2.37}$$

where,

X_{jk} *represent* the process of producing the kth crop with jth irrigation activity.

w_{ijk} *represent* crop consumptive use

$E(L)$ represent the application efficiency expressed as a function of the labour – intensity.

They presented the model which assumes a multiplicative relationship between water stress sustained in each of the four growth periods and may be expressed as

$$Y_a / Y_m = \prod_{i=1}^{4} \left[1 - K_{yi} \left(1 - \frac{ET_{a_i}}{ET_{p_i}} \right) \right] \tag{2.38}$$

where;

K_{y_i} *represent* crop – response factor for ith period

ET_{a_i} *represent* actual evapotranspiration in period i.

The results indicate that significant opportunities exist for conserving water in the study area under surface and centre pivot irrigations. They concluded that the modelling approach presented provides improved guidance for irrigation management under conditions of limited water supply.

Liudong *et al.* (2014) proposed optimal reservoirs operation for multi-crop deficit irrigation under fuzzy stochastic uncertainties under conjecture use of underground and surface water for water resources optimization management. They were able to optimize the total crop yield of the entire irrigation districts. They represent their model equation as:

$$\max F = \sum_{k=1}^{K}\sum_{i=1}^{N}\left[Y^{n}{}_{\max}\prod_{t=1}^{T}\left(\frac{ET_t^{k,n}}{ET_{t\max}^{n}}\right)^{\lambda_t^n} \times A^{k,n}\right] A_i + \sum_{i=1}^{I}\sum_{n=1}^{N}\left[Y^{n}{}_{\max}\prod_{t=1}^{T}\left(\frac{ET_t^{i,n}}{ET_{t\max}^{n}}\right)^{\lambda_t^n} \times A^{i,n}\right] \quad (2.39)$$

where,

$A^{k,n}$ *represnt* irrigated crop area

$ET^{k,n}$ *represent* actual crop evapotranspiration

$ET_{t\max}^{n}$ *represent* potential maximum crop evapotranspiration

λ_i^n *represent* sensitive index of the crop to water stress during the stage

F *represnt* expected crop yields of the entire irrigation district

i *represent* well irrigation district index

K *represent* river irrigation district index

n *represent* crop index

$Y_{\max}^{n}$ *represent* crop maximum yield of crop n under the condition of full irrigation method t *represent* time period index

The crop yields are estimated by deficit irrigation. Crop water production function, they introduced deficit irrigation Jasen model to reflect crop yields linked to sensitivity of water shortage. Their modelled equations are subjected to n constraints. The water balance equations of the reservoirs located in the mountain pass of the rivers.

$$V_t^k = V_{t-1}^k + I_t^k - R_t^k - O_t^k - E_t^k \quad (2.40)$$

V_{t-1}^k *represent* reservoir storage at the beginning of time period t

V_t^k *represent* reservoir storage at the end of time period t

R_t^k *represent* water consumption in the irrigation district

O_t^k *represent* reservoir overflow

E_t^k *represent* reservoir losses

I_t^k *represent* inflow to the reservoir in time period t

Reservoir capacity constraint

$V_t^k \leq C^k$

where;

C^k *represent* effective capacity of reservoir

Multiplying water withdrawal for agriculture by utilization coefficient of irrigation water is irrigation requirement:

$$R_t^k = AW_t^k + DW_t^k \tag{2.41}$$

and

$$IR_t^k = AW_t^k Y^k \tag{2.42}$$

where;

AW_t^k *represent* agricultural water requirement in river irrigation

DW_t^k *represent* non-agricultural water requirement in river irrigation district

$IR_t^k =$ net irrigation water requirement

Y_t^k *represent* utilization coefficient of irrigation water combining the rainfall with water consumption of crops in river irrigation district during growing stages.

Net irrigation water requirement is calculated as follows:

$$IR_t^k = \sum_{n=1}^{N} \left(ET_t^{k,n} - P_t^{k,n} \right) \times A^{k,n} \tag{2.43}$$

where;

$P_t^{k,n}$ *represent* effective precipitation.

When they subtracted seepage and evaporation loss reservoirs overflow, flows into downstream. The measuring data, the river water loss is large in river irrigation district.

$$GO_t = \sum_{k=1}^{K} \left[\alpha E_t^k + \beta RL_t^k + \gamma AW_t^k \left(1 - Y^k\right) \right] + GL_t \tag{2.44}$$

GO_t *represent* recharge of groundwater from river irrigation district

RL_t^k *represent* Loss of river water

GL_t *represent* Lateral groundwater inflow from mountain.

α, β, γ *represent* groundwater recharge coefficient of rivers and agricultural irrigation in well irrigation district.

The ground water quantity balance shown as:

$$VG_t = VG_{t-1} + GO_t + w\sum_{k=1}^{K} RO_t^k + \sum_{i=1}^{I} \left[\gamma Aw_t^i \left(1 - Y^i\right) \right] - \sum_{i=1}^{I} R_t^i - GwD_t - GwO_t \tag{2.45}$$

The result of their analysis indicates that deficit irrigation should be adopted primarily in river irrigation district. The optimization model can reach crop water requirement in each irrigation intervals.

They concluded that the model can provide decision makers with different irrigation water optimal staged allocation schedules of upstream and downstream and agricultural and non-agricultural water resources and so forth.

2.4 Groundwater Aquifer Withdrawal

An Aquifer is defined as a geological material that is capable of transmitting water to well placed in them in sufficient quantities to be consider economically (Idris- Nda, 2013).

Namsik *et al.* (2015), developed optimization model for groundwater withdrawal in the coastal region, they developed a computer model to assess optimal groundwater pumping states in a coastal region, they used sharp interface model to identify the optimal solution. The objective function is stated as follows:

Maximize $\sum_{i=1}^{Nopt} Q_i$, freshwater

(2.46)

where,

Q_i, represent freshwater withdrawal from pumping well

N_{opt} *represent* number of pumping wells where optimization is required

They optimized the objective function by the following constraints:

$R_{\max} < R_t, C_{\max} < C_t, D_{\max} < D_t$

(2.47)

where,

R represents degree of salt water intrusion

D represents the depression of groundwater level

C represents a relevant contaminant concentration in groundwater

Max represents maximum quantity

t indicates a pre – specified allowable value.

They based their mathematical models on two vertically integrated governing equations; one – describing freshwater flow and the other describing salt water flow in an aquifer layer. Their model is in the form:

$$\left.\begin{aligned} &\frac{\partial}{\partial x_i}\left[K_{ijm}^f b_m^f \frac{\partial h_m^f}{\partial x_j}\right] + \underset{T}{\overset{f}{\alpha}} \lambda_{m+1}^t \left(h_{m+1}^f - h_m^f\right) + \alpha_B^f + \gamma_m^f \left(h_{m-1}^f - h_m^f\right) = b_m^f + S_{sm}^f \frac{\partial h_m^f}{\partial t} - \theta \frac{\partial \xi_m}{\partial t} - Q_m^f \\ &\frac{\partial}{\partial x_i}\left[K_{ijm}^s b_m^s \frac{\partial h_m^s}{\partial x_j}\right] + \alpha_T^s \lambda_{m+1}^s \left(h_{m+1}^s - h_m^s\right) + \alpha_B^s \lambda_m^s \left(h_{m-1}^s - h_m^s\right) = b_m^s + S_{sm}^s \frac{\partial h_m^s}{\partial t} + \theta \frac{\partial \xi_m}{\partial t} - Q_m^s \end{aligned}\right\} \qquad (2.48)$$

Where,

Superscripts f represents freshwater

Superscripts s represents saltwater

i,j represents horizontal directions

B represents thickness of the freshwater and saltwater zone in aquifer unit m

K_m^f represents hydraulic conductivities with respect to freshwater

$K_{i,jm}^s$ represents hydraulic conductivities with respect to saltwater

$\lambda_m^l \left(l = f, s\right)$ *represents* leakiness of aquifer unit m

$\lambda_m^l = K_m^l \left(b\right), \alpha_T^l$ *represents* dimension factor indicating the top leakage for the aquifer unit

$\alpha_m^l \left(l = f, s\right)$ *represent* dimension factor indicating the bottom leakage for the aquifer unit

S_m^f represents aquifer specific storage coefficient in the freshwater zone

S_m^s *represent* aquifer specific storage coefficient in the saltwater zone

θ *represent* effective porosity

ξ_m *represents* height of the saltwater – freshwater interface above the datum

Q_m^f *represents* volumetric flux of freshwater due to pumping (or recharge)

Q_m^s *represents* volumetric flux of saltwater due to pumping (or recharge)

$h_{m-1}^l \left(l = f, s\right)$ *represents* Corresponding to the head at the top of underlying aquifer

Meanwhile, they defined the thicknesses of the freshwater and saltwater zones in aquifer (unit m) as:

$$b_m^f = Z_{Tm} - \xi_m \qquad (2.49)$$

$$b_m^s = \xi - Z_{Bm} \quad (2.50)$$

where,

Z_{Bm} *represents* elevation base of the aquifer

Z_{Tm} *represents* elevation top of the aquifer

They determined the interface position as

$$\xi_m = \frac{1}{\xi}\left[\left(P_s \big/ P_f \right) h_m^s - h_m^f \right] \quad (2.51)$$

Where,

P_f = freshwater densities

P_s *represents* saltwater densities

ξ *represents* density difference ratio and was given as

$$\xi = \frac{\left(P_s - P_f\right)}{P_f} \quad (2.52)$$

They used optimization techniques called genetic algorithm, which is designed for unconstrained optimization problems. The penalty function introduced to convert constrained to unconstrained is

Maximize

$$\sum_{i=1}^{Npw} Q_i, \ freshwater - P \quad (2.53)$$

Where, the penalty function

$$P = W_1 R + W_2 D + W_3 C \quad (2.54)$$

W = weighting function for each constrain

$$R = R_{\max} - R_t \qquad if\ R_{\max} < R_t \\ otherwise \tag{2.55}$$

$$D = D_{\max} - D_t \qquad if\ D_{\max} < D_t \\ otherwise \tag{2.56}$$

$$C = C_{\max} - C_t \qquad if\ C_{\max} < C_t \\ otherwise \tag{2.57}$$

$$C_{\max} = Max\left[\frac{Q_j\ saltwater}{Q_j\ freshwater + Q_j\ saltwater}\right] \tag{2.58}$$

Their result shows that the optimal solution is identified and they concluded that a preliminary application to a hypothetical case indicates that the model can be a useful tool for optimal management of groundwater in coastal areas. The model can also be used to design facilities to prevent saltwater intrusion.

Sidiropoulos *et al.* (2016) used an Optimization method to solve groundwater withdrawal problems. They estimate the optimum withdrawal rate, the number of well and locations of irrigation and domestic water supply for the city of Valos have been studied and optimised. The mathematical models for groundwater extraction are formulated as:

$$\sum_{i=1}^{T}\sum_{N=1}^{N} Q^t{}_n \tag{259}$$

Subjected to the following constraints:

$$\left.\begin{array}{l} h^{2044} \geq h^{1987},\ i = 1,...m \\ h^{2044} \geq 0m, i =,1,..,m \\ Q_n \geq 0m^3,\ \ n =,1,..,N \end{array}\right\} \tag{2.59a}$$

where;

Q_n = groundwater withdrawal flow rate of the manage well in m^3 / d

N = the total number of manage wells

T = number of monthly stress period for the management period 2012 – 2044

h = hydraulic head constraints

m = number of the head constraints point

The non-linearity of the objective function is solved by sequential Linearity programming algorithm (SLP). The results of their research work indicated that:

i) none of the two constraints can be satisfied, if the total water supply demand is extracted

ii) the first constraint cannot be satisfied if the domestic water supply well is located and operate in Zone 1 and 2. Zone 1 is the location where the forty water supply wells will be installed according to the Lake Karla restoration project

iii) A total annual volume of 1978 h_m of the renewable groundwater restoration under the first constraints.

They concluded that water table level equal to sea water level.

Christos and Sofia (2015) used mathematical models as a useful tool for the sustainable groundwater management. Their model is based to the numerical solution of the equations, which describes the movement in a confined or unconfined aquifer. Their models are for both confined and unconfined aquifer respectively. The equations are the following:

Confined aquifer:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right)+q'(x,y,t)=S(x,y)\frac{\partial h}{\partial t} \tag{2.60}$$

Unconfined aquifer

$$\frac{\partial}{\partial x}\left(K_x H \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y H \frac{\partial h}{\partial y}\right)+q'(x,y,t)=S_u(x,y)\frac{\partial h}{\partial t} \tag{2.61}$$

where;

T_x, T_y = the transmissivities of aquifer $\left(L^2/T \right)$

h represents the peizometric head (L)

$q'(x,y,t)$ *represents* sink – source term $\left(L/T \right)$

$S(x,y)$ *represents* the aquifer storability

K_x, K_y *represents* hydraulic conductivities of the aquifer $\left(L/T \right)$

H represents aquifer thickness (L)

The model was based on numerical of the partial differential equations that describe the water movement in a confined or unconfined aquifer. They claimed that the model can accept any type of boundary conditions and any type of recharge or discharge scheme. They solved both the equations for confined and unconfined by the alternating direction implicit method. According to this method the solution proceeds in two steps.

1st step:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x} \right)^{n+1/2} + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y} \right)^{n} + q = S_{ij} \frac{\left(h_{i,j} \right)^{n+1/2} - \left(h_{i,j} \right)^{n}}{\Delta t/2} \tag{2.62}$$

2nd step:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x} \right)^{n+1/2} + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y} \right)^{n+1} + q = S_{ij} \frac{\left(h_{i,j} \right)^{n+1} - \left(h_{i,j} \right)^{n+1/2}}{\Delta t/2}$$

(2.63)

where,

superscript n = time step

subscript i,j represents nodal point indexing in x and y directions

They calibrated the following parameters – the hydraulic conductivity of the phreatic aquifer in x and y directions, the transmissivity of the confine aquifer in x and y

directions and the storage coefficient and the inflow from the interval boundaries. They solve the problem minimizing the objective function F:

$$F = \sqrt{\frac{\sum_{n=1}^{Nm}\left(\sum_{k=1}^{Nn}\left(\left(h_k\right)^n{}_m\right)-\left(h_k\right)^n{}_c\right)^2}{\sum_{n=1}^{Nm} N_n}} \qquad (2.64)$$

where,

$\left(h_k\right)^n{}_m$ represents measured piezometric head in each piezometric (k)

$\left(h_k\right)^n{}_c$ *represents* estimated piezometric head in each piezometer (k)

N_m represents Number of monthly piezometers used in nth measurement.

The result pointed out that the calibration process, was terminated when the difference between two subsequent values of the objective function was smaller than a convergence criterion which was set equal to 10^{-9} convergence was reached after 4,058 iterations.

They concluded that the calibrated model was also validated with data collected in the same period. Both calibration and validation process are considered very satisfactory. Therefore, the model can constitute a very good tool for groundwater management of the aquifer.

Bredehoeft *et al.* (1982) presented mathematical framework, for groundwater model which are structured to solve a partial differentia equation. They made it known that, the basic equations of flow through porous material are the subject of a number of lengthy treatises. They listed the following equation of flow:

$$\nabla \cdot \frac{PK}{\mu}\left(\nabla P - \rho g \nabla Z\right) - q = \frac{\partial}{\partial t}\left(\phi P\right) \qquad (2.65)$$

Composition equation

$$\nabla \cdot \left(\rho c \frac{K}{\mu} \left(\nabla P - \rho g \nabla Z \right) \right) + \nabla \cdot \left(\rho E \right) \cdot \nabla c - qc = \frac{\partial}{\partial t} \left(P \phi c \right) \tag{2.66}$$

Internal energy fluid equation

$$\nabla \cdot \left(\frac{PK}{\mu} H \left(\nabla P - \rho g \nabla Z \right) \right) + \nabla \cdot K \cdot \nabla T - q_L - qcpT = \frac{\partial}{\partial t} \left[\phi \rho \upsilon + (1 - \phi)(\rho c P) RT \right]$$

(2.67)

where;

c *represents* concentration, mass fraction

C_p *represents* specific heat

E *represents* dispersion coefficient

g *represents* acceleration due to gravity

H *represents* enthalpy

k *represents* permeability

K = thermal conductivity of the aquifer

P r*epresents* pressure

q *represents* mass rate of production or injection of liquid per unit volume

q_l *represents* rate of heat loss per unit volume

R *represents* refers to rok phase

t *represents* time

T *represents* temperature

υ *represents* internal energy

Z *represents* elevation above a reference plane

ϕ *represents* porosity

ρ *represents* density

μ *represents* viscosity

∇ *represents* gradient

i.e vector of component:

$$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$$

i.e

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right)\text{X}, \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right)\text{y}, \frac{\partial}{\partial z}\left(T_z \frac{\partial h}{\partial z}\right)Z \qquad (2.68)$$

These set of equations or three equations are couple and must be solve simultaneous in many instances, coupling may be negligible and the equation can be treated uncoupled and solve separately.

They further, presented data requirements for digital simulation models, which they described as computer model used to solve a set of two simultaneous partial differential equations. One equation is the equation of flow, which describes the head distribution. In the aquifer, given the head distribution the flow velocity can be calculated by using Darcy's law. The second equation is the solute transport equation, which describes the chemical concentration in the system.

Table 2.3 Summary of data requirement

S/N	System characteristics	Stresses
1.	Transmissivity	Quality of surfac-water inflow
2.	Specific yield	Ground water withdrawal
3.	Saturated thickness	Precipitation
4.	Boundaries	Surface-water application
5.	Effective porosity	Consumptive use
6.	Dispersivity	Recharge (quantity and quality) stream stage

7.	Initial water table elevation	Stream stage
8	Initial dissolved-solids concentration	Tribulatory inflow

They determined the specific yield of the alum from aquifer tests and neutron moisture data to be about 0.20 (Moore and Wood, 1967). This common value for the type of deposited and was assumed to be constant throughout aquifer. No field data were available to determine either the effective porosity or the longitudinal dispersivity of the porous medium.

2.5 An Aquifer System

Melanie (2014) defined an aquifer as a body of rock and sediment that is saturated - water is in it and around it. In addition, water can move through it. It can be made of sand and gravel, sandstone, sandstone and carbonate, and other rocks. Each made up of permeable material. Aquifer can be confined or unconfined, The tidal fluctuation in the surface water causes a sinusoidal fluctuation of the ground water level. The response of ground water levels to sinusoidal boundary condition in a homogenous and isotropic confine aquifer is been studied by analogy to a one – dimensional heat – conduction equation and is given by Todd (1959) as:

$$h(x,t) = h_0 \exp\left(- x\beta \sin\left(\frac{2\pi t}{t_0} - x\beta \right)\right) \tag{2.69}$$

where;

$h(x,t) =$ the rise or fall of piezo metric surface at distance x with respect to a mean surface water level at time (t)

h_0 represents amplitude of the tide

t_0 represents the period of a complete tidal cycle.

β represents physical properties of the aquifer

and is defined as:

$$\beta = \sqrt{\pi s / (t_0 T)} \tag{2.70}$$

where;

s represents the storage coefficient,

T represents transmissivity

$$t_0 = KB \tag{2.71}$$

where;

K represents hydraulic conductivity

B represents thickness of the confined aquifer.

It is assumed that the response of $h(x,t)$ given in (2.71) in a confined aquifer can also be applied to an unconfined aquifer, provided that h_0 is sufficiently smaller than the saturated thickness of the aquifer (Todd, 1959). For simplicity, the saturated depth is denoted by B as well, for unconfined aquifers, S is the specific yield rather than storage coefficient.

Peter *et al.* (1995) developed the scheme for deriving intermediate equations where they consider the propagation of water table waves in one horizontal direction in an unconfined aquifer over horizontal impermeable base at mean depth on Darcy flow assumption (Peter, 1995):

$$u = -k\nabla h^* \qquad (2.72)$$

where;

$U(u,w) =$ discharge per unit area

$$h^* = h^*(x,z,t) = z + \frac{p}{\rho g} \qquad (2.73)$$

h^* represents piezometric head

K represents saturated hydraulic conductivity

The vertical co - ordinate z is reckoned from the impermeable base. Wenke *et al.* (2016) came up with a quantitative analysis of hydraulic interaction process in stream – aquifer systems. It revealed that, both the theoretical and laboratory tests have demonstrated that, the hydraulic connectedness of the stream aquifer system can reach a critical disconnection state. When the horizontal hydraulic gradient at the free water surface is equal to zero and the vertical is equal to one, for simplicity, they assumed that the soil hydraulic conductivity follows the exponential relative conductivity mode:

$$K(h) = k_s \exp(-\beta h) \quad (2.74)$$

where;

h represents soil – water pressure per head

K represents unsaturated water hydraulics conductivity;

β represents an arbitrary constant.

According to Meinzer (1949), Porosity is a dimensionless value that expresses the ratio of the volume of pores to the total volume of a porous material and usually expressed as a percentage:

$$p = \frac{v_p}{v} 100 \quad (2.75)$$

where;

p represents porosity

v_p represents volume of pore space $[L^3]$,

v represents volume of material $[L^3]$,

Primary porosity is attributable to the soil or rock matrix and secondary porosity is attributable to such phenomena as secondary solution or structurally controlled regional fracturing (Freez and cherry, 1979)

According to US Geology Survey (2004), the ratio of openings (voids) to the total volume of a soil or rock referred to as its porosity. Porosity expressed either as a decimal fraction or as a percentage. Thus,

$$n = \frac{v_t - v_s}{v_t} = \frac{v_v}{v_t} \qquad (2.76)$$

where;

n represents porosity as a decimal fraction,

v_t represents total volume of a soil or rock sample,

v_s represents volume of solids in the sample and

v_v represents volume of opening (voids).

2.6 Groundwater Flow

According to Mulligan *et al.* (1998) Groundwater flow in the subsurface is driven by difference in energy water flows from high-energy areas to low energy areas. The energy content of a unit volume of water is determined by the sum of gravitational potential energy, pressure energy, and kinetic energy. Energy per unit volume is given by Mulliga *et al* . (1999) as:

$$\rho gz + p + \frac{\rho v^2}{2}$$

(2.77) where;

ρ represents fluid density

g represents gravitational acceleration,

z represents elevation of the measuring point relative to a datum,

p represents fluid pressure at the measurement point,

v represents fluid velocity.

