ANALYSIS OF PARTIAL AND TOTAL BLOCKAGE OF UNUSED ENGINE OIL IN A RADIALLY SYMMETRIC CYLINDRICAL PIPE USING DIFFUSION MAGNETIC RESONANCE EQUATION.

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Abstract
An analysis of the Magnetic Resonance Imaging MRI technique requires a detailed understanding of how signals evolve in the medium of interest in the absence of obstructions or perturbation and the nature of the emerging signal when the original signal is perturbed by obstructions that might exist in the medium. Properties of the signal are then used to estimate the level of blockage. In this study, MRI is used to detect partial and total blockage of unused engine oil in a cylindrical pipe. The Bloch Nuclear Magnetic Resonance (NMR) flow equations are solved analytically in cylindrical coordinates for flow of fluid in a radially symmetric cylindrical pipe. Based on the appropriate boundary conditions, the radial axis was varied to depict partial and total blockage in the pipe. Mathematical techniques were then used to analyze the results (NMR signals) obtained. This method is a viable alternative to other methods of detecting blockage in fluid pipelines in oil and gas industry.

Keywords: Bloch NMR Equations, DMR equation, Cylindrical pipe, Magnetization.

Introduction

Diffusion Magnetic Resonance Imaging (DMRI) is one of the most rapidly evolving techniques in the MRI field. It provides accurate assessment of the individual component or multi-component systems in a matter of minutes whereas traditional radioactive tracer techniques may take weeks for each component (Awojoyogbe et al., 2011). Diffusion and flow can be measured very delicately and accurately using Magnetic Resonance Imaging (Hazlewood et al., 1974). Diffusion coefficient of a substance, defined as the amount of material that diffuses in a certain time plays a vital role in the detection of blockage in a pipe using MRI. Random diffusion motion of fluid molecules has intriguing properties. This is the principle being exploited by the method of DMRI. Though not widely known, it has been noted for long that nuclear magnetic resonance is capable of quantifying diffusion movement of molecules as a result of uniqueness in relaxation rates - 1 T and 2 T (Yusuf et al., 2010).

Some attempts have been made in the past to detect blockage. Yuan used time splitting algorithms and Godunov mixed format to simulate the pulse propagation in the blocked pipelines (Yuan et al., 2014). Another technique used by Sattar is by the system frequency response. This is a technique whereby the frequency response is used in the detection of partial blockages in a pipeline (Sattar et al., 2008). Similar to this is the method adopted by Mohapatra for the detection of partial blockages in single pipelines by the frequency response method (Mohapatra et al., 2006). Wang also investigated analytically the effects of a partial blockage on pipeline transients. A partial blockage is simulated using an orifice equation, and the influence of the blockage on the unsteady pipe flow is considered in the equation using a Dirac delta function (Wang et al., 2005).

In this work, the principle of Magnetic Resonance is applied to a cylindrical oil pipe under the influence of radiofrequency field as a probe to perturb the molecules of the unused engine oil. This causes the nuclei to absorb energy from the applied electromagnetic (EM) pulse(s) and radiate this energy at a specific resonance frequency which depends on the strength of the magnetic field and other factors. This allows the observation of specific magnetic properties of an atomic nucleus. A Radio Frequency (RF) transmitter is needed to transmit energy into the fluid under consideration in the cylinder in order to "activate" the nuclei so that they emit a signal (Waldo & Arnold, 1983). The relaxation process itself is referred to as the free induction decay (FID). It is the observable NMR signal generated by non-equilibrium nuclear spin magnetization precessing about the magnetic field conventionally along z direction (Hopf et al., 1973). This time-domain signal is typically digitized and then Fourier transformed in order to obtain a frequency spectrum of the NMR signal i.e. the NMR spectrum (Duer, 2004). The study of the behaviour of diffusion or flow of engine oil at the point of partial and total blockage is here now undertaken.

1.0 Introduction

Magnetic resonance imaging is a recent approach adopted in the diagnosis of ailments and diseases in humans without surgical invasion. It can also be used to determine problems associated with blockage in cylindrical pipes. It provides accurate self-diffusion coefficient for the individual component or multi-component systems in a matter of minutes whereas traditional radioactive tracer techniques may take weeks for each component (Awojoyogbe et al 2011). This is possible because fluids exhibit random molecular motion of spins and through magnetic resonance coupled with the fact that the molecules of the fluids carry magnetic moments with them, the rate of their signal loss or signal attenuation, which could be easily detected, go a long way to signify whether or not a problem exists at any point in any cylindrical object.

Diffusion Magnetic Resonance Imaging (DMRI) is one of the most rapidly evolving techniques in the MRI field. Diffusion and flow can be measured very delicately and accurately using Magnetic Resonance Imaging (Hazlewood et al 1974). Coefficient of diffusion of a substance, defined as the amount of material that diffuses in a certain time, plays a vital role in the detection of blockage using MRI. Though not widely known, it has been noted for long that nuclear magnetic resonance is capable of quantifying diffusion movement of molecules.

