**Mathematical Analysis of Discontinuities in the Flow Field of Gas in a Cylindrical Pipe Using Diffusion MRI**

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**Abstract**

*In this study, Magnetic Resonance Imaging (MRI) is used to detect partial and total blockage of hydrogen gas in a cylindrical pipe. Diffusion Magnetic Resonance (DMR) equation is solved analytically for flow of fluid in a radially symmetric cylindrical pipe. Appropriate boundary conditions were imposed and the radial axis varied to depict partial and total blockage in the pipe. The results show that for free fluid flow, the magnetization is between 0.004 and 0.005. For partial blockage, the magnetization reduces (signal loss) in value to 0.00001 and for total blockage it is zero (0). This method is a viable alternative to other methods of detecting blockage in fluid pipelines in oil and gas industry due to its non-invasive analysis of flow in fluid. The MRI model also registers signal in its first few seconds or microseconds. The analysis can also be useful in process industries where different network of pipes are used or machines use cylindrical pipes or tubes in transporting materials especially when there is a partial or total blockage at any point in the network.*

**Keywords**: Bloch NMR Equations, DMR equation, Cylindrical pipe, Magnetization.

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**Introduction**

Diffusion Magnetic Resonance Imaging (DMRI) is one of the most rapidly evolving techniques in the MRI field. It provides accurate assessment of the individual component or multi-component systems in a matter of minutes whereas traditional radioactive tracer techniques may take weeks for each component (Awojoyogbe *et al.,* 2011). Diffusion and flow can be measured very delicately and accurately using Magnetic Resonance Imaging (Hazlewood *et al.,* 1974). From the Bloch MRI flow equations and solution, the coefficient of diffusion can be obtained in terms of the flow parameters. These parameters are very important in the investigation of flow in bounded geometries. This has been used in biological flow, catalysis and other materials as well as fluid movement in hydrocarbon reservoirs and ground water movement and pollution (Awojoyogbe *et al*., 2009). Coefficient of diffusion of a substance, defined as the amount of material that diffuses in a certain time, plays a vital role in the detection of blockage in a pipe using MRI. Random diffusion motion of water molecules has intriguing properties depending on the physiological and anatomical environment of the organisms being studied. This is the principle being exploited by the method of DMRI. Though not widely known, it has been noted for long that nuclear magnetic resonance is capable of quantifying diffusion movement of molecules as a result of uniqueness in relaxation rates -  and  (Yusuf *et al.,* 2010).

Some attempts have been made in the past to detect blockage. Yuan used time splitting algorithms and Godunov mixed format to simulate the pulse propagation in the blocked pipelines (Yuan *et al.,* 2014). Another technique used by Sattar is by the system frequency response. This is a technique whereby the frequency response is used in the detection of partial blockages in a pipeline (Sattar *et al.,* 2008). Similar to this is the method adopted by Mohapatra for the detection of partial blockages in single pipelines by the frequency response method (Mohapatra *et al.,* 2006). Wang also investigated analytically the effects of a partial blockage on pipeline transients. A partial blockage is simulated using an orifice equation and the influence of the blockage on the unsteady pipe flow is considered in the equation using a Dirac delta function (Wang *et al.,* 2005). Ma *et al.,* (2007) worked extensively on the presence of a layer inside a pipe. He further revealed that obstruction scatters the guided wave propagating inside the pipe. Both the reflection and transmission of the guided wave were later used to effectively discover and characterize the layer.

In this work, the principle of Magnetic Resonance is applied to a cylindrical pipe under the influence of radiofrequency field as a probe to perturb the molecules of hydrogen gas. This causes the nuclei to absorb energy from the applied electromagnetic (EM) pulse(s) and radiate this energy at a specific resonance frequency which depends on the strength of the magnetic field and other factors. A Radio Frequency (RF) transmitter is needed to transmit energy into the fluid under consideration in the cylinder in order to “activate” the nuclei so that they emit a signal (Waldo and Arnold, 1983). This allows the observation of specific magnetic properties of an atomic nucleus.