Because groundwater flows very slowly (on the order of 1 m/d or less), its kinetics energy is very small relative to its gravitational, potential and pressure energies and kinetic energy terms is therefore ignored. By removing the kinetic energy term and rearranging equation (2.77) to express energy in terms of mechanical energy per unit weight, the concept of hydraulic head developed:

Energy per unit weight $=$ hydraulic head $= z + \frac{p}{\rho g}$

(2.78) Ground water therefore flows from region of high hydraulic head to areas of low hydraulic head. Because ground water flows through a porous media, the rate of flow depend on soil properties such as the degree to which pore space are interconnected. The property of interest in ground water flow is the permeability, k, which is a measure of a ease with which a fluid flows through the soil matrix Ground water flow rate can then be calculated using Darcy ' s laws which says that flow rate is linearly proportional to the hydraulic gradient, (Slomp and Van Cappellen, 2004) :

$$q = -\frac{\rho gak}{\mu}(\nabla h) \qquad (2.79)$$

where;

q represents the Darcy flux, or flow rate per unit surface area,

μ represents fluid viscosity.

A more general of Darcy's law is:

$$q = \frac{k}{\mu}(\nabla p + \rho g \nabla z) \qquad (2.80)$$

In inland aquifers, the density of ground water is constant and (2.80) reduced to the simpler form of Darcy's law that is (2.79) in coastal aquifers, however, the presence of saline water along the coast means that the assumption of constant density is not valid and so the more inclusive form of Darcy, s law that is (2.80) is required.

Flecher (2005) explained that, the first experimental study of groundwater flow was performed with water, through porous media and is proportional to the cross-sectional area and to the head loss along the path, and inversely proportional to the length of the flow path Darcy's Law can be expressed as:

$$Q = -kA\frac{dh}{dl} \tag{2.81}$$

where;

Q represents volumetric discharge of water ($L^{3.}T^{-1}$)

K represents hydraulic conductivity $\left(LT^{-1}\right)$

A cross sectional area $\left[L^2\right]$

$\frac{dh}{dl}$ represents gradient of hydraulic head $\left[LL^{-1}\right]$

Song-Bae *et al.* (2003) developed a mathematical model for the transport of hydrophobic organic contaminants in an aquifer under simplistic riverbank filtration conditions. The model considers a situation where contaminants are present together with dissolved organic matter (DOM) and bacteria.

Abiola *et al.* (2009) conducted a study on groundwater potential and aquifer protective capacity of the overburden units in Ado – Ekiti, Ekiti state. The study revealed that aquifer protective capacity characterization based on the values of the longitudinal unit conductance of the rock matrix in the area. From their analysis it was found that the thickness of the top soil appeared responsible for the observed overlapping resistivity across the study area, and the higher the clay content, the lower the ground water yield, this conforms to the characteristics of\ clay as an aquitard.

Shehu *et al.* (2017) stressed that in minning exacavation as well as many engineering operations, groundwater situation in an area under operation should be characterized and considered in design of the operation. Water shortages occurequiteoftenin many area of the world, calling for optimal managementof both surface and groundwater resources.

2.7 Groundwater Recharge

Oluseyi *et al.* (2015) developed estimation of groundwater recharge using empirical formula in Odeda local government area of Ogun state, Nigeria. They determined groundwater and groundwater recharge coefficient through a case study using empirical methods applicable to the tropical zone. They collected the related climatological data between January 1983 and December 2014 in Ogun Osun River Basin Development Authority (OORBDA) Ogun State Nigeria.

The model for estimation of groundwater recharge of the study area was conducted using a modified version for tropical regions based on water level fluctuation and rainfall depth, thus the equation is given as

$$R = 1.35\left(p - 14\right)^{0.5} \tag{2.82}$$

Where R = net recharge due to precipitation in inches

P represents precipitation in inches

The recharge coefficients equation is given as the ratio of recharge to effective rainfall and is expressed in percentage as

$$R_{coefficient} = \frac{R}{P_c}\% \tag{2.83}$$

R represents recharge

P_c represents effective rainfall

The estimated runoff for water budget equation is given as

$$R_{off} = 0.85 \times P - 30.5$$

where,

R_{off} represents directed runoff

P represents precipitation

They used co-integration analysis which involves a unit root test performed on levels the first difference and the second difference to determine whether the individual input series and stationary and exhibit and similar statistical properties.

The augmented Dickey – fuller (ADF) test was used to test for the stationary of the data, the test consists the following regression.

$$\Delta Y_{\tau} = \beta_1 + \beta_{2\tau} + \delta Y_{\tau - Y} \alpha \sum_{\tau=1}^{n} \Delta Y_{\tau} + \varepsilon_{\tau} \quad (2.84)$$

where;

$$\left. \begin{array}{l} \varepsilon_{\tau} = \Delta Y_{t-1} = \left(Y_{t-1} - Y_{t-2} \right) \\ \Delta Y_{t-2} = \left(Y_{t-2} - Y_{t-3} \right) \end{array} \right\}$$

(2.85)

The co-integration model is given as

$$P_t = \beta_o + \beta_1 R_1 + \beta_2 SR_t + \beta_4 MT_t + \beta_5 XT_t + \beta_6 ET_t + U_t \quad (2.86)$$

They used Pearson correlation coefficient to evaluate the strength of the relationship between meteorological factors. The impact of the independent variables i.e, humidity, temperature, rainfall, wind speed and duration of daily solar radiation on estimate recharge was evaluated using linear regression. The result shows that groundwater recharge was 194.7mm per year, evapotranspiration was 1296.2mm per year and the recharge coefficient was 20.2% for the study area. The result shows that about 11% of rainfall infiltrated the aquifer, 73% was loss to evapotranspiration and 36% ended up as runoff. Correlation between recharge and rainfall, temperature, humidity, solar radiation and evapotranspiration were at the 0.01 significance level and the results of linear regression prove that precipitation has a significant effect (with R^2 = 0.983) on estimated recharge.

Behrouz *et al.* (2013) solved the linear form of a one dimensional Boussinesq equation and an analytical mathematical model was developed to estimate the water table profile between two parallel subsurface drains. The model they developed is a generalisation of the Glover –Dumm's mathematical model, as a result the new model is applicable for both homogenous and heterogeneous soil.

2.8 Crop Water Requirement

The determination of crop coefficient coefficient and reference crop evapotranspiration are important for estimating irrigation water requirement of any crop in order to have a better irrigation scheduling and water management (Falguni, 2013).

Following Sidiripoulos *et al.* (2016) in optimization of groundwater withdrawal in region we observed the following gaps:

(i) They consider groundwater withdrawal for domestic supply

(ii) They did not consider aquifer yield

(iii)Non considering of distance between well as an applied constraint

Following Someyah *et al.* (2014), we observed the following limitations:

(i) they did not consider deficit irrigation

(ii) they considered crop price without irrigation cost

(iii) they did not consider soil or water salinity as an applied constraint

(iv) The objective function which maximizes the total farm income is only considered for the optimal operation of the reservoir

(v) Number of crops planted is not inclusive in the objective function

(vi) Irrigation /farm water needs which would have depended on reservoir capacity to determine crop types is not consider.

In addressing these limitations, we considered the following:

(i) mass balance equation for estimation of the groundwater quantity available in Badeji, Bida, Lapai and Mokwa/ Kudu irrigation sites;

(ii) specific yield of an unconfined and semi confine aquifer as an applied constraint in which specific yield is compared to optimal withdrawal rate;

(iii) deficit irrigation;

(iv) Potential energy for groundwater withdrawal;

(v) Aquifer yield;

(vi) Crop yield;

(vii) Irrigation water supply;

(viii) Maximum and minimum aquifer discharge;

(ix) Irrigation area of the land A_i

(x) We considered dual crop coefficient where $K_c = (K_{cb} + K_e)$

These distinguish the researches theses from the earlier researches on Mathematical Model on irrigation scheduling incorporating aquifer withdrawal.

CHAPTER THREE

3.0 MATERIAL AND METHODS

3.1 Map of the Study Area

Bida Basin lies in the sedimentary terrain of the middle part of Nigeria. It has an area of coverage of about 27,000 km^2. The area falls under middle climatic belt which is mainly tropical with average rainfall of about 1250m. We are therefore considering aquifer withdrawal of two lithological groups: confined and semi- confined aquifer in our selected study area. The map of the selected study area is therefore shown below.

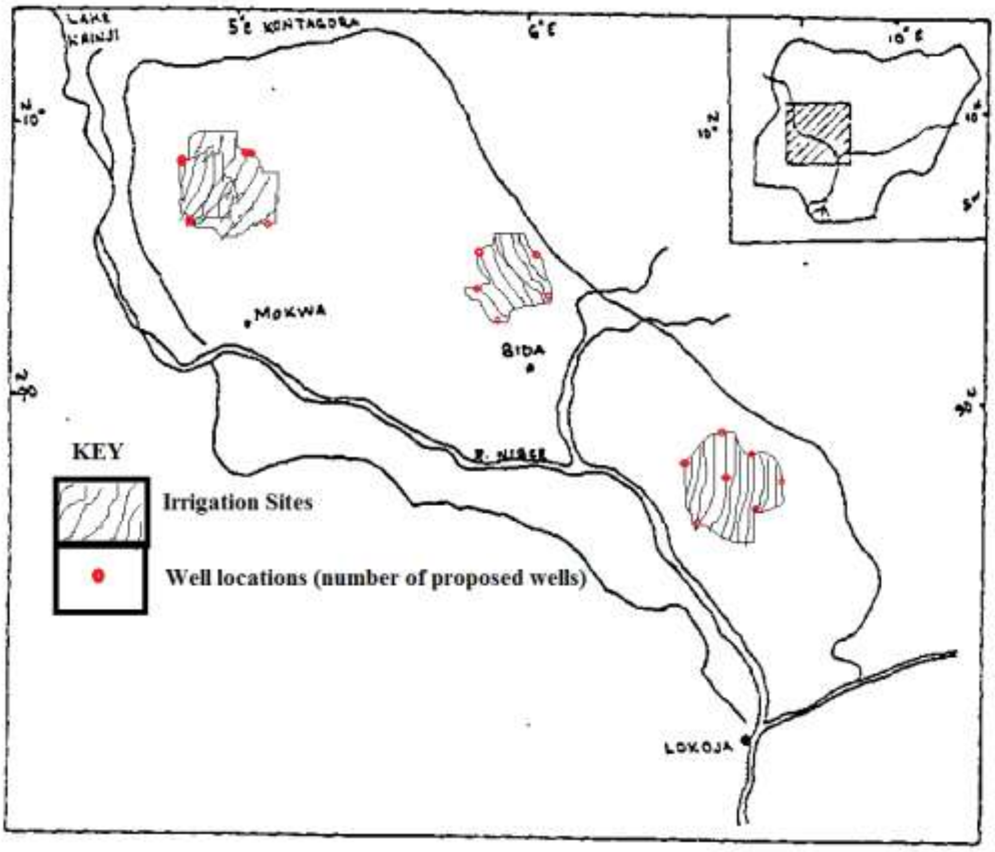

Figure 3.1 Irrigation Sites in Bida Basin (Idris-Nda, 2013)

The Figure 3.1 shows the map of the study area showing three irrigation sites in Bida (Badeji), Mokwa (kudu) and Lapai.

3.2 Groundwater Estimate

In this section groundwater level will be estimated during the Dry season (November to April), at the three different irrigation sites within Bida basin.Considering equation (3.1) below.

$$I = J \pm S\frac{dh}{dt}$$

(3.1)

where,

$I =$ rainfall (input)

$J =$ evapotranspiration (output).

$h =$ groundwater level

$t =$ time

$\frac{dh}{dt} =$ change in groundwater level with respect to time

$S = Storavity$

$S\frac{dh}{dt} =$ storage change of groundwater with respect to time.

For period without recharge, the groundwater level is expressed as:

$$\frac{dh}{dt} = c\left(h - h_0\right)$$

(3.2)

where,

$h_o =$ standard groundwater level

$c =$ constant

Integrate equation (3.2), we have

$$\int \frac{dh}{(\mathrm{h}-\mathrm{h}_0)} = \int cdt$$

(3.3)

$$e^{\ln(h-h_o)} = ct + k$$

(3.4)

$$h - h_0 = e^{ct}.\mathrm{e}^{k}$$

(3.5)

$$h = h_0 + Be^{ct}$$

(3.6)

where;

B = constant of the integration

Substituting $c(h-h_0)$ in equation (3.2) into (3.1) we have:

$$I = J \pm Sc(h-h_0)$$

(3.7)

suppose I = 0, in equation (3.7), then equation (3.7) becomes

$$J + Sc(h-h_0) = 0$$

(3.8)

Simplifying (3.8) we have,

$$J = -sc(h-h_0)$$

(3.9)

Considering the period of dry season under which our research is based, we adopt equation (3.9) and the computation of equation (3.9) is done using computer algebraic software package MATLAB. We have also used five climatic data namely: Relative humidity, evaporation, maximum and minimum Temperature hours of sunshine to trained and test the equation (3.9). The evaluation results for groundwater estimation during the dry season in the three different irrigation sites are presented in the next chapter and the algorithm design for (3.9) has been moved to appendix C.

The next model is formulated and solved to determine the optimum quantity of the groundwater to be drawn from each well located in each of the selected irrigation site.

3.3 Mathematical Model for Aquifer Withdrawal (Rate of Discharge)

Irrigation Famers are faced with the challenges of knowing the quantity of the groundwater needed for crops during the dry season, thus, the knowledge of the groundwater management is necessary for irrigation farming.The aim here is to extend on Sidiropolous et al. (2016) work for the optimum aquifer withdrawal for irrigation purpose using genetic algorithm

3.3.1 Model assumptions

The following are the groundwater assumptions:

- The groundwater withdrawal was estimated based on the weather effect in the area.
- The groundwater recharges from the irrigated water.
- Aquifer is compressible (stored water).
- Water table change during the withdrawal
- Aquifer bounded on top by aquitard and semi – confined aquifer

Based on the assumption made above and following Sidiropoulos *et al.* (2016) the groundwater irrigation withdrawal rate is computed as follows:

we let,

$S_{y_{i,j}}$ be a specific yield and $Q_{i,j}$ be a discharge rate. Considering energy required to drawn groundwater in each of the irrigation site, our energy is given as;

$$E_p = \frac{i\rho g A h_s}{eff_p},\tag{3.13}$$

E_p = potential energy

g = gravity, m/ $\sec^2$

h_s = withdrawal pressure m^2

ρ = density kg/ m^2

i = aquifer depth applied, m

A=irrigation field area, m^2

eff_p = efficiency of the withdrawal, diameter, dimensionless

then, from equation (3.6) we let,

$$Be^{ct} = h - h_0 = a_i \left(Q_{i,j}\right)^2 + b_i Q_{i,j} + c_i$$

(3.14)

where,

a_i, b_i = efficiency coefficients.

c = constant

h = groundwater level

h_0 = standard groundwater level

B = constant of the integration in equation (3.6)

e^{ct} = the quantity of groundwater available during the dry season in equation (3.6)

considering potential energy and groundwater withdrawal rate then;

$$Max\,Z = \frac{i\rho g A h_s}{eff_p} \sum_{i=1}^{npw} \sum_{j=1}^{n} \frac{Q_{i,j} S_{y_{i,j}}}{h - h_0}\tag{3.15}$$

From equation (314), equation (3.15) becomes;

$$Max\,Z = \frac{i\rho gAh_s}{eff_p}\sum_{i=1}^{npw}\sum_{j=1}^{n}\frac{Q_{i,j}S_{y_{i,j}}}{a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i} \tag{3.17}$$

Z = Objective function subject to the following constraints:

$$Q_{\min} \le Q_{i,j} \le Q_{\max}, \tag{3.18}$$

where;

$Q_{\min} = 0$

then,

$0 \le Q_{i,j} \le Q_{\max}$

$$\sum_{i=1}^{npw} Q_{i,j} = Q_d \tag{3.19}$$

where,

$Q_{i,j}$ = minimum aquifer discharge rate of well i at time j

Q_d = Irrigation water demand

$Q_{\max}$ = maximum aquifer discharge rate of well i at time

using Lagrange method of solution, we have:

$$\varphi = \sum_{i=1}^{npw}\sum_{j=1}^{n}\frac{Q_{i,j}S_{y_{i,j}}}{a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i} + \lambda_1\left(\sum_{i=1}^{npw}Q_{i,j} - Q_D\right) + \lambda_2\left(Q_{i,j} + x_i^{\,2} - Q_{\max}\right) \tag{3.20}$$

where,

$\lambda_1, \lambda_2 =$ rate of the change of the quality being optimised

Taking the derivative of equation (3.20) with respect to $Q_{i,j}$ we have:

$$\frac{\partial\varphi}{\partial Q_{i,j}} = 0 \Rightarrow \frac{\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)S_{y_{i,j}} - \left(2a_iQ_{i,j} + b_i\right)Q_{i,j}S_{y_{i,j}}}{\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)^2} + \lambda_1 + \lambda_2 = 0 \tag{3.21}$$

simplifying (3.21) we have:

$$\frac{a_i\left(Q_{i,j}\right)^2 S_{y_{i,j}} + b_i Q_{i,j} S_{y_{i,j}} + c_i S_{y_{i,j}} - 2a_i\left(Q_{i,j}\right)^2 S_{y_{i,j}} - b_i Q_{i,j} S_{y_{i,j}}}{\left(a_i\left(Q_{i,j}\right)^2 + b_i Q_{i,j} + c_i\right)^2} + \lambda_1 + \lambda_2 = 0 \quad (3.22)$$

simplifying (3.22) we have:

$$\frac{c_i S_{y_{i,j}} - a_i\left(Q_{i,j}\right)^2 S_{y_{i,j}}}{\left(a_i\left(Q_{i,j}\right)^2 + b_i Q_{i,j} + c_i\right)^2} + \lambda_1 + \lambda_2 = 0 \quad (3.23)$$

taking the derivative of equation (3.20) with respect to λ_1 we have:

$$\frac{\partial \varphi}{\partial \lambda_1} = 0 \Rightarrow \sum_{i=1}^{npw} Q_{i,j} - Q_d \quad (3.24)$$

Taking the derivative of equation (3.20) with respect to λ_2 we have:

$$\frac{\partial \varphi}{\partial \lambda_2} = 0 \Rightarrow Q_{i,j} + x^2 - Q_{\max} = 0 \quad (3.25)$$

taking the derivative of equation (3.20) with respect to x_2 we have:

$$\frac{\partial \varphi}{\partial x} = 0 \Rightarrow 2\lambda_2 x = 0 \quad (3.26)$$

from equation (3.26),

$\lambda_2 = 0, \; or \; x = 0$

(3.27)

Substitute $x = 0$ in equation (3.25) we have:

$$Q_{i,j} - Q_{\max} = 0 \quad (3.28)$$

simplifying (3.28)

$$Q_{i,j} = Q_{\max} \quad (3.29)$$

From (3.23),

$$\frac{\left(c_i - a_i\left(Q_{i,j}\right)^2\right)S_{y_{i,j}}}{\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)^2} + \lambda_1 + \lambda_2 = 0$$

(3.30)

Simplifying (3.30) we have:

$$\left(c_i - a_i\left(Q_{i,j}\right)^2\right)S_{y_{i,j}} + \lambda_1\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)^2 + \lambda_2\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)^2 = 0 \quad (3.31)$$

Substituting $\lambda_2 = 0$ in (3.31) we have;

$$\left(c_i - a_i\left(Q_{i,j}\right)^2\right)S_{y_{i,j}} + \lambda_1\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)^2 = 0 \quad (3.32)$$

Simplifying (3.34) we have:

$$\lambda_1\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)^2 = \left(a_i\left(Q_{i,j}\right)^2 - c_i\right)S_{y_{i,j}} \quad (3.33)$$

equation (3.33) yield

$$\lambda_1 = \frac{\left(a_i\left(Q_{i,j}\right)^2 - c_i\right)S_{y_{i,j}}}{\left(a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i\right)^2} \quad (3.34)$$

From equation (3.29),where, $Q_{i,j} = Q_{\max}$ substitute $Q_{i,j} = Q_{\max}$ into equation (3.34)

$$\lambda_1 = \frac{\left(a_i\left(Q_{\max}\right)^2 - c_i\right)S_{y_{i,j}}}{\left(a_i\left(Q_{\max}\right)^2 + b_iQ_{\max} + c_i\right)^2} \quad (3.35)$$

and $\lambda_2 = 0$ from equation (3.27) then, substitute for λ_1 equation (3.20) we have:

$$Max\,Z = \frac{i\rho gAh_s}{eff_p}\sum_{i=1}^{npw}\sum_{j=1}^{n}\frac{Q_{i,j}S_{y_{i,j}}}{a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i} + \frac{\left(a_i\left(Q_{\max}\right)^2 - c_i\right)S_{y_{i,j}}}{\left(a_i\left(Q_{\max}\right)^2 + b_iQ_{\max} + c_i\right)^2}\left(\sum_{i=1}^{npw}Q_{i,j} - Q_d\right)$$

(3.36)

where,

$$\varphi = \sum_{i=1}^{npw}\sum_{j=1}^{n}\frac{Q_{i,j}S_{y_{i,j}}}{a_i\left(Q_{i,j}\right)^2 + b_iQ_{i,j} + c_i} + \frac{\left(a_i\left(Q_{\max}\right)^2 - c_i\right)S_{y_{i,j}}}{\left(a_i\left(Q_{\max}\right)^2 + b_iQ_{\max} + c_i\right)^2}\left(\sum_{i=1}^{npw}Q_{i,j} - Q_d\right) \qquad (3.37)$$

If the government want to do the project that will benefit the people in the areas considered, then there is need to consider the net benefit and interest rate, we let:

$$\frac{p}{f} = \frac{1}{(1+r)^n} \qquad (3.38)$$

then,

$$B_{total} = B_pQ_{i,j} \qquad (3.39)$$

$$C_{total} = C_pQ_{i,j} \qquad (3.40)$$

where.