This same principle is applied under the influence of magnetic resonance caused by the introduction of radiofrequency field to perturb the fluid molecules. This causes the nuclei to absorb energy from the applied electromagnetic (EM) pulse(s) and radiate this energy back out. The energy radiated back out is at a specific resonance frequency which depends on the strength of the magnetic field and other factors. This allows the observation of specific quantum mechanical magnetic properties of an atomic nucleus.

A Radio Frequency (RF) transmitter is needed to transmit energy into the sample of the fluid under consideration in the cylinder in order to "activate" the nuclei so that they emit a signal.

The usual transmitter coil, in use, applies to the sample an RF magnetic field $B_1(t)$ where $B_1(t) = bB_1(t)coswt$. Such a field is said to be linearly polarized, since it oscillates in a single direction. w is called the irradiation frequency; it is also the reference frequency of the RF transmitter and the detection system. A typical value for w is 1.0 x10^8 rad.s^{-1} (Waldo and Arnold 1983).

On the whole, the process undergoes the following four stages: (1) a magnetic field, B_o is applied, (2) the sample responds to B_o (3) a radio frequency pulse or a train of radio frequencies pulses, is applied during a limited time and (4) the system relaxes.

The Free Induction Decay (FID) is the name given to the time-domain signal obtained during the relaxation process. The relaxation process itself is referred to as the free induction decay. It is the observable NMR signal generated by non-equilibrium nuclear spin magnetization precessing about the magnetic field conventionally along z (Hopf and Frederic 1973). This time-domain signal is typically digitized and then Fourier transformed in order to obtain a frequency spectrum of the NMR signal i.e. the NMR spectrum (Duer and Melinda 2004)

The following assumptions were made:

i. $B_o =$ fluid is magnetized by the static B_o field to an equilibrium magnetization.
ii. $f_o = \gamma B - \omega$ is the Larmor frequency of the resonating molecules as a result of static field.
iii. $M_y =$ transverse magnetization
iv. T_1 *and* T_2 are relaxation parameters.
v. $rfB_1(x,t) =$ radio frequency field
vi. NMR signal is the electro-motive force, (e.m.f.) induced by the precessing transverse magnetization M_y, of the flowing spins
vii. Fluid flows through a cylindrical vessel of uniform cross section with velocity V.
viii. Resonance condition exists within the excitor as well as the detector coils

2.0 The Bloch NMR Equations

The x, y, z components of magnetization of fluid flow are given by the Bloch equations below which are fundamental to understanding Magnetic Resonance Images:

$$\frac{DM_x}{Dt} = \frac{\partial M_x}{\partial t} + V.\nabla M_x = \gamma M_x B_1(x,t) - \frac{M_x}{T_2} \qquad (1)$$

$$\frac{DM_y}{Dt} = \frac{\partial M_y}{\partial t} + V.\nabla M_y = \gamma M_z B_1(x,t) - \frac{M_y}{T_2} \quad (2)$$

$$\frac{DM_z}{Dt} = \frac{\partial M_z}{\partial t} + V.\nabla M_z = -\gamma M_y B_1(x,t) + \frac{M_0 - M_z}{T_1} \quad (3)$$

where M_o = equilibrium magnetization

M_x = component of transverse magnetization along the x-axis

M_y = component of transverse magnetization along y-axis

M_z = component of magnetization along the field (z -axis)

γ = gyro-magnetic ratio of fluid spins

B_0 = static magnetic field

$B_1(x,t)$ = radio-frequency (RF) magnetic field

T_1 = Longitudinal or spin lattice relaxation time

T_2 = Transverse or spin-spin relaxation time

V = the flow velocity

$\frac{D}{Dt}$ = Differential operator

From the three fundamental Bloch equations, the diffusion equation was evolved with coefficient D evolving intrinsically without any additional term (Torrey 1956). The equation is

$$\Rightarrow \frac{\partial M_y}{\partial t} = D\frac{\partial^2 M_y}{\partial r^2} + \frac{F_o}{T_o}\gamma B_1(r,t) \quad (4)$$

where the diffusion coefficient $D = -\frac{V^2}{T_o}$ was accurately defined in terms of MRI flow parameters fluid velocity, V , T_1 and T_2 relaxation rates (as $T_0 = \frac{1}{T_1} + \frac{1}{T_2}$) and $F_o = \frac{M_o}{T_1}$.

The above diffusion equation with $D = -\frac{V^2}{T_o}$, called coefficient of diffusion was evolved as an intrinsic part of the Bloch Nuclear Magnetic Resonance (NMR) equations.