The relaxation process is referred to as the free induction decay (FID). It is the observable NMR signal generated by non-equilibrium nuclear spin magnetization precessing about the magnetic field conventionally along z direction (Hopf, *et al.,* 1973). This time-domain signal is typically digitized and then Fourier transformed in order to obtain a frequency spectrum of the NMR signal i.e. the NMR spectrum (Duer, 2004). The time interval of the NMR signal is in the end limited by T2 relaxation. However, damping occurs more frequently due to common interference of different NMR frequencies that are available. As the frequencies are properly resolved, which is naturally the case in the samples of nuclear magnetic resonance in solution, the total decay of the FID is relaxation-limited and the FID is more or less exponential (Richard and Shoemaker, 2001; Abdelrahim *et al*., 2013).

**Methods**

**The Bloch NMR Equations**

The  components of magnetization of fluid flow are given by the Bloch equations, which are fundamental to understanding magnetic resonance images:

=, (1)

=, (2)

. (3)

where = equilibrium magnetization, = component of transverse magnetization along the -axis, = component of transverse magnetization along -axis, = component of magnetization along the field ( -axis),  = gyro-magnetic ratio of fluid spins, = radio-frequency (RF) magnetic field,  = Longitudinal or spin lattice relaxation time,  = Transverse or spin-spin relaxation time.

Awojoyogbe *et al.,* (2011) evolved the diffusion equation with the diffusion coefficient, , from the fundamental Bloch equations stated in equations (1-3), as

  (4)

where the diffusion coefficient (evolved as an intrinsic part of the Bloch Nuclear Magnetic Resonance (NMR) equations) was accurately defined in terms of MRI flow parameters fluid velocity, ,and relaxation rates (as ) and . The equation was also applied by Dada *et al.,* (2015) in a Bloch NMR diffusion model for porous media.

**Solution of the Diffusion Equation in Radially Symmetric Cylinder**

It can be said that the cylinder under consideration is independent of because it is radially symmetric. Therefore can be expressed as

, (5)

where is the transverse magnetization. In cylindrical coordinates, Equation (4) transforms to

 (6a)

Assuming the transverse magnetization in equation (5) could be expressed as follows:

, (6b)

equation (6a) becomes

 (6c)

Setting

 (6d)

 (6e)

Using method of separation of variables such that

,

the solutions to equation (6e) are:

 . (7)

 (8)

. (9)

Using the solutions in equations (6d), (6e), (7), (8) and (9), we write

. (10)

. (11)

Setting the function to the reduced radio-frequency field applied to perturb the molecules of the fluid, we write:

. (12)

The radio frequency field (RF) field with a cosine function is defined as:

 = , (13)

which implies

. (14)

. (15)

Consequently,

. (16)

**Solution Using the Initial and Boundary Conditions**

We shall examine the behaviour of diffusion or flow at the point of free flow, partial and total blockage while applying the following conditions:

(17)

whereis the space depicting the blockage and is the direction of flow and both are defined as:

. (18)

Finally, the solution for the magnetization of any molecule of the fluid at any point and time is given as:

 (19)

**Analysis of Results**

The fluid under consideration is hydrogen gas and its diffusion coefficient is . and values of hydrogen gas were used and the following substitution made for free flow, partial and total blockage of the pipe:

 . (20)

Based on computational algorithm for free flow, partial and total blockage, the following figures were obtained respectively:

|  |  |
| --- | --- |
| **Figure 1a: Plot of Magnetization (My) against time** **(t = 0 sec.) for hydrogen gas when there is no blockage (r = 0)** | **Figure 1b: Plot of Magnetization (My) against time (t = 0.000000005 sec.) for hydrogen gas when there is no blockage (r = 0)** |
| **Figure 1c: Plot of Magnetization (My) against time** **(t = 0.00000001 sec.) for hydrogen gas when there is no blockage (r = 0)** | **Figure 1d: Plot of Magnetization (My) against time** **(t = 0.00000005 sec.) for hydrogen gas when there is no blockage (r = 0)** |
| **Figure 1e: Plot of Magnetization (My) against time** **(t = 0.0000001 sec.) for hydrogen gas when there is no blockage (r = 0)** | **Figure 1f: Plot of Magnetization (My) against time** **(t = 0.0000002 sec.) for hydrogen gas when there is no blockage (r = 0)** |
| **Figure 2a: Plot of Magnetization (My) against time** **(t = 0) when blockage is half the radius (r = 0.0125m)** | **Figure 2b: Plot of Magnetization (My) against time** **(t = 0.000000005 sec.) when blockage is half the radius (r = 0.0125m)** |
| **Figure 2c: Plot of Magnetization (My) against time** **(t = 0.00000001 sec.) when blockage is half the radius (r = 0.0125m)** | **Figure 2d: Plot of Magnetization (My) against time** **(t = 0.00000005 sec.) when blockage is half the radius (r = 0.0125m)** |
| **Figure 2e: Plot of Magnetization (My) against time** **(t = 0.00000001 sec.) when blockage is half the radius (r = 0.0125m)** | **Figure 2f: Plot of Magnetization (My) against time (t = 0.00000002 sec.) when blockage is half the radius (r = 0.0125m)** |
| **Figure 3a: Plot of Magnetization (My) against time** **(t = 0) when there is total blockage (r = 0.025m)** | **Figure 3b: Plot of Magnetization (My) against time** **(t = 0.000000005 sec.) when there is total blockage** **(r = 0.025m)** |
| **Figure 3c: Plot of Magnetization (My) against time** **(t = 0.00000001 sec.) when there is total blockage** **(r = 0.025m)** | **Figure 3d: Plot of Magnetization (My) against time** **(t = 0.00000005 sec.) when there is total blockage** **(r = 0.025m)** |
| **Figure 3e: Plot of Magnetization (My) against time** **(t = 0.0000001 sec.) when there is total blockage** **(r = 0.025m)** | **Figure 3f: Plot of Magnetization (My) against time** **(t = 0.0000002 sec.) when there is total blockage** **(r = 0.025m)** |

|  |
| --- |
| **Figure 4: Plot of magnetization against time when there is no blockage of hydrogen gas in the cylinder** |
| **Figure 5: Plot of magnetization against time when the blockage of hydrogen gas is half the radius in the cylinder** |
| **Figure 6: Plot of magnetization against time when there is total blockage of hydrogen in the cylinder** |

**Discussion of Results**

Figures 1a-1f showed that the magnetization recorded under the condition of free flow implied that there are no blockages in the pipe. This is further summarized in figure 4. It will be observed that the transverse magnetization is between 0.004A/m and 0.005A/m. As for Figures 2a-f, the magnetization recorded under the condition of partial blockage is 0.00001A/m, which is an indication of signal loss. This is also depicted in figure 5. For the condition of total blockage highlighted in Figures 3a-f, the magnetization registered is zero - a situation in which there is no MRI signal as can be clearly seen in figure 5. It is interesting to note that the MRI model is non-invasive and registers signals in microseconds. This underscores its appropriateness, effectiveness and efficiency to pipe imaging within small temporal resolution ranges.

**Conclusion**

We have developed a mathematical method for detailed analysis of flow of gases (containing free protons) in cylindrical pipe based on the analytical solution of the Bloch NMR diffusion equation with appropriate boundary conditions. The simple analytical expression obtained in equation (19) contains very important magnetic resonance flow parameters which can be useful for the non invasive analysis of flow in fluid blockage. The analysis can also be useful in process industries where different network of pipes are used or machines that use cylindrical pipes or tubes in transporting materials especially when there is a partial or total blockage at any point in the network. This leads to quick identification of problems whenever it arises to elicit immediate control and solution before much damage is done. What is interesting in this work is that the method developed may prove to be very useful in identifying, localizing and quantifying blockage components in 3D. It may provide a time friendly simulation platform for imaging, monitoring, testing and repetitive running of NMR in any complicated engineering set up.

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