B_{total} = total benefit

C_{total} = total cost

f = investment

p = cash flow

C_p = cost of groundwater withdrawal rate a unit volume per unit Head at well point i

B_p = benefit per unit supply of water at well point i

$Q_{i,j}$ = minimum aquifer discharge of well i at time j

The net benefit is given as

$$NB = \sum_{i=1}^{npw}\sum_{n=1}^{k}\frac{1}{(1+r)^n}\left(B_pQ_{i,j} - C_pQ_{i,j}\right) \qquad (3.41)$$

NB = net benefit

n = projects number of years (duration)

npw = number of pumping well

$$Q_{\max} = \left(\frac{\pi\left(R^2 + 4X_{\max}^2\right)\left(-R^2 - 2X_{\max}^2 - 2 + A\right)}{R^2 + 4X_{\max}^2 - A} \right) \frac{qy}{L} \quad (3.42)$$

Source: (Shehu *et al.*, 2012);

where;

$X_{\max}$ = aquifer withdrawal rate

$Q_{\max}$ = maximum flow rate for well i

A = depth of the mean sea level from the bed

qy = aquifer recharge through the irrigation

L = distance between the well

R = maximum withdrawal rate

$$A = \sqrt{4 + R^2}\sqrt{4X_{\max}^2 + R^2} \quad (3.43)$$

Source: (Shehu *et al.*, 2012);

then from equation (3.29) where $Q_{i,j} = Q_{\max}$

$$NB = \sum_{i=1}^{npw}\sum_{n=1}^{k} \frac{1}{(1+r)^n}\left(B_p Q_{\max} - C_p Q_{\max}\right) \quad (3.44)$$

This further simplifies to

$$NB = npw\left(-\frac{Q_{\max}\left(B_p - C_p\right)\left(\frac{1}{(1+r)}\right)^{k+1}(1+r)}{r} \right) + \frac{Q_{\max}\left(B_p - C_p\right)}{r} \quad (3.45)$$

where,

k = maintainance period

Substituting (3.42) and (3.43) into (3.44) we have:

$$NB = npw\left(-\frac{\pi\left(R^2+4X_{\max}^2\right)\left(-R^2-2X_{\max}^2-2+\sqrt{4+R^2}\sqrt{4X_{\max}^2+R^2}\right)qy\left(B_p-C_p\right)\left(\frac{1}{(1+r)}\right)^{k+1}(1+r)}{\left(R^2+4X_{\max}^2-\sqrt{4+R^2}\sqrt{4X_{\max}^2+R^2}\right)Lr} \right) + \frac{\pi\left(R^2+4X_{\max}^2\right)\left(-R^2-2X_{\max}^2-2+\sqrt{4+R^2}\sqrt{4X_{\max}^2+R^2}\right)qy\left(B_p-C_p\right)}{\left(R^2+4X_{\max}^2-\sqrt{4+R^2}\sqrt{4X_{\max}^2+R^2}\right)Lr} \qquad (3.46)$$

simplifying (3.46) yields;

$$NB = \frac{npw\pi yq\left(B_p-C_p\right)\left(\left(\frac{1}{1+r}\right)^k-1\right)}{\left(\sqrt{R^2+4}\sqrt{R^2+4X_{\max}^2}-R^2-4X_{\max}^2\right)Lr} \qquad (3.47)$$

Equation (3.46) can be used to determine the amout of groundwater to be drawn for the each well in irrigation field and equation (3.47) describes the values of net Benefit for fouteen well system.

3.4 Mathematical Model for Crop Water Requirement

Crops water need for two crops namely: Soya bean and Rice are considered as our samples in the three-irrigation field within Bida Basin.

3.4.1 Mathematical derivation of reference crop water need (ET_o)

With respect to an energy balance at the earth surface which equates all incoming and outgoing energy flux, the following governing equation are considered;

$$R_n = H + \lambda E + G \qquad (3.48)$$

(*F.A.O.*, 1998)

where,

R_n = energy flux density net incoming radiation (w/m^2),

H= flux density of latent heat into the air (w/m^2),

λE =flux density into the water body (w/m^2), G = heat flux density into the water body (w/m^2),

λ = the latent heat of vaporization of water,

E = the vapour flux density in kg/m^2s,

where,

H and λE from equation (3.48) is given as:

$$H = C_1 \frac{(T_s - T_a)}{r_a} \tag{3.49}$$

and

$$\lambda E = C_2 \frac{(e_s - e_d)}{r_a} \tag{3.50}$$

where;

C_1, C_2 = are constants

T_s =temperature at a certain height above the surface (kpa),

e_a = prevailing vapour pressure at the same height as T_a (kpa),

r_a = aerodynamic diffusion resistance,

Applying the similarity of transport heat and water vapour, we have;

$$\frac{H}{\lambda E} = \frac{C_1(T_s - T_a)}{C_2(e_s - e_d)} \tag{3.51}$$

Equation (3.51) becomes;

$$\frac{H}{\lambda E} = \gamma \left(\frac{T_s - T_a}{e_s - e_d} \right)$$

(3.52)

from equation (3.51)

$$\frac{c_1}{c_2} = \gamma$$

(3.53)

from equation (3.52)

$$\gamma = \frac{C_p \rho_a}{\lambda E}$$

(3.54)

making C_p the subject of the relation in (3.54) we have:

$$C_p = \frac{\gamma \lambda E}{\rho_a}$$

(3.55)

from equation (3.55)

$$\rho_a = \frac{P_a}{T_{vk} R}$$

(3.56)

where,

T_{kv} = the virtual temperature,

R = specific gas constant,

ρ_a = mean air density at constant pressure kgm^{-3},

C_p = specific heat at constant pressure $mj\, kg^{-1}\, {}^0C^{-1}$,

Equation (3.52) is referred to as Psychometric constant $(kpa / °C)$,

In determining the surface Temperature, we consider the Pen-man equation which is given as:

$$e_s - e_a = \Delta(T_s - T_a) \qquad (3.57)$$

Making Δ the subject of relation in (3.57) we have;

$$\Delta = \frac{e_s - e_a}{T_s - T_a} \tag{3.58}$$

From equation (3.57)

$$T_s - T_a = \frac{e_s - e_a}{\Delta} \tag{3.59}$$

Substituting equation (3.59) into equation (3.52), we have:

$$\frac{H}{\lambda E} = \frac{\gamma}{\Delta}\left(\frac{e_s - e_a}{e_s - e_d}\right) \tag{3.60}$$

Replacing $e_s - e_a$ by $e_s - e_d - e_a + e_d$ in (3.60) Equation (3.60) becomes

$$\frac{H}{\lambda E} = \frac{\gamma}{\Delta}\left(\frac{e_s - e_d - e_a + e_d}{e_s - e_d}\right) \tag{3.61}$$

Simplifying equation (3.63) to obtain

$$\frac{H}{\lambda E} = \frac{\gamma}{\Delta}\left(\frac{e_s - e_d}{e_s - e_d} - \frac{e_a - e_d}{e_s - e_d}\right)$$

(3.62)

equation (3.62) yield:

$$\frac{H}{\lambda E} = \frac{\gamma}{\Delta}\left(1 - \frac{e_a - e_d}{e_s - e_d}\right)$$

(3.63)

considering isothermal evaporation λE_a is giving as

$$\lambda E_a = C_2 \frac{e_s - e_d}{r_a}$$

(3.64)

Setting $\lambda = 1$ in equation (3.64), then equation (3.64) becomes

$$E_a = C_2 \frac{e_s - e_d}{r_a}$$

(3.65)

replacing C_2 by $\frac{\varepsilon\rho_a}{p_a}$ in equation (3.65) we have

$$E_a = \frac{\varepsilon\rho_a}{p_a}\left(\frac{e_s - e_d}{r_a}\right) = \frac{\varepsilon p_a}{p_a}\frac{e_a - e_d}{r_c} = \frac{\varepsilon p_a}{p_a}\frac{e_0 - e_d}{r_c + r_a}$$

(3.66)

dividing (3.64) by (3.49) and taking the fist term in equation (3.66) wehave

$$\frac{\lambda E_a}{\lambda E} = \frac{C_2\left(\frac{e_a - e_d}{r_a}\right)}{C_2\left(\frac{e_s - e_d}{r_a}\right)}$$

(3.67)

equation (3.67) yield

$$\frac{E_a}{E} = \frac{(e_a - e_d)}{(e_s - e_d)}$$

(3.68)

substituting equation (3.68) into equation (3.63), we have:

$$\frac{H}{\lambda E} = \frac{\gamma}{\Delta}\left(1 - \frac{E_a}{E}\right)$$

(3.69)

replacing E by ET_0 in equation (3.69), yield

$$\frac{H}{\lambda ET_0} = \frac{\gamma}{\Delta}\left(1 - \frac{E_a}{ET_0}\right)$$

(3.70)

making H the subject of relation in equation (3.70), we have

$$H = \frac{\lambda\gamma ET_0}{\Delta}\left(1 - \frac{E_a}{ET_0}\right)$$

(3.71)

From equation (3.48) which is

$$H = R_n - \lambda E - G \qquad (3.72)$$

substituting equation (3.71) for H in equation (3.72), we have:

$$\frac{\lambda\gamma ET_0}{\Delta}\left(1 - \frac{E_a}{E}\right) = (R_n - G) - \lambda ET_0$$

(3.73)

equation (3.73) yield

$$\frac{\lambda\gamma ET_0}{\Delta} - \frac{ET_0\lambda\gamma E_a}{\Delta ET_0} = (R_n - G) - \lambda ET_0$$

(3.74)

$$\Rightarrow \frac{\lambda\gamma ET_0}{\Delta} - \frac{\lambda\gamma E_a}{\Delta} = (R_n - G) - \lambda ET_0$$

(3.75)

$$\Rightarrow \frac{\lambda\gamma ET_0}{\Delta} + \lambda ET_0 = (R_n - G) + \frac{\lambda\gamma E_a}{\Delta}$$

(3.76)

$$\Rightarrow \lambda ET_0\left(\frac{\gamma}{\Delta} + 1\right) = \frac{(R_n - G)\Delta + \lambda\gamma E_a}{\Delta}$$

(3.77)

$$\Rightarrow \lambda ET_0\left(\frac{\gamma + \Delta}{\Delta}\right) = \frac{(R_n - G)\Delta + \lambda\gamma E_a}{\Delta}$$

(3.78)

$$\Rightarrow ET_0\left(\frac{\gamma + \Delta}{\Delta}\right) = \frac{\frac{(R_n - G)}{\lambda}\Delta + \frac{\lambda\gamma E_a}{\lambda}}{\Delta}$$

(3.79)

$$\Rightarrow ET_0\left(\frac{\gamma+\Delta}{\Delta}\right)=\frac{\frac{(R_n-G)}{\lambda}\Delta+\gamma E_a}{\Delta}$$

(3.80)

$$\Rightarrow \quad ET_0=\frac{\frac{(R_n-G)}{\lambda}\Delta+\gamma E_a}{\Delta}.\frac{\Delta}{\gamma+\Delta}$$

(3.81)

$$\Rightarrow ET_0=\frac{\frac{1}{\lambda}\left(R_n-G\right)\Delta+\gamma E_a}{\Delta+\gamma}$$

(3.82)

where;

ET_0 =open water evaporation rate (kg/m^2),

Δ = proportionality constant $(kpa/^o c)$,

R_n =net radiation (J/kg),

γ =Psychometric constant $(kpa/^o c)$,

E_a = isothermal evaporation rate (kg/m^2s).

the term $\frac{\frac{1}{\lambda}\left(R_n-G\right)\Delta}{\Delta+\gamma}$ in equation (3.82) is the radiation term.

the term $\frac{\gamma E_a}{\Delta+\gamma}$ in equation (3.82) is the aerodynamic term.

substituting equation (3.54) and (3.66) into equation (3.82) yields

$$ET_0=\frac{\frac{(R_n-G)}{\lambda}\Delta+\frac{C_pP_a}{\lambda\varepsilon}\frac{\varepsilon\rho_a}{P_a}\left(\frac{e_s-e_d}{r_a}\right)}{\gamma+\Delta}.$$

(3.83)

Simplifying equation (3.83) we obtain

$$ET_0 = \frac{\frac{(R_n - G)}{\lambda}\Delta + \frac{C_p\rho_a}{\lambda}\left(\frac{e_s - e_d}{r_a}\right)}{\gamma + \Delta}.$$

(3.84)

equation (3.84) yield

$$ET_0 = \frac{\frac{1}{\lambda}\left((R_n - G)\Delta + \frac{C_p\rho_a}{r_a}\left(e_s - e_d\right)\right)}{\left(\gamma + \Delta\right)}$$

(3.85)

where,

ε =ratio of molecular masses of water vapour to dry air (-),

p_a =density of moist air (kg/m^3),

ρ_a =atmospheric pressure (kpa),

c_p =Specific heat of dry air at constant pressure $(J/Kg.k)$.

equation (3.66) can be expressed as

$$E_a = \frac{\varepsilon p_a}{p_a}.\frac{e_a - e_d}{r_a} = \frac{\varepsilon p_a}{p_a}\frac{e_a - e_d}{r_c} = \frac{\varepsilon p_a}{p_a}\frac{e_0 - e_d}{r_c + r_a}$$

(3.86)

Simplifying equation (3.86), we have;

$$\frac{\varepsilon p_a}{p_a}\frac{e_a - e_d}{r_a} = \frac{\varepsilon p_a}{p_a}\frac{e_0 - e_d}{r_c + r_a}$$

(3.87)

$$\Rightarrow \frac{e_a - e_d}{r_a} = \frac{e_0 - e_d}{r_c + r_a}$$

(3.88)

divide through by r_a, we have:

$$e_a - e_d = \left(\frac{\frac{e_0 - e_d}{r_a + r_c}}{r_a} \right)$$

(3.89)

$$\Rightarrow e_a - e_d = \left(\frac{e_0 - e_d}{1 + \frac{r_c}{r_a}} \right)$$

(3.90)

where,

E_a =Isothermal evapotranspiration rate from canopy,

e_o =Internal saturated vapour pressure at (cp_a),

e_a =Saturated vapour pressure at the leaf surface,

e_d =Vapour pressure in the external air,

r_a =aerodynamic resistance (s/m),

r_c =Canopy diffusion resistance (s/m).

Substituting equation (3.90) into equation (3.85) we have

$$ET_0 = \frac{\frac{1}{\lambda}\left((R_n - G)\Delta + \frac{C_p \rho_a}{r_a} e_0 - e_d \right)}{\Delta + \gamma\left(1 + \frac{r_c}{r_a}\right)}$$

(3.91)

Let $$\gamma\left(1 + \frac{r_c}{r_a}\right) = \gamma^*$$

(3.92)

then equation (3.92) becomes

$$ET_0 = \frac{\frac{1}{\lambda}\left((R_n - G)\Delta + \frac{C_p \rho_a}{r_a} e_0 - e_d\right)}{\Delta + \gamma^*}.$$

(3.93)

where,

ET_o =evaporation rate from dry surface $(kg/m^2 s)$,

γ^* =Modified Psychometric constant $(kpa/^o c)$.

Substitute equations (3.55) and (3.56) into equation (3.93), we have

$$ET_0 = \frac{\frac{1}{\lambda}\left((R_n - G)\Delta + \frac{\frac{\gamma\varepsilon\lambda}{P_a}\frac{P_a}{T_{vk}R}}{r_a} e_0 - e_d\right)}{\Delta + \gamma^*}.$$

(3.94)

Simplifying equation (3.94) we obtained

$$ET_0 = \frac{\left(\frac{1}{\lambda}(R_n - G)\Delta + \frac{\gamma\varepsilon}{r_a T_{vk} R} e_0 - e_d\right)}{\Delta + \gamma^*}$$

(3.95)

where,

T_{kv} = the virtual temperature,

R = specific gas constant,

ρ_a = mean air density at constant pressure kgm^{-3},

C_p = specific heat at constant pressure $mj\,kg^{-1}\,{}^0C^{-1}$,

R_n = net radiation of the crop surface (w/m^2),

G = leaf flux density to the soil (w/m^2) zero for period of 10-30 days.

λ = latent heat of vapourization J/kg value 2.45×10^6.

From equation (3.95) with the following standard values r_a can be expressed as

$$r_a = \frac{\ln\frac{(z-d)}{z_{om}}\ln\frac{(z-d)}{z_{ov}}}{k^2u^2} \tag{3.96}$$

where,

Z = height at which wind speed is measured (m),

d = displacement height (m),

z_{om} = roughness length for momentum (m),

z_{ov} = roughness length for water vapour (m),

k = Von Karman Constant equal,

u_2 = wind speed measure at height (m/s), with the following standard values:

d=0.67h, z_{om} =0.123h, and z_{ov} =0.120m, $k = 0.41$, h = 0.12 (3.97)

substituting the standard values in equation (3.97) into equation (3.96), we have

$$r_a = \frac{208}{u_2} \tag{3.98}$$

From the equation (3.92) we let:

$$r_c = \frac{r_i}{LAI_{active}} = \frac{100}{(0.5)(24)(0.12)} = 70sm^{-1}$$

(3.99)

where,

$r_i = 100$ standard value,

$LAI_{active} = (0.5)(24)(0.12)$ standard values.

Substituting equation (3.98) and (3.99) into (3.92) we have

$$\gamma^* = \gamma\left(1+0.34u_2\right) \qquad (3.100)$$

from equation (3.58)

$$T_a = \frac{(T_{max} + T_{min})}{2} \qquad (3.101)$$

where,

T_a =average air temperature $({}^o c)$.

$$e_a = 0.6108\exp\left[\frac{17.27T_a}{T_a + 273.3}\right] \qquad (3.102)$$

where,

e_a =sat urated vapour pressure (kpa).

substituting equation (3.101) and (3.102) into (3.58) we have

$$\Delta = \frac{4098e_a}{(T_a + 273.3)^2} \qquad (3.103)$$

from equation (3.95)

$R_n = 86400$ standard value (3.104)

where,

The net long wave radiation with the standard values is given as:

$$R_n = \left(0.9\frac{n}{N} + 0.1\right)\left(0.34 - 0.139\sqrt{e_a}.f\left(\frac{T_k{}^4 \max + T_k{}^4 \min}{2}\right)\right) \qquad (3.105)$$

(F.A.O.,2010)

where,

R_n = net long-wave radiation (w/m^2),

n = daily dination of bright sunshine (h),

N=daily length (h),

e_d =actual vapour pressure (kpa),

T_k max = maximum absolute temperature (kpa),

T_k min = minimum absolute temperature (kpa),

f = Stefan Boltzmann Constant $(w/m^2)k^4$ which equals $5.6745*10^{-2}$.

From the equation (3.95),the actual vapour pressure e_d is given as:

$$e_d = \frac{RH}{100} e_a \quad (3.106)$$

where,

RH = relative humidity percentage

from the equation (3.82) aerodynamic evaporation E_a is given as:

$$E_a = \frac{900}{Ta + 273} u_2 (e_a - e_d) \quad (3.107)$$

where,

u_2 = wind speed measure at 2m height (m/s)

e_a = saturated vapour pressure (kpa)

e_d = actual vapour pressure (kpa)

from equation (3.56), we substitute the standard values and we have

$$\rho_a = \frac{p_a}{0.287(T_a + 275)} \quad (3.108)$$

where,

$T_{vk} = (T_a + 273)$ (standard values)

(3.109)

$R = (0.287)$ (standard values)

(3.110)

From equation (3.92)

$$\frac{1}{\lambda} = \frac{1}{2.45} = 0.408$$

(3.111)

where,

$\lambda = 2.45$ (standard value)

substituting equation (3.111), (3.107) and equation (3.100) into equation (3.95) we have

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{Ta + 273} u_2(e_0 - e_d)}{\Delta + \gamma(1 + 0.34u_2)} \quad (3.112)$$

The equation (3.112) is reference evapotranspiration equation.

3.4.2 Dual crop water coefficient

The dual crop coefficient is complicated and more intensive than single cropwater coefficient. Inorder to evaluate the performance of crop water coefficient, we let:

K_r = Dimensionless evaporation reduction coefficient dependent on the cumulative depth of water depleted (evaporated) from the top soil.

K_{cb} = is the value for $(K_{cb})_{mid}$ or $(K_{cb})_{end}$,

then

$$K_r K_e = K_r\left(K_c - K_{cb}\right)$$

(3.113)

(*F.A.O.*, 1998)

where,

K_e = soil evaporation coefficient,

k_c = crop coefficient value of K_c following rain or irrigation,

K_r = Dimensionless evaporation reduction coefficient,

simplifying the equation, (3.113) we have

$$K_r K_e = K_r K_c - K_r K_{cb}$$

(3.114)

(3.114) yield

$$\left(k_c\right) = \frac{K_r K_e + K_r\left(K_{cb}\right)}{K_r}$$

(3.115)

Equation (3.115) yield

$$K_c = K_e + K_{cb}$$

(3.116)

where,

$K_r = 1$ (standard value)

K_{cb} = Basal crop coefficient

3.4.3 Mathematical formulation of irrigated area of land (A_i)

The net irrigated Area of the Land is the actual Land area on which irrigation was used for growing crops as many times in an agricultural year. Irrigated Area of land can be regular forms (rectangular, Triangular) or Irregular form. we therefore consider rectangular form of Irrigated Surface Area of Land.

we let:

L = length of the farm

B = Breadth of the farm

l = length of the spacing on the farm land

b = breadth of the spacing on the farm land

The spacing area of the farm land is considered to be

$$lb = \left(\frac{LB}{P_n} \right) N \qquad (3.117)$$

(*Adeboye et 'al.*, 2006)

where,

P_n = number of plants on the farm land,

N = number of seeds per stand.

also, plants on two adjacent rows is;

$$S = \left(\frac{L}{l} + \frac{B}{b} + I \right) N \qquad (3.118)$$

the accurate plant population formula becomes

$$P_n = \left(\frac{lb}{LB}\right)N + S \tag{3.119}$$

Substituting (3.117) and (3.118) into (3.119) and simplifies we obtain

$$P_n = \left(\frac{LB}{lb} + \frac{L}{l} + \frac{B}{b} + I\right)N = \left(\frac{LB + Lb + lB + lbI}{lb}\right)N \tag{3.120}$$

furthermore,

$$LB = lbP_n - (Lb + lB + lbI) \tag{3.121}$$

Replacing LB with A_i;

then

$$A_i = lbP_n - (Lb + lB + lbI) \tag{3.122}$$

where;

A_i = irrigated area of land.

$$ET_{cwn} = ET_0 \times K_c \times A_i$$

(3.123)

where,

ET_{cwn} = crop water need

ET_0 = reference crop evapotranspiration

K_c = crop water coefficient

A_i = crop irrigated area

combining equation (3.112), (3.116) and (3.122), the crop water need equation becomes;

$$ET_{cwn} = \left(\frac{0.408\Delta(R_n - G) + \gamma\frac{900}{Ta + 273}u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)}\right) \times \left(K_{cb} + K_e\right) \times \left(lbP_n - (Lb + lB + lbI)\right) \tag{3.124}$$

Equation (3.118) describes the following:

(i) Reference Crop evapotranspiration.

(ii) Dual crop water Coefficient.

(iii) Irrigated Area of land

The three equations combined together to evaluate crop water need for two crops which is Rice and Soya Bean. The crop yield for the two crops can be computed from the following equations ET_0 = (3.112), A_i = (3.116), ET_{cwn} = (3.124).

the crop yield is computed as:

$$CY = K_y(1 - \frac{ET_0}{ET_{cwn}})$$

(3.125)

(*F.A.O.*, 1998)

we compute the Equation (3.124) for estimation of the crop water need in Lapai irrigation site, Mokwa irrigation site and Bida irrigation site. The equation (3.125) is used to estimate crop yield. See the Appendix for the computation.