3.0 Solution of the Diffusion Equation in Radially Symmetric Cylinder

Since the cylinder under consideration is radially symmetric, then it is independent of θ. Therefore M_y can be expressed as

$$M_y = M_y(r, z, t) \qquad (5)$$

M_y is the transverse magnetization.

In cylindrical coordinates, equation (4) transforms to

$$\frac{\partial M_y}{\partial t} = D\left(\frac{\partial^2 M_y}{\partial r^2} + \frac{1}{r}\frac{\partial M_y}{\partial r} + \frac{\partial^2 M_y}{\partial z^2}\right) + \frac{F_o}{T_o}\gamma B_1(t)$$

(6)

This can be expressed in the form:

$$M_y = F(r, z)U(t) + w_c(t) \qquad (7)$$

Where $\qquad w_c(\dot{t}) = \frac{F_o}{T_o}\gamma B_1(t) \qquad (8)$

$\Rightarrow \qquad w_c(t) = \int_0^{t_0} \frac{F_o}{T_o}\gamma B_1(t)\, dt \qquad (9)$

Using the method of separation of variables (MSV) and equating the two equations to a constant, say $-\lambda^2$, we have,

$$M_y = F(r, z)U(t) \qquad (10)$$

$$\frac{dU(t)}{dt} + \lambda^2 D U(t) = 0 \qquad (11)$$

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} + \frac{\partial^2 F}{\partial z^2} + \lambda^2 F = 0 \qquad (12)$$

By integrating equation (11), the general solution below is obtained:

$$U(t) = C_1 e^{-\lambda^2 D t} \qquad \lambda = 1, 2, \dots, \dots, \qquad (13)$$

where C_1 is the arbitrary constant of integration.

Using the method of separation of variables (MSV) and following the same procedure again with a constant, say $-\mu^2$ we have the following two differential equations:

$$\frac{\partial^2 Q}{\partial r^2} + \frac{1}{r}\frac{\partial Q}{\partial r} + \mu^2 Q = 0 \qquad (14)$$

and

$$\frac{\partial^2 Z}{\partial z^2} - \beta^2 Z = 0 \tag{15}$$

where we have $\beta^2 = \mu^2 - \lambda^2$ (16)

It could be noted that from equations (14), a Bessel differential equation evolves and its solution is given as

$$F(r) = C_2 J_0(\mu r) + C_3 Y_m(\mu r) \tag{17}$$

where $J_0(\mu r)$ is the Bessel function of the first kind, of order zero and $Y_m(\mu r)$ is the Bessel function of the second kind, of order m. $C_2 and C_3$ are constants.

Also from (15), $Z(z) = C_4 e^{\beta z} + C_5 e^{-\beta z}$ (18)

Consequently, the solutions to the equations are:

$$U(t) = C_1 e^{-\lambda^2 D t} \qquad \lambda = 1, 2, \dots, \dots, \tag{19}$$

$$F(r) = C_2 J_0(\mu r) + C_3 Y_m(\mu r) \tag{20}$$

$$Z(z) = C_4 e^{\beta z} + C_5 e^{-\beta z} \tag{21}$$

Combining the solution to the diffusion equation (6), this gives the product of the quantities in (19), (20) and (21) plus $\int_0^{t_0} w_c(\dot{t})\, dt$ i.e.

$$M_y = M_y\,(r, z, t) = F(r) Z(z)\, U(t) + \int_0^{t_0} w_c(\dot{t})\, dt \tag{22}$$

$$M_y\,(r, z, t) = \{C_2 J_0(\mu r) + C_3 Y_m(\mu r)\}\{C_4 e^{\beta z} + C_5 e^{-\beta z}\}\{C_1 e^{-\lambda^2 D t}\} + \int_0^{t_0} w_c(\dot{t})\, dt \tag{23}$$

The last function on the right hand side $\int_0^{t_0} \frac{F_o}{T_o} \gamma B_1(t)\, dt = \int_0^{t_0} w_c(\dot{t})\, dt$ is the radio-frequency field applied to perturb the molecules of the fluid. Therefore for the solution of

$$w_c(t) = \int_0^{t_0} \frac{F_o}{T_o} \gamma B_1(t)\, dt \tag{24}$$

The radio frequency field (rf) field is defined as

$$B_1(t) = bB_1(t) coswt \tag{25}$$

which implies $w_c(t) = \int_0^{t_0} \frac{F_o}{T_o} b\gamma B_1(t) coswt\, dt$ (26)

$$\int_0^{t_0} \frac{bF_o}{T_o} \cos\,(wt)\, dt = \frac{bF_o}{wT_o} \gamma \sin\,(wt) \tag{27}$$

Consequently,

$$M_y\,(r,z,t) = \{C_2 J_0(\mu r) + C_3 Y_m(\mu r)\}\{C_4 e^{\beta z} + C_5 e^{-\beta z}\}\{C_1 e^{-\lambda^2 Dt}\} + \frac{bF_o}{wT_o}\gamma \sin\,(wt) \qquad (28)$$