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Analysis of Results

In this chapter, we presented the data used for a mathematical model for estimating the quantity of groundwater level, for optimum aquifer withdrawal for irrigation, crop water requirement, the resulting outputs and the graphical interpretation of the models.

Computational results were processed using Intel® Pentium® Dual T3200@4.00GH 500MBMemory and MATLAB7.9

4.2 Meteorological Data

The weather data was collected by the Nigeria Meteorological Station in Abuja. The data used for reference Evapotraspiration ET_o computation are meteorological data obtained from the Nigeria meteorological station in Abuja.; for instance, minimum and maximum temperatures, wind speed (in km) per day, the relative humidity (maximum and minimum, in %) and the hours of sunshine, and the physical data such as altitude, latitude and longitude.The rainfall data was also obtained from the meteorological station. Rainfall records from range of years (10–15) were collected to allow for a calculation crop water need.

In order to implement our algorithm, each iteration of training entails the groundwater values $h = 0.7$ (groundwater level), $h_o = 0.9$ (standard groundwater level), while J = $X_{i.}$ and $X_i = x_1, x_2, ..., x_6$ .are all input data which includes: minimum and maximum temperatures, wind speed in km per day, the relative humidity (maximum and minimum, in %) and the hours of sunshine, S = 4.5 (storativity) and C = 15.5 (constant) in equation (3.9) are computed in MATLAB7.9 in Artificial Neural Network and the results are shown in the tables 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8

4.3 Groundwater Level Estimate Results

The model (3.1) was solved, and the data on Rainfall, Humidity, and evapotranspiration, the minimum and maximum temperature were used to train and test equation (3.9) by Artificial Neural Network to estimate the Groundwater available for the period of the dry season in Mokwa, Lapai, and Badeji (Bida) study area.

4.3.1 Groundwater level estimate in Lapai

Table 4.1 shows the evaluation of all fifteen networks performance for the estimation of the groundwater level as given by the equation (3.9) which is trained and tested by Artificia neural network

Table 4.1: Shows the achievements by artificial neural network (ANN) model in Lapai irrigation sites

Neural	N1	N2	Epoch	LR	R-Train	R-Test	R-All	MSE	Data %
FFLM	5	5	100	0.9	0.75	0.88	0.85	3.94	80-20
FFRP	5	5	100	0.9	0.65	0.70	0.66	4.35	80-20
FFSCG	5	5	100	0.9	0.45	0.62	0.55	4.07	80-20
FFBFG	5	5	100	0.9	0.44	0.55	0.50	4.02	80-20
FFCGF	5	5	100	0.9	0.50	0.63	0.61	4.04	80-20
RNLM	5	5	100	0.9	0.45	0.56	0.60	4.11	80-20
RNRP	5	5	100	0.9	0.51	0.53	0.56	4.16	80-20
RNSCG	5	5	100	0.9	0.64	0.44	0.64	4.04	80-20
RNBFG	5	5	100	0.9	0.52	0.57	0.60	4.08	80-20
RNCGF	5	5	100	0.9	0.60	0.56	0.65	4.49	80-20
CFLM	5	5	100	0.9	0.55	0.43	0.56	4.26	80-20
CFRP	5	5	100	0.9	0.56	0.62	0.68	4.01	80-20
CFSCG	5	5	100	0.9	0.35	0.54	0.45	4.20	80-20
CFBFG	5	5	100	0.9	0.43	0.56	0.55	4.32	80-20
CFCGF	5	5	100	0.9	0.56	0.51	0.60	4.11	80-20

N1=numbers of neurons in the first hidden layer, N2= numbers of neurons in the second hidden layer, LR = learning rate, MSE = mean square error, R= correlation coefficient between network output and network target outputs in training and testing.

The depth to groundwater for all fifteen networks by various training algorithm are compared. It is observed that Feed Forward Levenberg Marquardt (FFLM) is the best overall performance for groundwater estimated in Lapai with mean square error of (3.94) and the corresponding correlation coefficient of 0.85. The Cascade Forward network with Resilient Back propagation (CFRP) trained with the same algorithm is the second best with mean square error (MSE) of 4.01 and corresponding correlation coefficient of 0.68.

The following graphs are the first best performance algorithms shown in the table (4.1) and the solution given by the equation (3.9) in Lapai irrigation site.

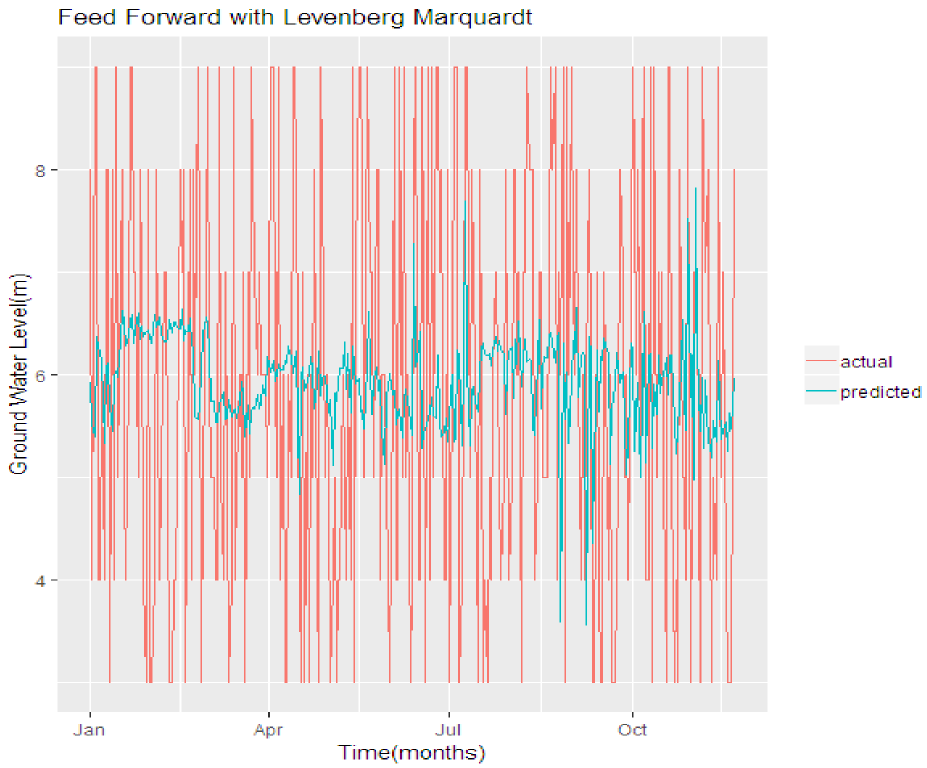

Figure 4.1 Hydrograph of the groundwater for the best Performance algorithm in Lapai

Figure 4.1 shows the hydro - graph of groundwater level (m) against time (month) in Lapai. The actual groundwater level in Lapai irrigation area during the dry season as shown by the hydrograph from November to April is $5.93\times10^{6}\, ft^{3}$ (cubic feet) and the estimated groundwater level in Lapai study area during the dry season as shown from the hydrograph is $5.94\times10^{6}\, ft^{3}$ (cubic feet).The graph shows that the estimated groundwater level found in Lapai irrigation sites is enough to practice inter - cropping farming system during the dry season. however, the estimated groundwater volume can be use for planning a large-scale irrigation farming as the standard groundwater consumptive use for rice crop is within 200mm per day. the depth to groundwater increases from `December and reached its highest level in April but it is lowest in September.

Comment [OOIS1]:

Comment [OOIS2]:

Figure 4.2 below is the graph of the second-best performance algorithm as shown in Table 4.1 with Mean Square Error of (4.01) and given by the equation (3.9).

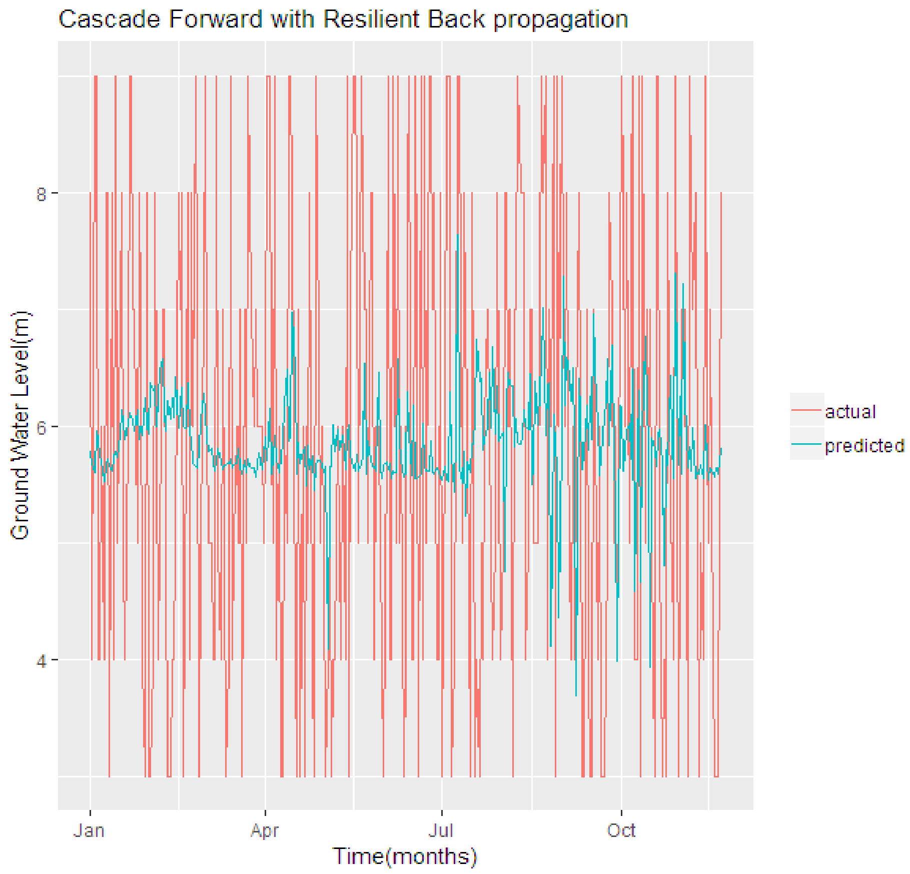

Figure 4.2: Hydrograph of the groundwater for the second-best performance Algorithm

Figure 4.2 shows the hydrograph of groundwater level (m) against time (month)in Lapai.it is observed from the graph that the Cascade Forward with Resilient Back propagation is the second best performance algorithm and it is observed from the graph that depth to groundwater increases from `December and reached its highest level in April it is lowest in September.

Figure 4.3 below is the graph of the third-best performance algorithm as shown in Table 4.1 with Mean Square Error of (4.02) and given by the equation (3.9).

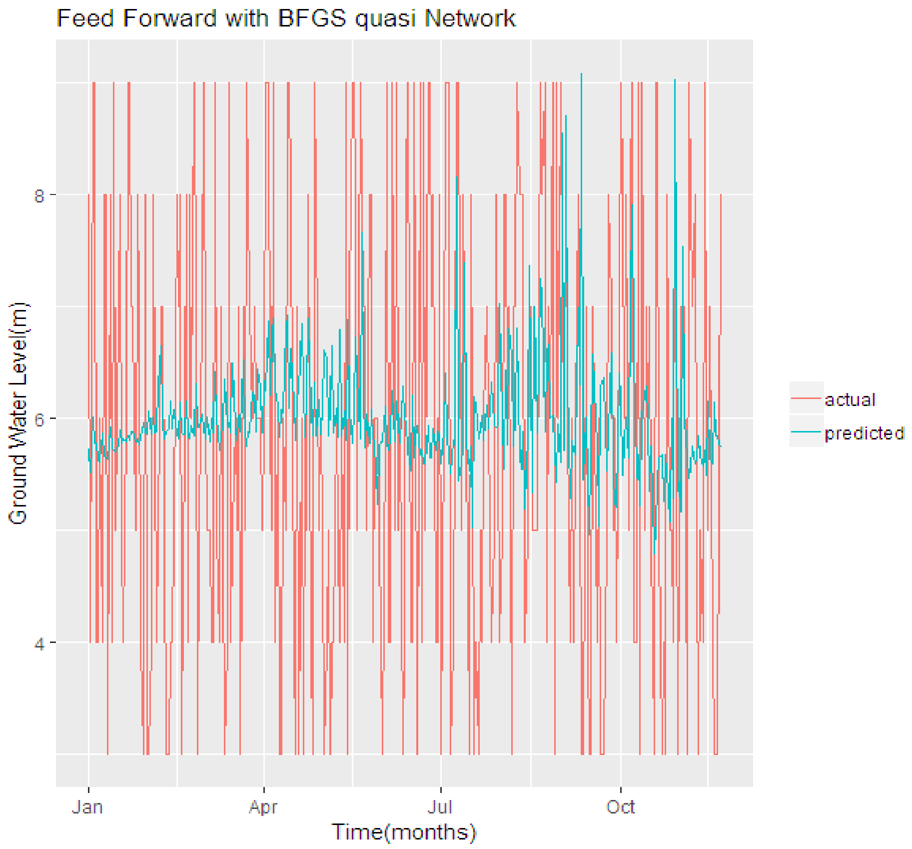

Figure 4.3: Hydrograph of the groundwater for the third best performance algorithm Lapai

Figure 4.3 shows the hydrograph of groundwater level (m) against time (month) in Lapai.it is observed that the Feed Forward Neural Network with BFCG quasi Network is the third best algorithm that estimated groundwater levels in Lapai, the depth to groundwater increases from `December and reached its highest level in April but it is lowest in September.

Figure 4.4 below is the graph of the fouth-best performance algorithm as shown in Table 4.1 with Mean Square Error of (4.04) and given by the equation (3.9).

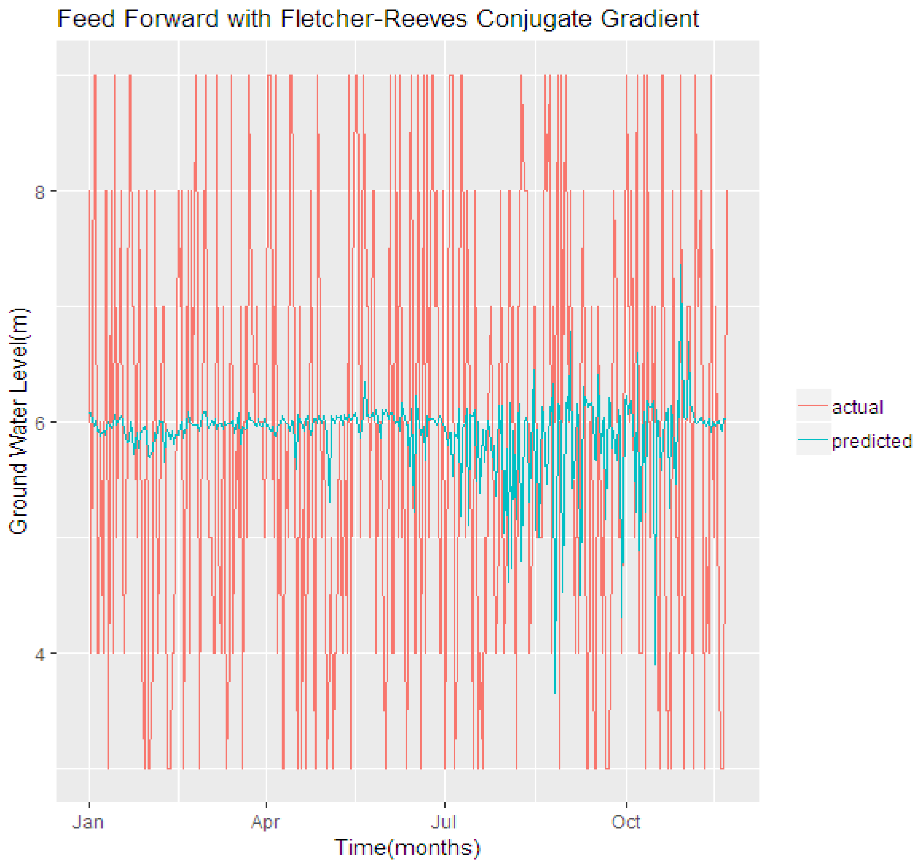

Figure 4.4: Hydrograph of the groundwater for the fourth best performance algorithm Lapai

Figure 4.4 shows the hydrograph of groundwater level (m) against time (month) in Lapai. It is observed that the Feed Forward Neural Network with Fletcher Reeves Conjugate Gradient is the fourth best performance algorithm and it is observed from the graph that depth to groundwater increases from `December and reached its highest level in April it is lowest in September.

Figure 4.5 below is the graph of the fifth-best performance algorithm as shown in Table 4.1 with Mean Square Error of (4.07) and given by the equation (3.9).

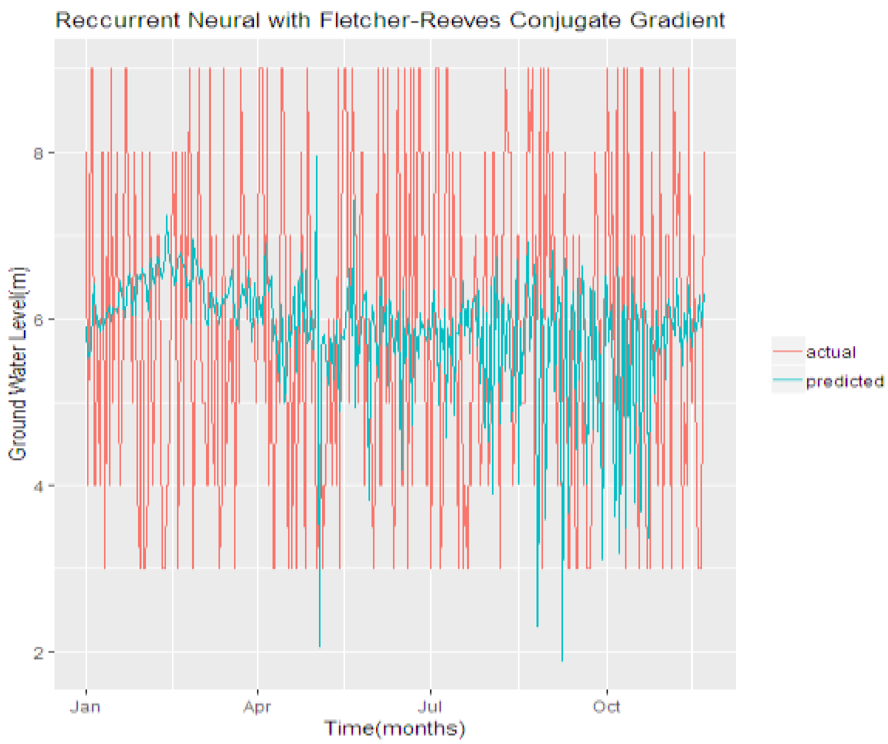

Figure 4.5: Hydrograph of the groundwater for the fifth best performance algorithm Lapai

Figure 4.5 shows the hydrograph of groundwater level (m) against time (month) in Lapai. It is observed that the Feed Forward with Fletcher Reeves Conjugate Gradient is the Fifth best performance algorithm and it is observed from the graph that depth to groundwater increases from `December and reached its highest level in April it is lowest in September.

4.3.2. Groundwater extraction results in Lapai irrigation site

The model equation (3.17) is solved and the solution given by equation (3.36) was encoded by Genetic Algorithm to determine the quantity of the Groundwater to be drawn from each fourteen well system, considering groundwater maximum and minimum flow rate, Aquifer Specific yield, Irrigation water demand, Irrigation area of the land, potential energy required and water withdrawal efficiency in Lapai irrigation site. As shown in table 4.4

Table 4.2 Optimum withdrawal of irrigation water demand in Bida basin (Lapai irrigation site)

Month	Irrig. Demand (QD) (m^3/month)	Irrig. Areaof land A_i/ha	Specific yield (S_y)	Discharge rate ($Q_{i,j}$) (m^3/d)	Max. Discharge rate (Q_{max}) (m^3/d)	PotentialEnergy Required (kilowatts/Month)	Groundwater level ($Q_{i,j}$ - QD) (m^3/d)	Aquifer depth to Water (m) (i)
January	38145	284	13.5	1,415	7300	627.66	11640	66.0
February	38690	284	10.4	1,401	7300	425.72	6650	69.0
March	38690	284	8.9	1295	7300	761.58	6100	78.0
April	33120	284	7.5	1160	7300	4683.08	3360	89.1
November	32175	284	14.3	1700	7300	13303	37650	58.2
December	32865	284	13.7	1550	7300	1144.7	27270	64.8

$npw = 20$,
$n = 7$
,
$h_s = 0.5$,
$\rho = 0.6$
$g = 0.5$
,
$efficiency = 64.5$
$a_i = b_i = c_i = 1$

Table 4.2 gives the values for irrigation water demand (Qd), E_p (potential energy required), efficiency (irrigation water withdrawal efficiency), groundwater discharge rate $\left(Q_{i,j}\right)$, maximum groundwater discharge $\left(Q_{\max}\right)$, area of the irrigation land A_i, groundwater level, $\left(Q_{i,j} - Qd\right)$, aquifer depth to water (i), for twenty well as used in equation (3.38). It is observed from the table 4.2 that inequitable potential energy kilowatts (E_p) are needed for the groundwater withdrawal in the study area.The higher the increases in the depth to aquifer the higher the potential energy required. For a maximum groundwater discharge ($Q_{\max}$) of 7300 m^3/d the groundwater discharge rate ($Q_{i,j}$) is decreasing, this occurs due to an increase in solar radiation and surface pressure $\left(h_s\right)$ in the study area. We note from the table that the monthly groundwater level in the research area is the result of the difference of the monthly irrigation water demand and the monthly Aquifer Discharge Rate in the study area. The groundwater level ranges from 37650 m^3/d to 3360 m^3/d and the optimum groundwater level occurs in December which is 27270 m^3/d. The depth to aquifer increases as a result of evapotranspiration and groundwater withdrawal. The specific yield of the aquifer shows apparently that the underlying aquifer in the area can sustain irrigation of any crops irrespective of their irrigation water demand.

4.3.3 Analysis of groundwater extraction in Lapai

The model eqution (3.36) is solved and the solution given by equation (3.47) was used to estimate the quantity of the groundwater to be drawn from each fourteen well system as shown in Table. 4.3.

Table 4.3 Result of the groundwater to be drawn from the each fourteen well in Lapai

(Npw) number of active wells	(H_{max}) withdrawal rate	(yq) aquifer recharge through irrig.	(R) maximum withdrawal rate	L(m) distance between the well	(k) management period in years
1	2700	800	7000	45	7
2	2320	800	7000	60	7
3	4300	800	7000	40	7
4	2800	800	7000	200	7
5	3700	800	7000	50	7
6	4432	800	7000	120	7
7	3400	800	7000	90	7
8	3000	800	7000	110	7
9	2100	800	7000	170	7
10	5466	800	7000	80	7
11	5100	800	7000	50	7
12	3323	800	7000	60	7
13	3600	800	7000	100	7
14	2765	800	7000		7

Table 4.3 shows the analysis of the outcome of groundwater extraction in Lapai. Fourteen active wells are considered and we note from the table that the Groundwater withdrawal rate (H_{max}) increases or decreases in values, these occur due to the effects of distance between the well, topographical and variabilities in aquifer depth to water of the each well. the optimum Aquifer withdrawal rate occurs at an observation well number (11) with corresponding well distance of (80m). The discharge rate of the well

number (11) is high to the moderate than other wells. The withdrawal rate enhanced the quality of groundwater which provides healthy crops and raised the value of crop yield.

4.4 Analysis of the net Benefit for groundwater withdrawal in Lapai

The model equation (3.36) is solved and the solution of equation (3.47) was used to determine the values of the Net benefit (N_B) arising from the economic Analysis of Groundwater withdrawal from twenty well system as shown in table 4.4

Table 4.4 Result analysis of the net benefit for groundwater extraction in Lapai.

Npw	L (m) Distance between the well	B_p (₦)	(C_p) (₦)	r (₦)	NB (₦)
1	65	150000	120000	23.5	27662.4
2	100	160000	125000	23.5	12412.84
3	70	150000	120000	23.5	13831.2
4	110	150000	120000	23.5	8801.6
5	130	160000	121000	23.5	7447.5
6	130	160000	120000	23.5	7447.5
7	90	160000	120000	23.5	10757.4
8	150	160000	120000	23.5	6454.5
9	100	160000	120000	23.5	9681.8
10	80	180000	140000	23.5	12102.3
11	90	250000	160000	23.5	10757.4
12	40	160000	162000	23.5	24204.6
13	90	270000	165000	23.5	10757.4
14		300000	170000	23.5	10757.4

$\pi yq = 0.7, \; R = 2500, \; X_{max} = 719, \; k = 7$

Table 4.4 Shows the Economic analysis of the outcome of groundwater to be drawn in Mokwa. The results show that twenty active wells are considered and as the cost of Groundwater withdrawal rate increased, so is the Benefit per unit supply. We note from the table that the Net Benefit of the project in the research area is the result of the difference of cost of the extraction of the groundwater withdrawal rate in the study area and the Benefit per unit supply in the study area at the recurring interest rate of 23.5million Naira.

4.5 The analysis of the computational method for rice crop water requirement in Lapai

The model equation (3.48) is solved and the solution given by equation (3.124) was used to estimate the crop water need (ET_{cwn}), evapotranspiration (ET_c), crop water coefficient ($K_{cb}+K_e$), and irrigation area of the land (A_i) as shown in table 4.5

Table 4.5 Shows the results of the computational method for rice crop water need in Lapai

Months	ET_o	K_c	ET_{cwn} Mm	A_i Hectares	Crop Yield $K_y\left(1-\frac{ET_0}{ET_{cwn}}\right)A_i$
November	3.95	3.41	5381.96	400	399.72
December	4.32	2.73	4718.28	400	399.64
January	4.41	4.3	7578.44	400	399.88
February	4.84	4.04	7818.09	400	397.44
March	4.07	1.49	2424.31	400	397.6
Total			**5584.22**		**398.87**

Table 4.5 shows the results analysis of the Computation method for Rice irrigation crop water needs. It is observed from the table that the values of reference evapotranspiration (ET_o) through growth season indicate that it was low at the beginning of season and increase gradually till havesting period. This may be due to changes in climatologicall norm in Lapai irrigation sites, as the cultivation beggins with Solar radiation and relatively high temperature and ended by an increase than it it was. The crop coefficient (K_c) result decreases at initial stage of growth (December) and became higher in (February) mid-season with 4.04 (K_c) this indicates that the crop is at the harvesting period which is when the crops need little or no water. The total average rice water need for the period of four months on 400 hectares of land is 55584.22 mm and the expected total average crop yield is estimated to be 398.87 per hectar. The Rice water need is values gotten conform with crop water requirement estimated under standard condition and indicates that the crop is disesas-free, under the optmum soil conditions and achieving full production under the given climatic conditions.

4.6 The Analysis of the Computational Method for Soya Bean Crop Water Requirement in Lapai

The model equation (3.48) is solved and the solution given by equation (3.124) was used to estimate the crop water need (ET_{cwn}), evapotranspiration (ET_c), crop water coefficient (K_{cb}+K_e), and irrigation area of the land (A_i) as shown in table 4.6

Table 4.6 shows the result of the computation method for soya beans crop Lapai

Months	ET_o	K_c	ET_{cwn} Mm	A_i Hectares	Crop Yield $K_y\left(1-\frac{ET_0}{ET_{cwn}}\right)A_i$
November	4.31	0.82	1370.64	450	448.61
December	4.59	1.64	3629.89	450	449.42
January	4.19	0.3	517.29	450	446.31
February	5.53	0.3	550.27	450	405
March	4.31	0.82	1370.64	450	436.1
Average			769.03		**2185.44**

Table 4.6 shows the results analysis of the Computation method for Soya bean irrigation crop water needs. It is observed from the table that the values of reference evapotranspiration (ET_o) through growth season indicates that it was 5.31 at the initial stage at the beginning of season and increase gradually till harvesting period. This may be due to changes in climatological norm in Lapai irrigation sites, as the cultivation beggins with Solar radiation and relatively high temperature and ended by an increase than it was. The crop coefficient (K_c) result decreases at initial stage of growth (December) and became higher in (February) mid-season with 5.53 (K_c) this indicates that the crop is at the harvesting period which is when the crops need little or no water. The total average rice water need for the period of four months on 450 hectares of land is 769.03mm and the expected total average crop yield is estimated to be 2185.44 per hectar. The Rice water need is values gotten conform with crop water requirement

estimated under standard condition and indicates that the crop is disesas-free, under the optmum soil conditions and achieving full production under the given climatic conditions.

4.6.1 Groundwater estimation results in Mokwa

The model (3.1) was solved, and data on Rainfall, Humidity, evapotranspiration, the minimum and maximum temperature) were used to train and test equation (3.9) by Artificial Neural Network to estimate the Groundwater available for the period of the dry season in Mokwa study area.

Table 4.7 Shows the evaluation of all fifteen networks for the observation well.

Neural	N1	N2	Epoch	LR	R-Train	R-Test	R-All	MSE	Data %
FFLM	5	5	100	0.9	0.50	0.63	0.60	141.97	80-20
FFRP	5	5	100	0.9	0.46	0.56	0.59	148.04	80-20
FFSCG	5	5	100	0.9	0.53	0.63	0.62	144.08	80-20
FFBFG	5	5	100	0.9	0.60	0.67	0.70	139.85	80-20
FFCGF	5	5	100	0.9	0.61	0.63	0.54	145.60	80-20
RNLM	5	5	100	0.9	0.73	0.78	0.85	137.96	80-20
RNRP	5	5	100	0.9	0.70	0.78	0.80	138.79	80-20
RNSCG	5	5	100	0.9	0.52	0.56	0.59	146.92	80-20
RNBFG	5	5	100	0.9	0.49	0.53	0.61	141.74	80-20
RNCGF	5	5	100	0.9	0.54	0.60	0.60	143.51	80-20
CFLM	5	5	100	0.9	0.40	0.47	0.51	145.84	80-20
CFRP	5	5	100	0.9	0.45	0.57	0.50	146.10	80-20
CFSCG	5	5	100	0.9	0.51	0.50	0.56	144.59	80-20
CFBFG	5	5	100	0.9	0.50	0.56	0.60	140.21	80-20
CFCGF	5	5	100	0.9	0.61	0.54	0.62	142.63	80-20

N1=numbers of neurons in the first hidden layer, N2= numbers of neurons in the second hidden layer, LR = learning rate, MSE = mean square error, R= correlation coefficient between network output and network target outputs in training and testing.

Table 4.7 Shows the achievements by Artificial Neural Network (ANN) model in Mokwa irrigation sites. The depth to groundwater for all fifteen networks by various training algorithm are compared. It is observed from the Table 4.2 that Recurrent Neural Network with Levenberg Marquardt (RNLM) is the best overall performance for groundwater estimated in Mokwa with mean square error of 137.96 and the corresponding correlation coefficient of 0.85 and by the Recurrent Neural Network with Resilient Back propagation (RNRP) trained with the same algorithm known as the second best with mean square error (MSE) of 138.79 and corresponding correlation coefficient of 0.80.

Figure 4.6 below is the graph of the best performance algorithm as shown in Table 4.7 with Mean Square Error of (137.964) and given by the equation (3.9).

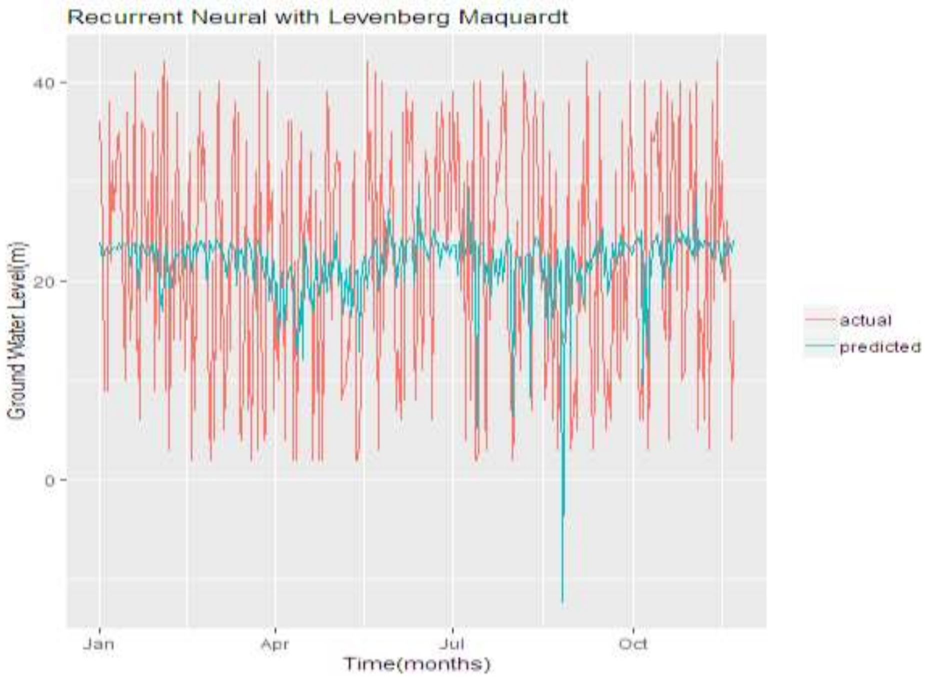

Figure 4.6 Hydrograph of the groundwater for the best performance algorithm in Mokwa

Figure 4.6 shows the hydrograph of groundwater level (m) against time (month) in Mokwa irrigation site The actual groundwater level in Mokwa study area during the dry season (November – April) is $27.19\times10^{6}\,ft^{3}$ (cubic feet) and the estimated groundwater level in Mokwa study area during the dry season is $26.78\times10^{6}\,ft^{3}$ (cubic feet).The estimated groundwater values in Mokwa higher than groundwater estimated value in Lapai and Bida. The factor that responsible for this is that the area of Land in Mokwa is closer to River Niger. The graph shown that the estimated groundwater is sufficient to practice intercropping farming system during the dry season and livestock farming. However, the estimated groundwater volume can be used for planning a large scale irrigation farming as the standard groundwater consumptive use for rice crop is within 200mm per day. It is observed from the graph that the Feed Forward Neural Network with Levenberg Marquardt is the best algorithm that estimated groundwater levels in Lapai, the depth to groundwater increases from `December and reached its highest level in April but it is lowest in September.

Figure 4.7 below is the graph of the second-best performance algorithm as shown in Table 4.7 with Mean Square Error of (138.79) and given by the equation (3.9).

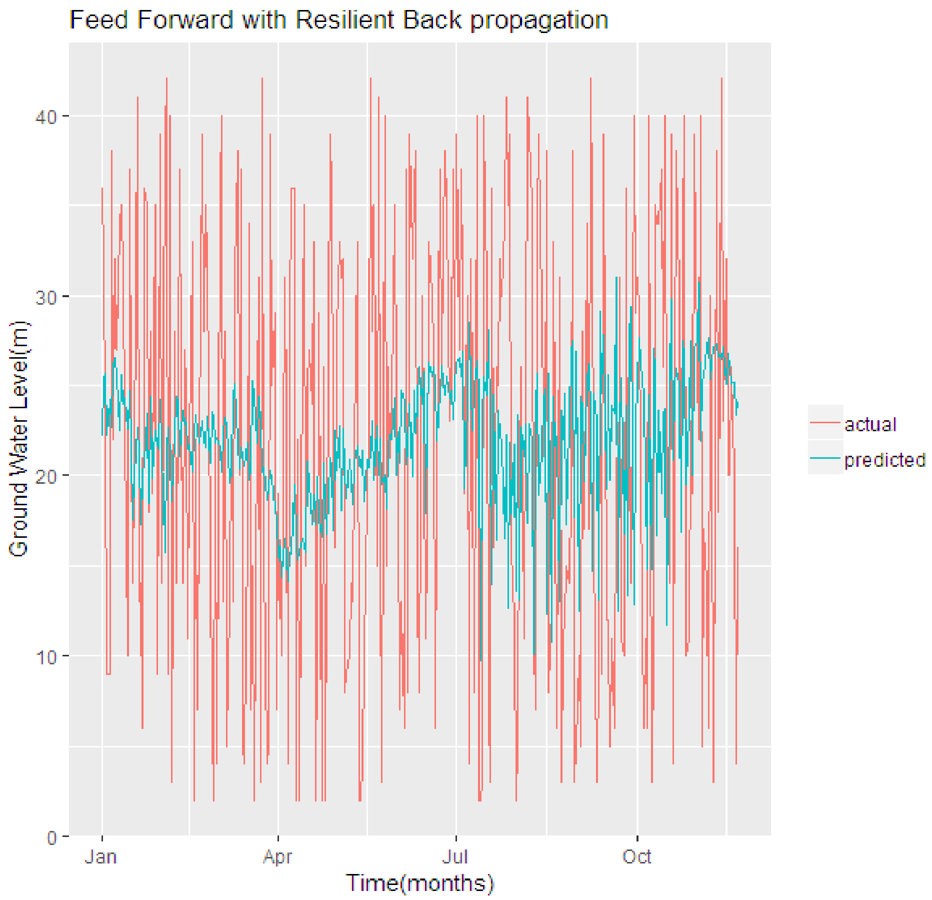

Figure 4.7 Hydrograph of the groundwater for the second-best performance algorithm in Mokwa

Figure 4.7 shows the hydrograph of groundwater level (m) against time (month). It is observed that the Resilient Back propagation is the second best algorithm that estimated groundwater levels, the depth to groundwater increases from `December and reached its highest level in April but it is lowest in September.

Figure 4.8 below is the graph of the third-best performance algorithm as shown in Table 4.7 with Mean Square Error of (139.85) and given by the equation (3.9).

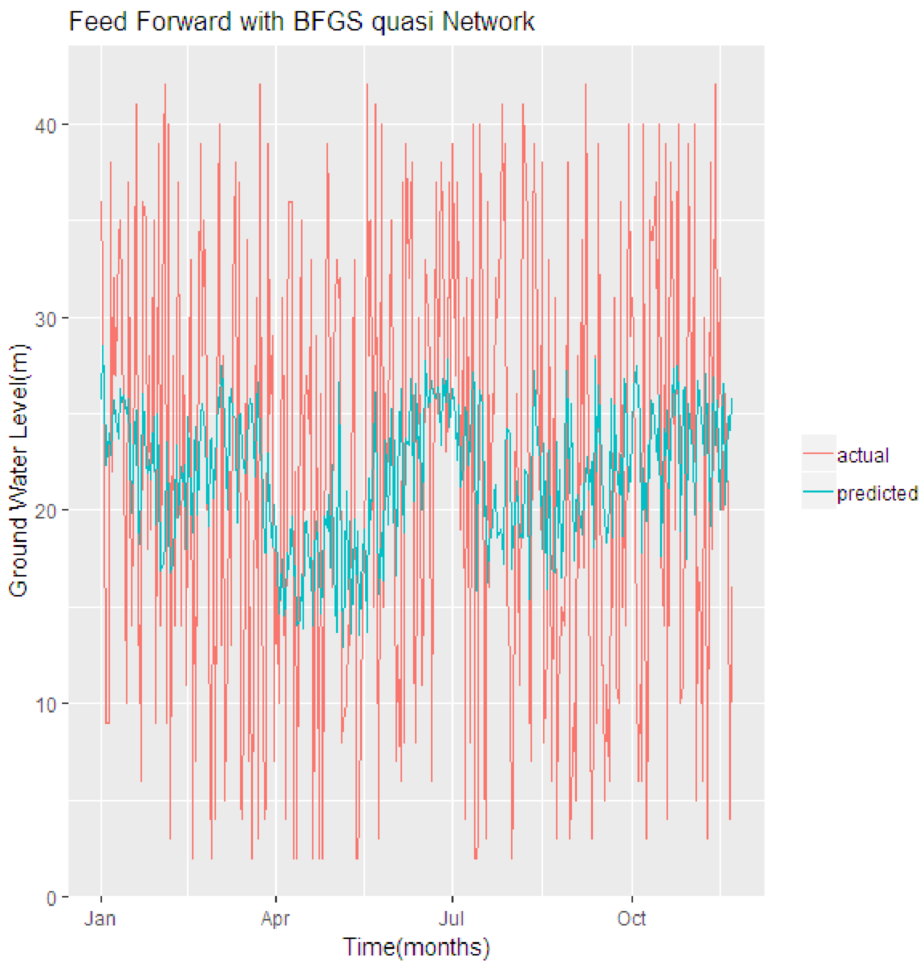

Figure 4.8 Hydrograph of the groundwater for the third best performance algorithm in Mokwa

Figure 4.8 shows the hydrograph of groundwater level (m) against time (month). It is observed that the Feed Forward with Scaled Conjugate gradient is the third best algorithm that estimated groundwater levels, the depth to groundwater increases from `December and reached its highest level in April but it is lowest in September.

Figure 4.9 below is the graph of the fourth-best performance algorithm as shown in Table 4.7 with Mean Square Error of (140.21) given by the equation (3.9).

Figure 4.9: Hydrograph of the groundwater for the fourth best performance algorithm in Mokwa

Figure 4.9 shows the hydrograph of groundwater level (m) against time (month). It is observed that the Cascade Forward with BFGS Quasi Network is the fourteenth best algorithm that estimated groundwater levels in Mokwa, the depth to groundwater increases from `December and reached its highest level in April but it is lowest in September.

Figure 4.10 below is the graph of the fifth-best performance algorithm as shown in Table 4.1 with Mean Square Error of (141.79) and given by the equation (3.9).

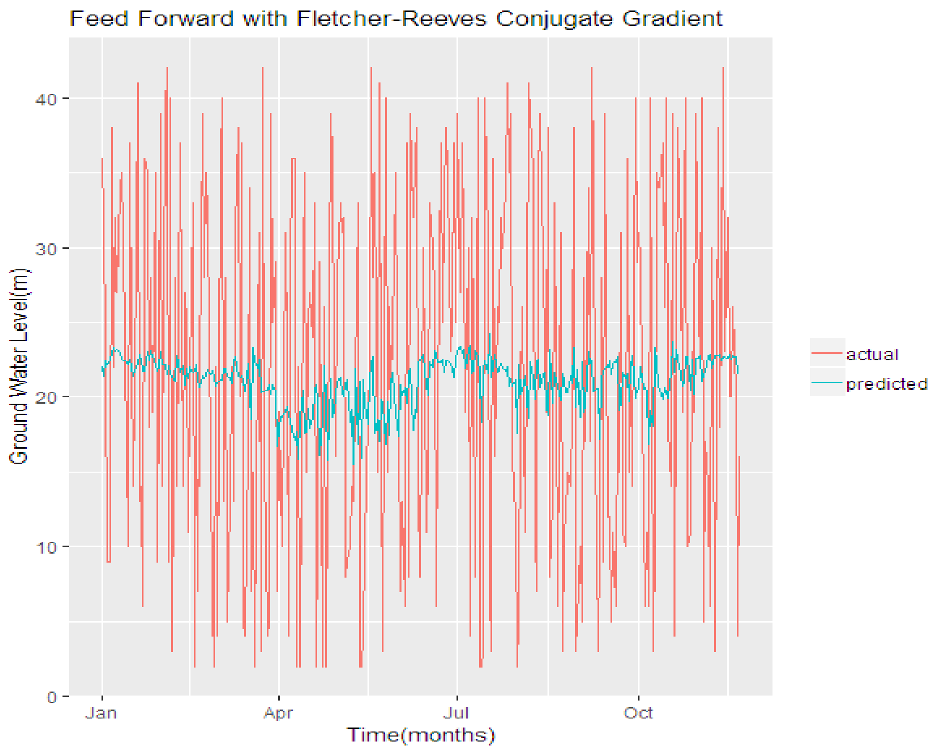

Figure 4.10: Hydrograph of the groundwater for the fifth best performance algorithm in Mokwa

Figure 4.10 shows the hydrograph of groundwater level (m) against time (month). It is observed that the Feed Forward with Fletcher Conjugate Gradient is the tenth best algorithm that estimated groundwater levels in Mokwa, the depth to groundwater increases from `December and reached its highest level in April but it is lowest in September.

4.7 Groundwater Extraction Results in Mokwa Irrigation Site

The model equation (3.17) is solved and the solution given by equation (3.36) was encoded by Genetic Algorithm to determine the quantity of the Groundwater to be drawn from each fourteen well system, considering groundwater maximum and minimum flow rate, Aquifer Specific yield, Irrigation water demand, Irrigation area of the land, potential energy required and water withdrawal efficiency in Lapai irrigation site. As shown in table 4.8.

Table 4.8 Optimum withdrawal of irrigation water demand in Bida basin (Mokwa irrigation site)

Month	Irrig. Demand (QD) (m^3/month)	Irrig. Areaof land A_i/ha	Specific yield (S_y)	Discharge rate ($Q_{i,j}$) (m^3/d)	Max. Discharge rate (Q_{max}) (m^3/d)	PotentialEnergy Required (kilowatts/Month)	Groundwater level ($Q_{i,j}$ - QD) (m^3/d)	Aquifer depth to Water (m) (i)
January	76290	400	26.3	2,931	7300	6276.66	46980	73.0
February	77380	400	21.4	2,801	7300	4255.72	49370	78.0
March	77570	400	17.3	2789	7300	6611.58	49680	96.0
April	66240	400	14.9	2320	7300	45823.08	43040	99.1
November	64350	400	28.3	3400	7300	26606	30350	51.2
December	65730	400	27.1	3100	7300	2299.5	34730	66.8

$npw = 20$,
$n = 7$,

$h_s = 0.05$,
$\rho = 0.8$
$g = 0.5$
,

$efficiency = 80.5$
$a_i = b_i = c_i = 1$

Table 4.8 gives the values for irrigation water demand (QD), E_p (potential energy required), efficiency (irrigation water withdrawal efficiency), groundwater discharge rate $(Q_{i,j})$, maximum groundwater discharge $(Q_{\max})$, area of the irrigation land A_i, groundwater level, $(Q_{i,j} - QD)$, aquifer depth to water (i), for twenty well as used in equation (3.36). It is observed from the table 4.8 that inequitable potential energy kilowatts (E_p) are needed for the groundwater withdrawal in the study area increases from November to April as the depth to aquifer increases. For a maximum groundwater discharge ($Q_{\max}$) of 7300 m^3/d, the groundwater discharge rate ($Q_{i,j}$) is decreasing, this occurs due to an increase in solar radiation and surface pressure (h_s) in the study area. We note from the table that the monthly groundwater level in the research area is the result of the difference of the monthly irrigation water demand and the monthly Aquifer Discharge Rate in the study area. The groundwater level ranges from 30350 m^3/d to 43040 m^3/d and the optimum groundwater level occurs in December which is 49370 m^3/d. The depth to aquifer increases as a result of evapotranspiration and groundwater withdrawal. The specific yield of the aquifer shows apparently that the underlying aquifer in the area can sustain irrigation of any crops irrespective of their irrigation water demand

4.8 Analysis of Groundwater Withdrawal in Mokwa

The model eqution (3.36) is solved and the solution given by equation (3.47) was used to estimate the quantity of the groundwater to be drawn from each fourteen well system as shown in Table. 4.9.

Table 4.9 Analysis of the net benefit for groundwater extraction in Mokwa

(Npw) number of active wells	(H_{max}) withdrawal rate	(yq) aquifer recharge through irrig.	(R) maximum withdrawal rate	L(m) distance between the well	(k) management period in years
1	2900	1100	7000	65	7
2	2820	1100	7000	100	7
3	4200	1100	7000	70	7
4	2800	1100	7000	110	7
5	3900	1100	7000	130	7
6	4232	1100	7000	130	7
7	4400	1100	7000	90	7
8	4000	1100	7000	150	7
9	4100	1100	7000	100	7
10	2466	1100	7000	80	7
11	5100	1100	7000	90	7
12	3223	1100	7000	40	7
13	3800	1100	7000	90	7
14	2365	1100	7000	90	7
15	2900	1100	7000	10	7
16	5900	1100	7000	60	7
17	3700	1100	7000	30	7
18	4600	1100	7000	40	7
19	6100	1100	7000	50	7
20	5100	1100	7000		7

$\pi = 3.142,\ B_p = 1500,\ C_p = 120,\ N_B = 7234.5$

Table 4.9 shows the analysis of the outcome of groundwater extraction in Mokwa. The results show that twenty active wells are considered and we note from the table that the Groundwater withdrawal rate $\left(H_{\max}\right)$ increases or decreases in values, these occur due to the effects of distance between the well, topographical and variabilities in aquifer depth to water of the each well. the optimum Aquifer withdrawal rate occurs at an observation well number (19) with corresponding well distance of (50m). the discharge rate of the well number (19) is high to the moderate than others.

4.9 Analysis of Groundwater Withdrawal in Mokwa

The model equation (3.36) is solved and the solution given by equation (3.47) was used to determine the values of the Net benefit (N_B) arising from the economic Analysis of Groundwater withdrawal from twenty well system as shown in table 4.10

Table 4.10 Results of the net benefit for groundwater extraction in Mokwa.

Npw	L (m)	B_p (₦)	(C_p) (₦)	r (₦)	NB (₦)
1	65	150000	120000	13.5	2067
2	100	160000	125000	13.5	2411.5
3	70	150000	120000	13.5	2067
4	110	150000	120000	13.5	2067
5	130	160000	121000	13.5	2687.1
6	130	160000	120000	13.5	2756
7	90	160000	120000	13.5	2756
8	150	160000	120000	13.5	2756
9	100	160000	120000	13.5	2756
10	80	180000	140000	13.5	2756
11		250000	160000	13.5	6201

	90				
12		160000	162000	13.5	6752.2
	40				
13		270000	165000	13.5	7234.5
	90				
14		300000	170000	13.5	8957
	90				
15		300000	171000	13.5	8888.1
	10				
16		320000	180000	13.5	9646
	60				
17		320000	181000	13.5	9577.1
	30				
18		320000	180000	13.5	9646
	40				
19		320000	180000	13.5	9646
	50				
20		320000	180000	13.5	9646

$\pi yq = 0.7,\ \ R = 250,\ \ X_{\max} = 6080,\ \ k = 7$

Table 4.10 Shows the Economic analysis of the outcome of groundwater extraction in Mokwa. The results show that twenty active wells are considered and as the cost of Groundwater withdrawal rate increased, so is the Benefit per unit supply. We note from the table that the Net Benefit of the project in the research area is the result of the difference of cost of the extraction of the groundwater withdrawal rate in the study area and the Benefit per unit supply in the study area at the recurring interest rate of 13.5million Naira.

4.10 The Analysis of the Computaional Method for Rice Crop Water Requirement in Mokwa

The model equation (3.48) is solved and the solution given by equation (3.124) was used to estimate the crop water need (ET_{cwn}), evapotranspiration (ET_c), crop water coefficient (K_{cb}+K_e), and irrigation area of the land (A_i) as shown in table 4.11

Table 4.11 Shows the result of the computaional method for rice crop water need in Mokwa

Months	ET_o	K_c	ET_{cwn} Mm	A_i Hectares	Crop Yield $K_y\left(1-\frac{ET_0}{ET_{cwn}}\right)A_i$
November	4.93	3.4	6718.89	400	399.71
December	4.32	2.7	4718.28	400	399.63
January	4.26	4.3	7320.48	400	399.76
February	4.08	4.0	6600.76	400	399.78
March	4.06	1.4	4444.31	400	399.63
Total			**5960.5**		**399.702**

Table 4.11 shows the results analysis of the Computation method for Rice irrigation crop water needs. It is observed from the table that the values of reference evapotranspiration (ET_o) through growth season indicate that it was high at the beginning of season and decrease gradually till havesting period. This may be due to changes in climatologicall norm in Lapai irrigation sites, as the cultivation beggins with Solar radiation and relatively high temperature and ended by an increase than it it was. The crop coefficient (K_c) result decreases at initial stage of growth (December) and became higher in (February) mid-season with 4.08 (K_c) this indicates that the crop is at the harvesting period which is when the crops need little or no water. The total average rice water need for the period of four months on 400 hectares of land is 5960.5 mm and

the expected total average crop yield is estimated to be 399.71 per hectar. The Rice water need is values gotten conform with crop water requirement estimated under standard condition and indicates that the crop is disesas-free, under the optmum soil conditions and achieving full production under the given climatic conditions.

4.10.1 The Analysis of the Computaional Method for Rice Crop Water Requirement in Lapai

The model equation (3.48) is solved and solution as given by equation (3.124) was used to estimate the crop water need (ET_{cwn}), evapotranspiration (ET_c), crop water coefficient ($K_{cb}+K_e$), and irrigation area of the land (A_i) as shown in table 4.12

Table 4.12 shows the results of the computation method for soya beans crop in Mokwa.

Months	$\mathbf{ET_o}$	$\mathbf{K_c}$	$\mathbf{ET_{cwn}}$ **Mm**	$\mathbf{A_i}$ **Hectares**	**Crop Yield** $K_y\left(1-\frac{ET_0}{ET_{cwn}}\right)A_i$
November	4.33	0.23	517.29	400	396.68
December	4.32	0.3	518.49	400	366.68
January	4.25	0.82	347.58	400	395.08
February	4.08	1.64	666.34	400	397.56
March	4.07	1.49	2424.31	400	399.33
Total			**894.802**		391.06

Table 4.12 shows the results analysis of the Computation method for Soya bean irrigation crop water needs. It is observed from the table that the values of reference evapotranspiration (ET_o) through growth season indicates that it was 4.32 at the initial

stage at the beginning of season and increase gradually till havesting period. This may be due to changes in climatologicall norm in Lapai irrigation sites, as the cultivation beggins with Solar radiation and relatively high temperature and ended by an increase that it was. The crop coefficient (K_c) result low at initial stage of growth (December) and increases in (February) mid-season with 4.08 (K_c) this indicates that the crop is at the harvesting period which is when the crops need little or no water. The total average rice water need for the period of four months on 400 hectares of land is 894.802mm and the expected total average crop yield is estimated to be 391.06 per hectar. The value gotten for Soya bean water need conform with crop water requirement estimated under standard condition and that indicates that the crop is disesas-free, under the optmum soil conditions and achieving full production under the given climatic conditions.

4.10.2 Groundwater estimation results in Bida

The model (3.1) was solved, and the data on Rainfall, Humidity, and evapotranspiration, the minimum and maximum temperature were used to train and test equation (3.9) by Artificial Neural Network to estimate the Groundwater available for the period of the dry season in Badeji (Bida) study area. As shown in table 4.13.

Table 4.13 The best networks and corresponding performance measure on Bida groundwater estimation

Neural	N1	N2	Epoch	LR	R-Train	R-Test	R-All	MSE	Data %
FFLM	5	5	100	0.9	0.61	0.53	0.67	76.67	80-20
FFRP	5	5	100	0.9	0.45	0.51	0.56	83.12	80-20
FFSCG	5	5	100	0.9	0.62	0.67	0.70	75.54	80-20
FFBFG	5	5	100	0.9	0.63	0.69	0.72	75.51	80-20
FFCGF	5	5	100	0.9	0.52	0.58	0.60	76.04	80-20
RNLM	5	5	100	0.9	0.62	0.56	0.65	73.94	80-20
RNRP	5	5	100	0.9	0.47	0.46	0.54	76.32	80-20

RNSCG	5	5	100	0.9	0.49	0.51	0.58	76.10	80-20
RNBFG	5	5	100	0.9	0.80	0.85	0.92	73.90	80-20
RNCGF	5	5	100	0.9	0.61	0.64	0.69	75.89	80-20
CFLM	5	5	100	0.9	0.81	0.75	0.85	73.14	80-20
CFRP	5	5	100	0.9	0.42	0.53	0.58	81.25	80-20
CFSCG	5	5	100	0.9	0.35	0.43	0.50	79.71	80-20
CFBFG	5	5	100	0.9	0.38	0.45	0.51	80.62	80-20
CFCGF	5	5	100	0.9	0.43	0.42	0.45	79.76	80-20

N1=numbers of neurons in the first hidden layer, N2= numbers of neurons in the second hidden layer, LR = learning rate, MSE = mean square error, R= correlation coefficient between network output and network target outputs in training and testing.

Table 4.13 Shows the achievements by Artificial Neural Network (ANN) model in Bida irrigation sites. The depth to groundwater for all fifteen networks by various training algorithm are compared. It is observed from the table 4.3 that Feed Cascade Forward Network with Leverberg Marquardt (CFLM) is the best overall performance for groundwater estimated in Bida with mean squre error of 73.14 and the corresponding correlation coefficient of 0.85 and by the Recurrent Neural Network with Back Fletcher Gradient (RBNFG) trained with the same algorithm known as the second best with mean square error (MSE) of 73.90 and corresponding correlation coefficient of 0.92.

Figure 4.11 below is the graph of the best performance algorithm as shown in Table 4.13 with Mean Square Error of (73.14) and given by the equation (3.9).

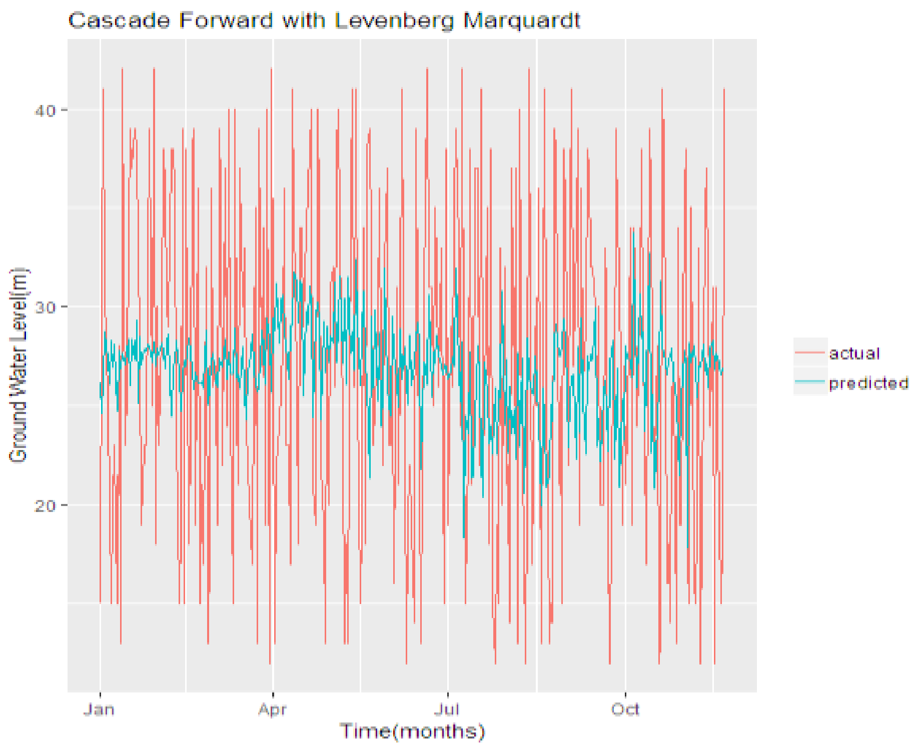

Figure 4.11: Hydrograph of the groundwater for the best performance algorithm in Bida

Figure 4.11 shows the hydrograph of groundwater level (m) against time (month) in Bida. The actual groundwater level in Lapai study area during the dry season (November – April) is $21.93\times10^{6}\ ft^{3}$ (cubic feet) and the estimated groundwater level in Lapai study area during the dry season is $21.81\times10^{6}\ ft^{3}$ (cubic feet).The graph shown that the estimated groundwater is surffificient to practice intercropping farming system during the dry season and live stock farming. However, the estimated groundwater volume can be use for planning a large scale irrigation farming as the standard groundwater consumptive use for rice crop is within 200mm per day. It is observed from the graph that the Cascade Forward Network with Levenberg Marquardt is the best algorithm that estimated groundwater levels in Bida, the depth to groundwater increases from `November and reached its highest level in April but it is lowest in September.

Figure 4.12 below is the graph of the second-best performance algorithm as shown in Table 4.1 with Mean Square Error of (73.90) and is given by the equation (3.9).

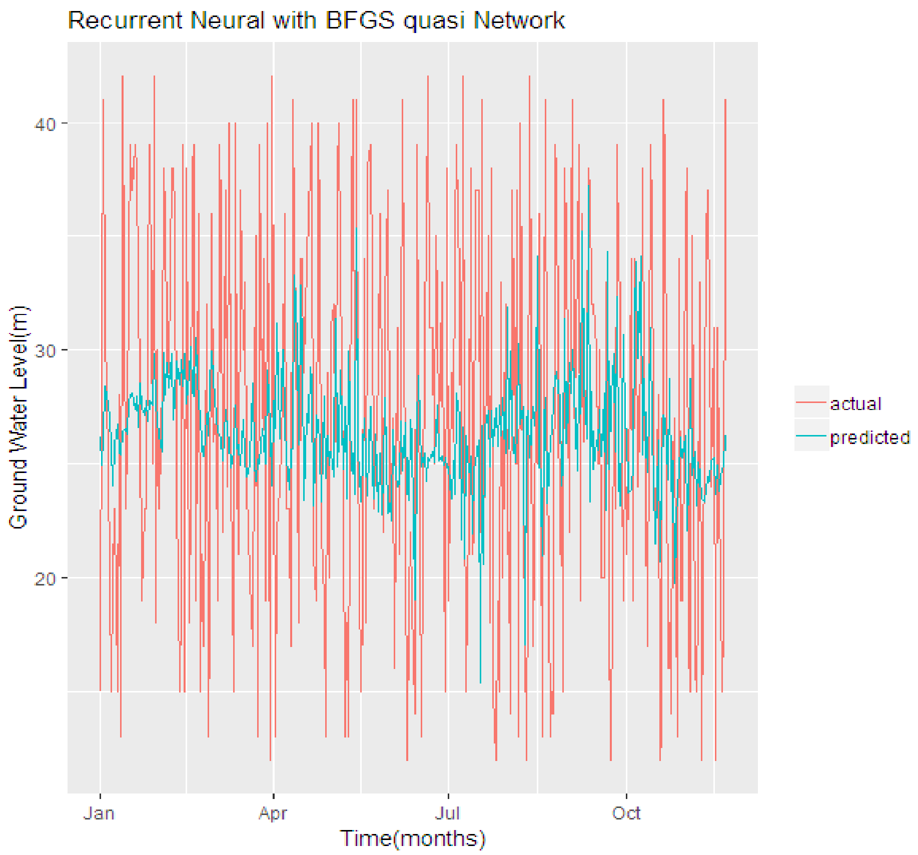

Figure 4.12: Hydrograph of the groundwater for the second best performance algorithm in Bida

Figure 4.12 shows the hydrograph of groundwater level (m) against time (month). It is observed that the Recurrent Neural Network with BFCG quasi Network is the second best algorithm that estimated groundwater levels in Bida, the depth to groundwater increases from `November and reached its highest level but it is lowest September.

Figure 4.13 below is the graph of the third-best performance algorithm as shown in Table 4.13 with Mean Square Error of (73.94) and given by the equation (3.9).

Figure 4.13: Hydrograph of the groundwater for the third best performance algorithm in Bida

Figure 4.13 reveals the hydrograph of groundwater level (m) against time (month). It is observed that the Recurrent Neural Network with Levenberg Marquardt is the third best algorithm that estimated groundwater levels in Bida, the depth to groundwater increases from `November and reached its highest level in April but it is lowest in September.

Figure 4.14 below is the graph of the fourth-best performance algorithm as shown in Table 4.13 with Mean Square Error of (75.51) and given by the equation (3.9).

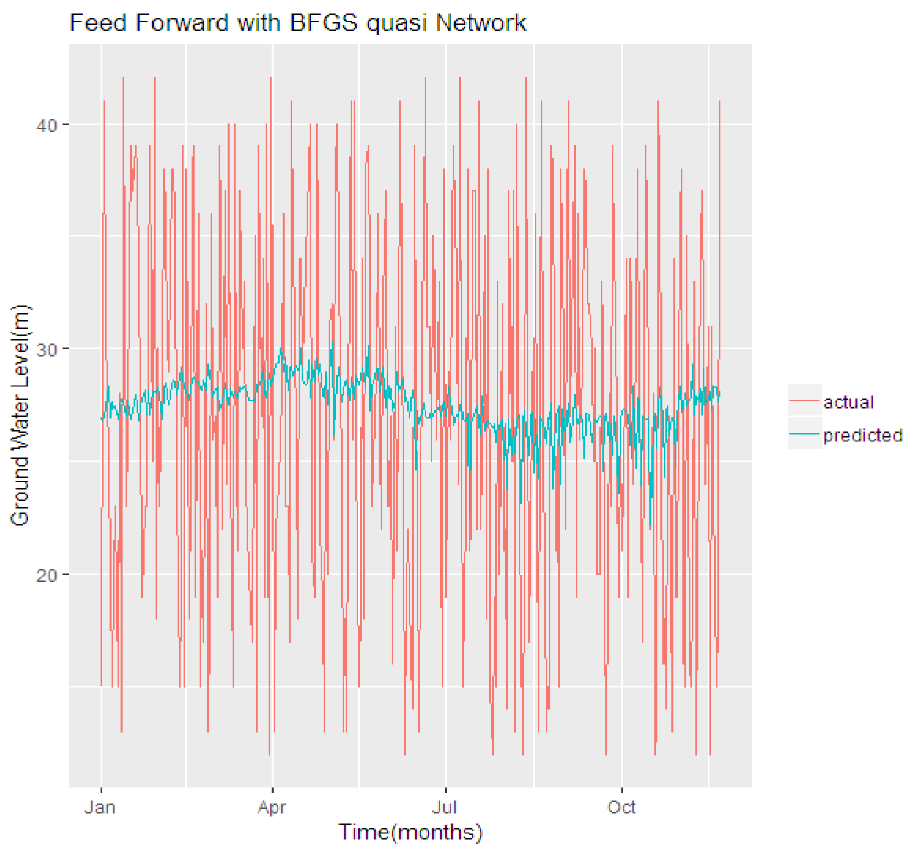

Figure 4.14: Hydrograph of the groundwater for the fouth best performance algorithm Bida

The Figure 4.14 shows the hydrograph of groundwater level (m) against time (month). It is observed that the Feed Forward Neural Network with BFCG quasi Network is the fourth best algorithm that estimated groundwater levels in Bida, the depth to groundwater increases from `November and reached its highest level in April but it is lowest in September.

Figure 4.15 below is the graph of the fifth-best performance algorithm as shown in Table 4.13 with Mean Square Error of (75.54) and is given by the equation (3.9).

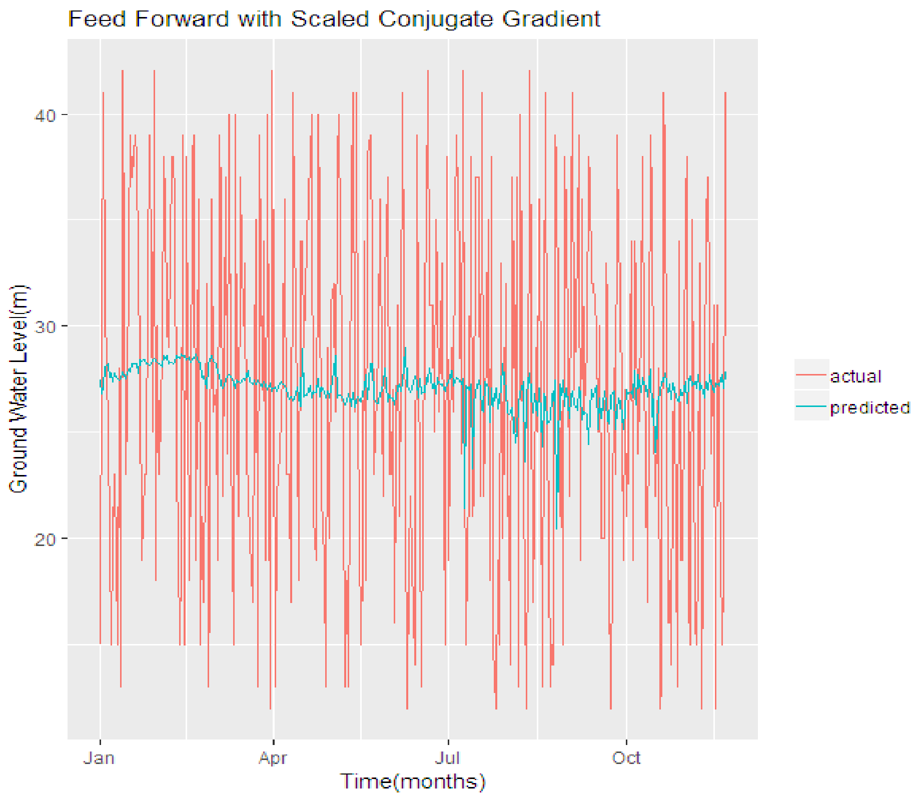

Figure 4.15: Hydrograph of the groundwater for the fifth best performance algorithm in Bida

Figure 4.15 unveils the hydrograph of groundwater level (m) against time (month). It is observed that the Feed Forward Neural Network with Scaled Conjugate Gradient is the fifth best algorithm that estimated groundwater levels in Bida, the depth to groundwater increases from `November and reached its highest level in April but it is lowest in September.

4.10.3 Optimum withdrawal of irrigation water demand in Bida Basin (Bida irrigation site)

the model equation (3.17) is solved and the solution given by equation (3.36) was encoded by Genetic algorithm to determine the quantity of the Groundwater to be drawn from each fourteen well system, considering groundwater maximum and minimum flow rate, Aquifer Specific yield, Irrigation water demand, Irrigation area of the land, potential energy required and water withdrawal efficiency in Lapai irrigation site. As shown in table 4.14.

Table 4.14 Optimum withdrawal of irrigation water demand in Bida basin (Bida irrigation site)

Month	Irrig. Demand (QD) (m^3/month)	Irrig. Areaof land A_i/ha	Specific yield (S_y)	Discharge rate ($Q_{i,j}$) (m^3/d)	Max. Discharge rate (Q_{max}) (m^3/d)	PotentialEnergy Required (kilowatts/Month)	Groundwater level ($Q_{i,j}$ - QD) (m^3/d)	Aquifer depth to Water (m) (i)
January	77280	430	26.3	2,931	7300	6276.66	47970	73.0
February	78481	430	26.8	2,801	7300	4255.72	50471	75.0
March	79470	430	17.3	2789	7300	6611.58	51580	58.0
April	67540	430	10.9	2320	7300	45823.08	44340	55.1
November	64350	430	22.3	3500	7300	26606	60850	54.2
December	66830	430	25.1	3100	7300	2299.5	35830	61.8

$npw = 20,$
$n = 7$
,

$h_s = 0.7,$
$\rho = 0.8$
$g = 0.9$
,

$efficiency = 70.5$
$a_i = b_i = c_i = 1$

Table 4.14 gives the values for irrigation water demand (QD), E_p (potential energy required), efficiency (irrigation water withdrawal efficiency), groundwater discharge rate $\left(Q_{i,j}\right)$, maximum groundwater discharge $\left(Q_{\max}\right)$, area of the irrigation land A_i, groundwater level, $\left(Q_{i,j} - QD\right)$, aquifer depth to water (i), for twenty well as used in equation (3.36). It is observed from the table 4.8 that inequitable potential energy kilowatts (E_p) are needed for the groundwater withdrawal in the study area increases from November to April as the depth to aquifer increases. For a maximum groundwater discharge ($Q_{\max}$) of 7300 m^3/d, the groundwater discharge rate ($Q_{i,j}$) is decreasing, this occurs due to an increase in solar radiation and surface pressure $\left(h_s\right)$ in the study area. We note from the table that the monthly groundwater level in the research area is the result of the difference of the monthly irrigation water demand and the monthly Aquifer Discharge Rate in the study area. The groundwater level ranges from 60850 m^3/d to 44340 m^3/d and the optimum groundwater level occurs in December which is 50471 m^3/d. The depth to aquifer increases as a result of evapotranspiration and groundwater withdrawal. The specific yield of the aquifer shows apparently that the underlying aquifer in the area can sustain irrigation of any crops irrespective of their irrigation water demand

4.10.4 Analysis of groundwater withdrawal in Bida

The model eqution (3.36) is solved and the solution given by equation (3.47) was used to estimate the quantity of the groundwater to be drawn from each fourteen well system as shown in Table. 4.15.

Table 4.15 Result of the net benefit for groundwater extraction in Bida.

(Npw) number of active wells	(H_{max}) withdrawal rate	(yq) aquifer recharge through irrig.	(R) maximum withdrawal rate	L(m) distance between the well	(k) management period in years
1	2800	950	7000	60	7
2	3320	950	7000	25	7
3	4700	950	7000	40	7
4	2900	950	7000	70	7
5	3700	950	7000	90	7
6	4432	950	7000	130	7
7	3600	950	7000	90	7
8	3000	950	7000	110	7
9	3100	950	7000	50	7
10	4466	950	7000	90	7
11	4100	950	7000	50	7
12	2323	950	7000	70	7
13	4600	950	7000	80	7
14	3765	950	7000	100	7
15	2900	950	7000	40	7
16	6800	950	7000		7

$\pi = 3.142,\ B_p = 1500,\ C_p = 1200,\ N_B = 4234.5$

Table 4.15 shows the analysis of the outcome of groundwater extraction in Mokwa. The results show that twenty active wells are considered and we note from the table that the Groundwater withdrawal rate (H_{max}) increases or decreases in values, these occur due to the effects of distance between the well, topographical and variabilities in aquifer depth to water of the each well. the optimum Aquifer withdrawal rate occurs at an observation well number (16) with corresponding well distance of (40m). the discharge rate of the well number (16) is high to the moderate than others. The well withdrawal rate depends on well yield capacity, the major constraint of farming is water and the results shown by the research indicates that each of the well has the capacity to feed irrigation crops. The result also shown that there are moderates withdrawal of the groundwater which nipped in the board the drainage problem that can cause the washing away of the soil minerals.

4.10.5 Analysis of groundwater withdrawal in Mokwa

The model equation (3.36) is solved and the solution given by equation (3.47) was used to determine the values of the Net benefit (N_B) arising from the economic Analysis of Groundwater withdrawal from twenty well system as shown in table 4.16.

Table 4.16 Result of the net benefit for groundwater extraction in Bida

Npw	L (m) Distance Between the well	B_p (₦)	(C_p) (₦)	r (₦)	NB (₦)
1	65	150000	120000	20.5	14895.1
2	100	160000	125000	20.5	11295.4
3	70	150000	120000	20.5	13831.2
4	110	150000	120000	20.5	8801.6
5	130	160000	121000	20.5	9681.8
6	130	160000	120000	20.5	9930.1
7	90	160000	120000	20.5	14343.4
8	150	160000	120000	20.5	8606.0
9	100	160000	120000	20.5	12909.1
10	80	180000	140000	20.5	16136.4

11		250000	160000	20.5	32272.8
	90				
12		160000	162000	20.5	18441.6
	40				
13		270000	165000	20.5	32272.8
	90				
14		300000	170000	20.5	18441.6
	90				
15		300000	171000	20.5	7178.1
	10				
16		320000	180000	20.5	64545.6

$\pi yq = 0.7,\ R = 2500,\ X_{\max} = 719,\ \ k = 7$

Table 4.16 Shows the Economic analysis of the outcome of groundwater extraction in Mokwa. The results show that twenty active wells are considered and as the cost of Groundwater withdrawal rate increased, so is the Benefit per unit supply. We note from the table that the Net Benefit of the project in the research area is the result of the difference of cost of the extraction of the groundwater withdrawal rate in the study area and the Benefit per unit supply in the study area at the recurring interest rate of 20.5million Naira.

4.10.6 The analysis of the computational method for rice crop water requirement in Bida

The model equation (3.48) is solved and is given by equation (3.124) was applied to estimate the crop water need (ET_{cwn}), evapotranspiration (ET_c), crop water coefficient (K_{cb}+K_e), and irrigation area of the land (A_i) as shown in table 4.17

Table 4.17 Shows the results of the computational method for rice crop water need in Bida

Months	ET_o	K_c	ET_{cwn} Mm	A_i Hectares	Crop Yield $K_y\left(1-\frac{ET_0}{ET_{cwn}}\right)A_i$
November	3.94	3.41	6054.71	450	449.55
December	4.32	2.73	5308.07	450	449.63
January	4.41	4.3	8525.74	450	449.75
February	4.83	4.04	8795.35	450	449.86
March	4.06	1.49	2727.35	450	449.33
Average			**2326.75**		**449.62**

Table 4.17 shows the results analysis of the Computation method for Rice irrigation crop water needs. It is observed from the table that the values of reference evapotranspiration (ET_o) through growth season indicate that it was low at the beginning of season and increase gradually till harvesting period. This may be due to changes in climatological norm in Lapai irrigation sites, as the cultivation begins with Solar radiation and relatively high temperature and ended by an increase than it was. The crop coefficient (K_c) result decreases at initial stage of growth (December) and became higher in (February) mid-season with 4.04 (K_c) this indicates that the crop is at the harvesting period which is when the crops need little or no water. The total average rice water need for the period of four months on 450 hectares of land is 2326.75mm and the expected total average crop yield is estimated to be 449.62.87 per hectare. The Rice

water need is values gotten which is conform with crop water requirement estimated under standard condition and that indicates that the crop is disease-free under the optimum soil conditions and achieving full production under the given climatic conditions.

4.10.7 The analysis of the computational method for soya bean crop water requirement in Bida

The model equation (3.48) is solved and the solution given by equation (3.124) was used to estimate the crop water need (ET_{cwn}), evapotranspiration (ET_c), crop water coefficient ($K_{cb}+K_e$), and irrigation area of the land (A_i) as shown in table 4.18

Table 4.18 Shows the results of the computational method for soya bean water need in Bida

Months	ET_o	K_c	ET_{cwn} Mm	A_i Hectares	Crop Yield $K_y\left(1-\frac{ET_0}{ET_{cwn}}\right)A_i$
November	4.31	3.41	1370.63	450	387.6
December	4.59	2.73	3629.89	450	395.2
January	4.19	0.82	517.29	450	396.8
February	5.53	1.64	550.27	450	360
Total			**6068.08**		**1539.6**

Table 4.18 shows the results analysis of the Computation method for Soya bean irrigation crop water needs. It is observed from the table that the values of reference evapotranspiration (ET_o) through growth season indicates that it was 5.31 at the initial stage at the beginning of season and increase gradually till havesting period. This may be due to changes in climatological norm in Lapai irrigation sites, as the cultivation

begins with Solar radiation and relatively high temperature and ended by an increase than it was. The crop coefficient (K_c) result decreases at initial stage of growth (December) and became higher in (February) mid-season with 4.04 (K_c) this indicates that the crop is at the harvesting period which is when the crops need little or no water. The total average rice water need for the period of four months on 450 hectares of land is 6068.08 mm and the expected total average crop yield is estimated to be 1539.6 per hectare. The Rice water need is values gotten conform with crop water requirement estimated under standard condition and indicates that the crop is disease-free, under the optimum soil conditions and achieving full production under the given climatic conditions.

The summary of the results analysis in the three selected study areas are presented in table 4.19

Table 4.19. Results a of the three study areas

Novem – April	Mokwa	Bida	LapaI
Average Actual Groundwater level	$26.78\times10^6\ ft^3$	$21.78\times10^6\ ft^3$	$5.93\times10^6\ ft^3$
Average estimated	$27.19\times10^6\ ft^3$	$21.81\times10^6\ ft^3$	$5.94\times10^6\ ft^3$
TotalAverage cwn for Rice and Soya bean	R=5960.5 mm S=894.802mm	R=2326.75mm S=6068.08mm	R=5584.22mm S=769.03mm
Crop yield In thousand metric tonns	R=399.702mm S=391.06mm	R=449,62mm S=1539.6mm	R=387.98mm S=2185.44m

			m
Irrigation water demand	71260.7mm	74465.3mm	35614.16mm

The table 4.19 shows the summary of the results from the three selected study areas. The values for the average estimated groundwater level in Mokwa is higher than the two remaining irrigation sites. This indicates that the overlying aquifer in Mokwa irrigation sites is closer to the river Niger. The levelness of the land is another factor that responsible for the higher quantity of groundwater in Mokwa. The irrigator farmers in Mokwa have advantage of practicing inter-cropping and recycling the practice of irrigation farming before rainfall season than other two remaining study areas. The crop required more water for growth in Mokwa and Lapai than Bida, the factor responsible for this is the relative high temperature and solar radiation. The farmers tend to produce more crops and gain profit in Mokwa than Bida and Lapai as the aquifer there gave higher values than Bida and Lapai. Bida has more crop yield as shown by the research work than Mokwa and Lapai. This is due to the fact that the optimum soil water condition there are better and higher than Mokwa and Bida. The irrigation water demand in Bida is higher than Mokwa and lapai, the changes in the climatological norm is responsible for it.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this research work, the optimal irrigation schedule and aquifer withdrawal system during the dry season was proposed and analysed. The study proffered solutions to estimation of the quantity of groundwater available during the dry season in Bida Basin and determined the optimum groundwater to be drawn from the well at a given time so as to avoid over withdrawal or under withdrawal of the groundwater in the irrigation sites.

Five climatic data obtained from Nigeria metrological station, Abuja were used to train and test in Artificial Neural Network. Result from the analysis showed that the Feed Forward Levenberg algorithm has the best overall performance out of the fifteen algorithms being compared in Lapai irrigation site.The actual and the estimated groundwater levels are $5.93 \times 10^6\ ft^3$ and $5.94 \times 10^6\ ft^3$ respectively. Similar results were obtained for actual and the estimated groundwater level in Mokwa and Bida. To proffer solution to groundwater to be drawn from the well at a given time, different population sizes from 50 to 200 of the Genetic Algorithm are explored and exploited.We investigated other key parameters in the Genetic Algorithm which include Two-point crossover (TPC), Single point crossover (SPC), scattered crossover (SC), Arithmetic crossover (AC), Heuristic crossover (HC) and intermediate crossover (IT) were also carried out. Results from the analysis showed that the groundwater withdrawal in each of the well located in the study areas were moderate as irrigation water demand and depth to groundwater could be confirmed from Tables 4.3, 4.9 and 4.15.

The computational method results analysis for estimated crop water needs show tha the highest crop water need for rice crop was observed during mid- stage and highest crop water need for soya bean was observed during the developmental stage.The information to irrigator famers is that if the groundwater level is estimated and optimum groundwater to be withdrawn at a given time is accurately determined and also the crop water need results are compared and found to conformed to the obtained results under standard conditions, the crop yield would drastically increase. The computation result with the area of the land given can then be used to forecast crop yield.

Our computational method results also show that crop water need for crops can better be estimated if the size of the land for irrigation is considered as it was done in equation (3.118). The irrigator farmers can have knowledge of the estimated crop water need for a given size of the Land before commencing irrigation exercise

5.2 Contribution to Knowledge

The research work established the cropwater needs, the quantity of the groundwater available and optimum groundwater to be drawn for irrigation practices during the dry season. The model estimated groundwater available to be $5960.5mm$ in Mokwa; $2326mm$ in Bida and 5584.22 in Lapai and the crop water need for Soya Bean are $984mm$ 894mm in Mokwa,Bida, and $769.03mm$ in Lapai respectively. It is also use to estimate crop yield for a given area of land. The crop yield for rice are $399.702mm$ in mokwa, $449.6mm$ in Bida and $387.98mm$ in Lapai and the copyield for Soya Bean are $391.06mm$ in Mokwa , 2185.44 in Bida and $2185mm$ in Lapai respectively. The estimation of groundwater available is $27.19\times10^{6}\ ft^{3}$ (Mokwa), $21.81\times10^{6}\ ft^{3}$ (Bida) and 5.93×10^6 ft^3 (Lapai) during the dry season provides an important information to farmers which is useful in proper planning for optimum crop yield.

5.3 Recommendations

The followings recommendations are hereby made:

(i) Estimation of the groundwater during the rainfall and dry seasons (full irrigation) can be carried out as further sturdy.

(ii) Further research on crop water needs can be carried out on field with irregular shape.

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APPENDIX A

Analysis of the Net Benefit for groundwater extraction

> $restart;$

> $n := 20; C[p] := 12000; B[p] := 15000; r := 1.5; k := 7;$

$$n := 20$$

$$C_p := 12000$$

$$B_p := 15000$$

$$r := 1.5$$

$$k := 7$$

> $NB := \dfrac{n \cdot \pi \cdot y[q] \cdot (B[p] - C[p]) \cdot \left(\left(\frac{1}{1+r} \right)^{-k} - 1 \right)}{\left(\sqrt{R^2+4} \cdot \sqrt{R^2 + 4 \cdot X^2} - R^2 - 4 X^2 \right) \cdot L}$

$$NB := \frac{3.656109375\, 10^7\, \pi\, y_q}{\left(\sqrt{R^2+4}\, \sqrt{R^2+4 X^2} - R^2 - 4 X^2 \right) L}$$

>

> $sol1 := eval(NB, \{y[q] = 0.25, X = 0.1080, L = 0.65\})$

$$sol1 := \frac{1.406195913\, 10^7\, \pi}{\sqrt{R^2+4}\, \sqrt{R^2 + 0.04665600} - R^2 - 0.04665600}$$

> $sol2 := eval(NB, \{y[q] = 0.25, X = 0.2580, L = 0.65\})$

$$sol2 := \frac{1.406195913\, 10^7\, \pi}{\sqrt{R^2+4}\, \sqrt{R^2 + 0.26625600} - R^2 - 0.26625600}$$

> $sol3 := eval(NB, \{y[q] = 0.25, X = 0.3980, L = 0.65\})$

$$sol3 := \frac{1.406195913\, 10^7\, \pi}{\sqrt{R^2+4}\, \sqrt{R^2 + 0.63361600} - R^2 - 0.63361600}$$

Appendix B

Optimum withdrawal of irrigation water demand in Bida Basin

> *restart*

> $Q_{i,j} := 2531$

$$Q_{i,j} := 2531$$

> $S_y := 26.3$

$$S_y := 26.3$$

> $a_i := 1$

$$a_i := 1$$

> $b_i := 1$

$$b_i := 1$$

> $c_i := 1$

$$c_i := 1$$

> $Q_{\max} := 5300$

$$Q_{\max} := 5300$$

> $\rho := 0.1$

$$\rho := 0.1$$

> $A := 10000$

$$A := 10000$$

> $h_s := 200$

$$h_s := 200$$

> $e := 70$

$$e := 70$$

> $g := 8$

$$g := 8$$

> $l := 5$

$$l := 5$$

> $Q_{\mathrm{D}} := 10$

$$Q_{\mathrm{D}} := 10$$

> $npw := 2$

$$npw := 2$$

> $n := 2$

$$n := 2$$

> $Z = \frac{l \cdot \rho \cdot g \cdot A \cdot h_s}{e} \cdot \left(\frac{Q_{i,j} S_y}{a_i \cdot Q_{i,j}^2 + b_i \cdot Q_{i,j} + c_i} + \frac{\left(a_i \cdot Q_{\max}^2 - c_i\right) \cdot S_y}{\left(a_i \cdot Q_{i,j}^2 + b_i \cdot Q_{i,j} + c_i\right)^2} \left(Q_{i,j} - Q_{\mathrm{D}}\right) \right)$

$$Z = 6369.842866$$

> $Z = \frac{l \cdot \rho \cdot g \cdot A \cdot h_s}{e} \cdot \sum_{i=1}^{npw} \sum_{j=1}^{n} \left(\frac{Q_{i,j} S_y}{a_i \cdot Q_{i,j}^2 + b_i \cdot Q_{i,j} + c_i} + \frac{\left(a_i \cdot Q_{\max}^2 - c_i\right) \cdot S_y}{\left(a_i \cdot Q_{i,j}^2 + b_i \cdot Q_{i,j} + c_i\right)^2} \left(\sum_{i=1}^{npw} Q_{i,j} - Q_{\mathrm{D}} \right) \right)$

$$Z = 46292.61313$$

APPENDIX C

An Algorithm design for equation (3.9)

Initialize weights;

While not stop criterion do

Calculate $(h-h_0)e^{ct}(w)$ for each pattern;

$$ei=\sum_{p=1}^{p}(h-h_0)e^{ct}(w)^T h-h_0 e^{ct}(w);$$

Calculate $J^p(w)$ for each pattern;

Repeat

Calculate Δw_j

$$e_2=\sum_{p=1}^{p}h-h_0 e^{ct}(w+\Delta w)^T h-h_0 e^{ct}(w+\Delta w);$$

If $(e_1=e_2)$ then

$\mu := \mu * B$

End if;

Until ($(e_2<e_1)$

$\mu := \frac{\mu}{B}$

$w=w+\Delta w$;

End while

$\forall$ i; $\Delta_{k,i}$ (0)=Δ_0, $\frac{\partial c}{\partial w_{k,i}}$(0)=0

For t←1 to T

For all parameters (weights) do.

Δ update each weight

Calculated derivative of each weight $\frac{\partial c}{\partial w_{k,i}}$ like in EBP

If $\frac{\partial c}{\partial w_{k,i}}$(t-1)*$\frac{\partial c}{\partial w_{k,i}}$(t)>0, then

$\Delta_{k,i}$(t)=min($\Delta_{k,i}$(t-1)* η^+, Δ max)

$\Delta w_{k,i}$ (t)= -sign $\left(\frac{\partial c}{\partial w_{k,i}}(t)\right)\Delta_{k,i}(t)$

$\Delta w_{k,i}(t+1) = w_{k,i}(t) + \Delta w_{k,i}(t)$

Else if $\frac{\partial c}{\partial w_{k,i}}(t-1) * \frac{\partial c}{\partial w_{k,i}}(t) < 0$ then

$\Delta_{k,i} = \max(\Delta_{k,i}(t-1) * \eta^{-}, \Delta_{\min})$

$w_{k,i}(t-1) = w_{k,i}(t) - \Delta w_{k,i}(t-1)$

$\frac{\partial c}{\partial w_{k,i}} = 0$

Else if $\frac{\partial c}{\partial w_{k,i}}(t-1) * \frac{\partial c}{\partial w_{k,i}}(t) = 0$ then

$\Delta w_{k,i}(t) = -sign\left(\frac{\partial c}{\partial w_{k,i}}(t)\right) * (\Delta_{k,i}(t)$

$w_{k,i}(t+1) = w_{k,i}(t) + \Delta_{k,i}(t)$

End if

End for

Algorithm

```
function out1 = layrecnet(varargin)
  persistent INFO;
  if isempty(INFO), INFO = get_info; end
  if (nargin > 0) && ischar(varargin{1}) ...
      && ~strcmpi(varargin{1},'hardlim') && ~strcmpi(varargin{1},'hardlims')
    code = varargin{1};
    switch code
      case 'info',
        out1 = INFO;
      case 'check_param'
        err = check_param(varargin{2});
        if ~isempty(err), nnerr.throw('Args',err); end
        out1 = err;
      case 'create'
        if nargin < 2, error(message('nnet:Args:NotEnough')); end
```

```
        param = varargin{2};
        err = nntest.param(INFO.parameters,param);
        if ~isempty(err), nnerr.throw('Args',err); end
        out1 = create_network(param);
        out1.name = INFO.name;
      otherwise,
            try
          out1 = eval(['INFO.' code]);
        catch %#ok<CTCH>
          nnerr.throw(['Unrecognized argument: ''' code ''''])
        end
    end
  else
    [args,param] = nnparam.extract_param(varargin,INFO.defaultParam);
    [param,err] = INFO.overrideStructure(param,args);
    if ~isempty(err), nnerr.throw('Args',err,'Parameters'); end
    net = create_network(param);
    net.name = INFO.name;
    out1 = init(net);
  end
end
 function v = fcnversion
  v = 7;
end
function info = get_info
  info = nnfcnNetwork(mfilename,'Layer Recurrent Neural Network',fcnversion, ...
    [ ...
    nnetParamInfo('layerDelays','Layer Delays','nntype.strictpos_delayvec',1:2,...
    'Row vector delays in each layers feedback connection.'), ...
    nnetParamInfo('hiddenSizes','Hidden Layer Sizes','nntype.strict_pos_int_row',10,...
    'Sizes of 0 or more hidden layers.'), ...
    nnetParamInfo('trainFcn','Training Function','nntype.training_fcn','trainlm',...
    'Function to train the network.'), ...
    ]);
```

```
end

function err = check_param(param)
  err = '';
end

function net = create_network(param)
  net = feedforwardnet(param.hiddenSizes,param.trainFcn);
  for i=1:(net.numLayers-1)
    net.layerConnect(i,i) = true;
    net.layerWeights{i,i}.delays = param.layerDelays;
  end
  if isdeployed
      net.trainParam.showWindow = false;
  else
      net.plotFcns = [net.plotFcns {'plotresponse','ploterrcorr','plotinerrcorr'}];
  end
end
```

Code for feed forward neural network

```
function out1 = feedforwardnet(varargin)
  persistent INFO;
  if isempty(INFO), INFO = get_info; end
  if (nargin > 0) && ischar(varargin{1}) ...
      && ~strcmpi(varargin{1},'hardlim') && ~strcmpi(varargin{1},'hardlims')
    code = varargin{1};
    switch code
      case 'info'
        out1 = INFO;
      case 'check_param'
        err = check_param(varargin{2});
        if ~isempty(err), nnerr.throw('Args',err); end
        out1 = err;
      case 'create'
        if nargin < 2, error(message('nnet:Args:NotEnough')); end
```

```
      param = varargin{2};
      err = nntest.param(INFO.parameters,param);
      if ~isempty(err), nnerr.throw('Args',err); end
      out1 = create_network(Zparam);
      out1.name = INFO.name;
     otherwise
      try
        out1 = eval(['INFO.' code]);
      catch
      nnerr.throw(['Unrecognized argument: ''' code ''''])
      end
    end
  else
    [args,param] = nnparam.extract_param(varargin,INFO.defaultParam);
    [param,err] = INFO.overrideStructure(param,args);
    if ~isempty(err), nnerr.throw('Args',err,'Parameters'); end
    net = create_network(param);
    net.name = INFO.name;
    out1 = init(net);
  end
end
 function v = fcnversion
  v = 7;
end
function info = get_info
  info = nnfcnNetwork(mfilename,'Feed-Forward Neural Network',fcnversion, ...
    [ ...
    nnetParamInfo('hiddenSizes','Hidden Layer Sizes','nntype.strict_pos_int_row',10,...
    'Sizes of 0 or more hidden layers.'), ...
    nnetParamInfo('trainFcn','Training Function','nntype.training_fcn','trainlm',...
    'Function to train the network.'), ...
    ]);
  end
 function err = check_param(~)
```

```
  err = '';
end
 function net = create_network(param)
 net = network;
  Nl = length(param.hiddenSizes)+1;
  net.numLayers = Nl;
  net.biasConnect = true(Nl,1);
  [j,i] = meshgrid(1:Nl,1:Nl);
  net.layerConnect = (j == (i-1));
  for i=1:Nl
    if i == Nl
      net.layers{i}.name = 'Output';
    else
      if (Nl == 2)
        net.layers{i}.name = 'Hidden';
      else
        net.layers{i}.name = ['Hidden ' num2str(i)];
      end
      net.layers{i}.size = param.hiddenSizes(i);
      net.layers{i}.transferFcn = 'tansig';
    end
    net.layers{i}.initFcn = 'initnw';
  end
  net.numInputs = 1;
  net.inputConnect(1,1) = true;
  net.inputs{1}.processFcns = {'removeconstantrows','mapminmax'};
  net.outputConnect(Nl) = true;
  net.outputs{Nl}.processFcns = {'removeconstantrows','mapminmax'};
  net.dividefcn = 'dividerand';
  net.trainFcn = param.trainFcn;
  net.performFcn = 'mse';
  net.adaptFcn = 'adaptwb';
  net.inputWeights{1,1}.learnFcn = 'learngdm';
  net.layerWeights{find(net.layerConnect)'}.learnFcn = 'learngdm';
```

```
  net.biases{:}.learnFcn = 'learngdm';
  net.plotFcns = iPlotFcns();
end
function plotFcns = iPlotFcns()
if isdeployed
    plotFcns = {};
else
    plotFcns = {'plotperform','plottrainstate','ploterrhist','plotregression'};
end
end
```

Code for cascade forward network

```
function out1 = cascadeforwardnet(varargin)
  persistent INFO;
  if isempty(INFO), INFO = get_info; end
  if (nargin > 0) && ischar(varargin{1}) ...
      && ~strcmpi(varargin{1},'hardlim') && ~strcmpi(varargin{1},'hardlims')
    code = varargin{1};
    switch code
      case 'info',
        out1 = INFO;
      case 'check_param'
        err = check_param(varargin{2});
        if ~isempty(err), nnerr.throw('Args',err); end
        out1 = err;
      case 'create'
        if nargin < 2, error(message('nnet:Args:NotEnough')); end
        param = varargin{2};
        err = nntest.param(INFO.parameters,param);
        if ~isempty(err), nnerr.throw('Args',err); end
        out1 = create_network(param);
        out1.name = INFO.name;
      otherwise,
        try
          out1 = eval(['INFO.' code]);
        catch           nnerr.throw(['Unrecognized argument: ''' code ''''])
        end
    end
```

```
  else
    [args,param] = nnparam.extract_param(varargin,INFO.defaultParam);
    [param,err] = INFO.overrideStructure(param,args);
    if ~isempty(err), nnerr.throw('Args',err,'Parameters'); end
    net = create_network(param);
    net.name = INFO.name;
    out1 = init(net);
  end
end

function v = fcnversion
  v = 7;
end
function info = get_info
  info = nnfcnNetwork(mfilename,'Cascade-Forward Neural Network',fcnversion, ...
    [ ...
    nnetParamInfo('sizes','Hidden Layer Sizes','nntype.strict_pos_int_row',10,...
    'Sizes of 0 or more hidden layers.'), ...
    nnetParamInfo('trainFcn','Training Function','nntype.training_fcn','trainlm',...
    'Function to train the network.'), ...
    ]);
end
function err = check_param(param)
  err = '';
end
function net = create_network(param)
net = feedforwardnet(param.sizes,param.trainFcn);
  net.inputConnect = true(net.numLayers,1);
  [j,i] = meshgrid(1:net.numLayers,1:net.numLayers);
  net.layerConnect = (j < i);
  if isdeployed
      % Do not show training GUI for deployed code
      net.trainParam.showWindow = false;
  end
```

APPENDIX D

Based on the evaluation of aquifer withdrawal in the selected areas, we design an algorithm

For equations (3.39) as follows:

Procedure for GA

Input: Problem data, GA Parameter

Output: Optimal Solutions Q_j

For Gen=1 to MAX_Gen

Begin

For j=1 to N

npw←0_j

Initialize p(npw) by encoding routine calculate objectives $F_j(p)$, j=1,2,…….q) bydecoding routine;

Create E (Q^{max});

Evaluate eval (Q^{max})by fitness assignment routine;

While (not terminating condition) do

Create Q_d (npw) by crossover routine;

Create Q_d (npw) by mutation routine;

Calculate objectives f_i (Q_d), j= 1…npw by decoding routine;

Update (Q_d(npw);

Evaluate eval Q_d (npw) by fitness assignment routine;

Select p(npw+1) from Q_d(npw) by selection routine;

npw←npw+1;

end

output optimal solutions E(Q^{max})

End

Begin

For i= 1 to N

M←0

Initialize p(npw) by encoding routine;

Create E(Qi^{max});

Evaluate eval (Qi^{max}) by fitness assignment routine;

While (not terminating condition) do

Create (npw) by crossover routine;

Create (npw) by mutation routine;

Create objective F_i(QD) i=1,…q by decoding routing;

Update $_{Qd}$ (npw);

Evaluate eval $_{Qd}$ (npw) by fitness assignment routine;

Select p(npw+1) from $_{Qd}$ (npw) by selection routine;

npw←npw+1;

for s_y←$ns_r^{-1}$to pop

if s_{r-1}< random () ≤$s_r$then

Code for genetic algorithms

```
function [x,fval,exitFlag,output,population,scores] =
ga(fun,nvars,Aineq,bineq,Aeq,beq,lb,ub,nonlcon,intcon,options)
defaultopt = struct('PopulationType', 'doubleVector', ...
    'PopInitRange', [], ...
    'PopulationSize', '50 when numberOfVariables <= 5, else 200', ...
    'EliteCount', '0.05*PopulationSize', ...
    'CrossoverFraction', 0.8, ...
```

```
'MigrationDirection','forward', ...

'MigrationInterval',20, ...

'MigrationFraction',0.2, ...

'Generations', '100*numberOfVariables', ...

'TimeLimit', inf, ...

'FitnessLimit', -inf, ...

'StallTest', 'averageChange', ...

'StallGenLimit', 50, ...

'StallTimeLimit', inf, ...

'TolFun', 1e-6, ...

'TolCon', 1e-3, ...

'InitialPopulation',[], ...

'InitialScores', [], ...

'NonlinConAlgorithm', 'auglag', ...

'InitialPenalty', 10, ...

'PenaltyFactor', 100, ...

'PlotInterval',1, ...

'CreationFcn',@gacreationuniform, ...

'FitnessScalingFcn', @fitscalingrank, ...

'SelectionFcn', @selectionstochunif, ...

'CrossoverFcn',@crossoverscattered, ...

'MutationFcn',{{@mutationgaussian 1  1}}, ...

'HybridFcn',[], ...

'Display', 'final', ...

'PlotFcns', [], ...
```

```
    'OutputFcns', [], ...
    'Vectorized','off', ...
    'UseParallel', false);
try
    narginchk(1,11);
catch ME
    error(message('globaloptim:ga:numberOfInputs', ME.message));
end
if nargin == 1 && nargout <= 1 && strcmpi(fun,'defaults')
    x = defaultopt;
    return
end

if nargin < 11, options = [];
    if nargin < 10,  intcon = [];
        if nargin < 9,  nonlcon = [];
            if nargin < 8, ub = [];
                if nargin < 7, lb = [];
                    if nargin <6, beq = [];
                        if nargin <5, Aeq = [];
                            if nargin < 4, bineq = [];
                                if nargin < 3, Aineq = [];
                                end
                            end
                        end
```

```
                    end
                end
            end
        end
    end
end
if nargin == 3 && (isstruct(Aineq) || isa(Aineq, 'optim.options.SolverOptions'))
    options = Aineq; Aineq = [];
end
if nargin == 10 && (isstruct(intcon) || isa(intcon, 'optim.options.SolverOptions'))
    options = intcon;
    intcon = [];
end
if nargin == 1
    if isa(fun,'struct')
        [fun,nvars,Aineq,bineq,Aeq,beq,lb,ub,nonlcon,intcon,rngstate,options] =
separateOptimStruct(fun);
        resetDfltRng(rngstate);
    else
error(message('globaloptim:ga:invalidStructInput'));
    end
end
options = prepareOptionsForSolver(options, 'ga');
if isfield(options, 'CreationFcn')
```

```
    options.CreationFcn=replaceEnumStringWithFcnHdl('GaOptions','CreationFcn',
options.CreationFcn);
end
if isfield(options, 'CrossoverFcn')
   options.CrossoverFcn=replaceEnumStringWithFcnHdl('GaOptions','CrossoverFcn',
options.CrossoverFcn);
end
if isfield(options, 'FitnessScalingFcn')
    options.FitnessScalingFcn = replaceEnumStringWithFcnHdl('GaOptions',
'FitnessScalingFcn', options.FitnessScalingFcn);
end
if isfield(options, 'HybridFcn')
    options.HybridFcn=replaceEnumStringWithFcnHdl('GaOptions', 'HybridFcn',
options.HybridFcn);
end
if isfield(options, 'MutationFcn')
    options.MutationFcn=replaceEnumStringWithFcnHdl('GaOptions','MutationFcn',
options.MutationFcn);
end
if isfield(options, 'PlotFcns')
    options.PlotFcns = replaceEnumStringWithFcnHdl('GaOptions', 'PlotFcn',
options.PlotFcns);
end
if isfield(options, 'SelectionFcn')
```

```
    options.SelectionFcn=replaceEnumStringWithFcnHdl('GaOptions','SelectionFcn',
options.SelectionFcn);
end
if iscell(fun)
    FitnessFcn = fun{1};
else
    FitnessFcn = fun;
end
if isempty(FitnessFcn) ||  ~(isa(FitnessFcn,'inline') || isa(FitnessFcn,'function_handle'))
    error(message('globaloptim:ga:needFunctionHandle'));
end
valid =  isnumeric(nvars) && isscalar(nvars)&& (nvars > 0) ...
    && (nvars == floor(nvars));
if ~valid
    error(message('globaloptim:ga:notValidNvars'));
end
defaultopt.PopInitRange = [-10;10];
if ~isempty(intcon)
    gaminlpvalidateoptions(options);
    defaultopt.PopulationSize = max(min(10*nvars, 100), 40);
    defaultopt.EliteCount = ceil(0.05*defaultopt.PopulationSize);
    defaultopt.PopInitRange = [-1e4 + 1; 1e4 + 1];
end
user_options = options;
if ~isempty(options) && ~isa(options,'struct')
```

```
        error(message('globaloptim:ga:optionsNotAStruct'));
elseif isempty(options)
    options = defaultopt;
end
options = gaoptimset(defaultopt,options);
options.UserSpecPopInitRange = isa(user_options, 'struct') && ...
    isfield(user_options, 'PopInitRange') && ~isempty(user_options.PopInitRange);
msg = isoptimargdbl('GA', {'NVARS','A',   'b',   'Aeq','beq','lb','ub'}, ...
                            nvars,  Aineq, bineq, Aeq,  beq,  lb,  ub);
if ~isempty(msg)
    error('globaloptim:ga:dataType',msg);
end
options.MultiObjective = false;

[x,fval,exitFlag,output,population,scores,FitnessFcn,nvars,Aineq,bineq,Aeq,beq,lb,ub,
...
    NonconFcn,options,Iterate,type]  =  gacommon(nvars,fun,Aineq,bineq,Aeq,beq,lb,ub,
...
                                              nonlcon,intcon,options,user_options);

if exitFlag < 0
    return;
end
if isempty(Aineq)
    Aineq = zeros(0,nvars);
```

```
end
if isempty(bineq)
    bineq = zeros(0,1);
end
if isempty(Aeq)
    Aeq = zeros(0,nvars);
end
if isempty(beq)
    beq = zeros(0,1);
end
 if ~isempty(options.OutputFcns) || ~isempty(options.PlotFcns)
    options.OutputPlotFcnOptions  = optimoptions(@ga);
    options.OutputPlotFcnOptions =
copyForOutputAndPlotFcn(options.OutputPlotFcnOptions,options);
end
 if ~isempty(intcon)
    [x,fval,exitFlag,output,population,scores] = gaminlp(FitnessFcn,nvars, ...
        Aineq,bineq,Aeq,beq,lb,ub,NonconFcn,intcon,options,output,Iterate);
else
    switch (output.problemtype)
        case 'unconstrained'
            [x,fval,exitFlag,output,population,scores] = gaunc(FitnessFcn,nvars, ...
                options,output,Iterate);
        case {'boundconstraints', 'linearconstraints'}
            [x,fval,exitFlag,output,population,scores] = galincon(FitnessFcn,nvars, ...
```

```
            Aineq,bineq,Aeq,beq,lb,ub,options,output,Iterate);
        case 'nonlinearconstr'
            if strcmpi(options.NonlinConAlgorithm,'auglag')
                [x,fval,exitFlag,output,population,scores] = gacon(FitnessFcn,nvars, ...
                    Aineq,bineq,Aeq,beq,lb,ub,NonconFcn,options,output,Iterate,type);
            else
                [x,fval,exitFlag,output,population,scores] = gapenalty(...
                    FitnessFcn,nvars,Aineq,bineq,Aeq,beq,lb,ub, ...
                    NonconFcn,[],options,output,Iterate,type);
            end
    end
end
```

Compute $\sum_{j=1}^{npw} Q\ j\ eval(s_y)$;

APPENDIX E

Computational Data Entry for Rice Crop Water Need

	Rn	G	Gamma	Ta	U2	Es – Ea	Delta	Kcb	Ke	l	b	Pn	L	B	num	dnum	ETo	K	A	ETc
January	13.366	0	0.056	20.9	2.229	0.62	0.152	0.15	0.67	0.5	0.5	80000	100	100	1.066	0.25	4.256	0.82	9950	34725
February	12.902	0	0.056	20.8	2.193	0.588	0.151	1.1	0.54	0.5	0.5	80000	100	100	1.016	0.25	4.085	1.64	9950	66651
March	12.714	0	0.056	20.2	2.339	0.702	0.146	0.3	0.42	0.5	0.5	80000	100	100	1.04	0.25	4.217	0.72	9950	30209
April	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
May	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
June	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
July	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
August	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
September	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
October	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
November	12.923	0	0.956	20.7	2.4	0.679	0.149	0.12	0.11	0.5	0.5	80000	100	100	1.1	0.25	4.311	0.3	9950	12765

December	**13.021**	0	0.056	20.9	2.448	0.697	0.152	0.15	0.15	0.5	0.5	80000	100	100	1.1	0.25	4.321	0.3	9950	12897
	5.4105	**0**	**0.0983**	**8.625**	**0.9674**	**0.274**	**0.063**	**0.15**	**0.1575**	**0.2083**	**0.2083**	**33333**	**41.67**	**41.7**	**0.443**	**0.1**	**1.766**	**0.32**	**4146**	**13104**
																				157247
																				963.21

Computational Data Entry for Soya Bean Crop Water Need

	Rn	G	Gamma	Ta	U2	Es - Ea	Delta	Kcb	Ke	l	b	Pn	L	B	num	dnum	ETo	K	A	ETc
January	13.4	0	0.056	20.9	2.23	0.6	0.152	3.63	0.67	0.2	0.2	5E+05	100	100	1.07	0.3	4.2561	4.3	9979.96	182645
February	12.9	0	0.056	20.8	2.19	0.6	0.151	3.5	0.54	0.2	0.2	5E+05	100	100	1.02	0.2	4.0846	4.04	9979.96	164688
March	12.7	0	0.056	20.2	2.34	0.7	0.146	1.07	0.42	0.2	0.2	5E+05	100	100	1.04	0.2	4.0676	1.49	9979.96	60486.4
April	12.4	0	0.056	19.3	2.22	0.6	0.136	0.85	0.34	0.2	0.2	5E+05	100	100	1.03	0.2	4.0442	1.51	9979.96	60486.4
May	12.2	0	0.056	18.2	2	0.6	0.124	0,62	0.22	0.2	0.2	5E+05	100	100	1	0.2	4.0221	1.54	9979.96	154323
June	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
July	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
August	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

September	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
October	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
November	13.3	0	0.056	21.1	2.74	1	0.154	3.26	0.15	0.67	0.2	5E+05	100	100	1.29	0.3	4.9259	3.41	56979.9	957105
December	**13**	0	0.056	20.9	2.45	0.7	0.152	2.58	0.15	0.67	0.2	5E+05	100	100	1.1	0.3	4.3208	2.73	56979.9	672118
	7.49	**0**	**0.033**	**11.8**	**1.35**	**0.4**	**0.085**	**1.354**	**0.21**	**0.2**	**0.117**	**3E+05**	**58.3**	**58.3**	**0.63**	**0.1**	**2.4768**	**1.59**	**13655**	**187654**
	89.9	0	0.392	141	16.2	4.8	1.015	14.89	2.49	2.34	1.4	4E+06	700	700	7.55	1.7	29.721	19	163860	2251851
																				13595.1

APPENDIX F

WORK PLAN

S/N	ACTIVITY	PERIOD
1	Properties of solution	July, 2018 to January 2019
2	Solutions of the models/ problem	Febuary,2019 to June.2019
3	Tables and graphical representation of the solution as our results	July,2019 to November, 2019
4	Editorial work	December,2019 to May 2020