4.0 SOLUTION USING THE INITIAL AND BOUNDARY CONDITIONS

In order to examine the behaviour of diffusion or flow of unused engine oil at the point of blockage, the following conditions shall be imposed:

$i)\quad M_y\,(r,z,0) = M_i(r,z);$

$ii)\quad M_y\,(r,0,t) = 0;$

$iii)\quad M_y\,(r,L,t) = 0;$

$iv)\quad M_y\,(a,z,t) = 0;$

$v)\quad \left|M_y(r,z,t)\right| = M,$

(29)

where r *is the space depicting the blockage* and z is the direction of flow and both are defined as below -

$$0 \le r < a; \quad 0 < z < L; \quad t > 0$$

Firstly from the boundedness condition, $r = 0, Y_m(\mu r) \to -\infty$; to keep the solution finite, C_3 must be zero. Thus the solution becomes

$$M_y\,(r,z,t) = \{e^{-\lambda^2 Dt}\}\{C_2 J_0(\mu r)\}\{C_4 e^{\beta z} + C_5 e^{-\beta z}\} \qquad (30)$$

From the second boundary condition, we see that

$$M_y\,(r,0,t) = \{e^{-\lambda^2 Dt}\}\{J_0(\mu r)\}\{C_4 + C_5\} = 0 \qquad (31)$$

So that we must have $C_4 + C_5 = 0 \; or \; C_5 = -C_4$

then (3.47) becomes $M_y\,(r,z,t) = \{e^{-\lambda^2 Dt}\}\{J_0(\mu r)\}\{e^{\beta z} - e^{-\beta z}\} = 0$ (32)

From the third condition, we have $M_y\,(r,L,t) = \{e^{-\lambda^2 Dt}\}\{J_0(\mu r)\}\{e^{\beta L} - e^{-\beta L}\} = 0$ (33)

which can be satisfied with $e^{\beta L} - e^{-\beta L} = 0$ (34)

$$\Rightarrow e^{\beta L}.\,e^{\beta L} = e^{-\beta L}.\,e^{\beta L} = 1 = e^{2k\pi i}$$

It follows that we must have $2\beta L = 2k\pi i$ or $\beta = \frac{k\pi i}{L}$ $\quad k = 0,1,2, \dots \dots$ (35)

Using this in equation (30), it becomes

$$M_y\,(r,L,t) = \{Ce^{-\lambda^2 Dt}\}\{J_0(\mu r)\} sin\frac{k\pi z}{L} = 0 \quad (36)$$

where C is a new constant.

From the fourth condition, we obtain

$$M_y\,(a,z,t) = \{Ce^{-\lambda^2 Dt}\}\{J_0(\mu a)\} sin\frac{k\pi z}{L} = 0 \quad (37)$$

which can be satisfied only if $\{J_0(\mu a)\} = 0$ (38)

$$\mu a = s_1, s_2, \ldots. \quad (39)$$

$$\mu = \frac{s_1}{a}, \frac{s_2}{a}, \ldots\ldots \quad (40)$$

where $\frac{s_m}{a}(m = 1,2, \ldots.)$ .is the positive root of the Bessel function $\{J_0(x)\} = 0$. Now from (16), (35) and (40), it follows that:

$$\lambda^2 = (\frac{s_m}{a})^2 - (\frac{k\pi i}{L})^2 \; = (\frac{s_m}{a})^2 + (\frac{k\pi}{L})^2 \quad (41)$$

so that a solution satisfying all the boundary conditions except the first is given by

$$M_y\,(r,z,t) = \left\{Ce^{-Dt(\frac{s_m}{a})^2+(\frac{k\pi}{L})^2}\right\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} sin\frac{k\pi z}{L} \quad (42)$$

where $k = 1,2,3, \ldots..; m = 1,2,3, \ldots.$

Replacing $C\ by\ C_{km}$ and summing over $k\ and\ m$ we obtain by the superposition principle the solution

$$M_y\,(r,z,t) = \sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\left\{C_{km}e^{-Dt(\frac{s_m}{a})^2+(\frac{k\pi}{L})^2}\right\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} sin\frac{k\pi z}{L} \quad (43)$$

The first condition in (3.46) now leads to

$$M_i\,(r,z) = \sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\{C_{km}\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} sin\frac{k\pi z}{L} \quad (44)$$

This can be written as

$$M_i\,(r,z) = \sum_{k=1}^{\infty}[\sum_{m=1}^{\infty}\{C_{km}\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\}] sin\frac{k\pi z}{L} = \sum_{k=1}^{\infty} b_k sin\frac{k\pi z}{L} \quad (45)$$

where $$b_k = \sum_{m=1}^{\infty}\{C_{km}\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} \quad (46)$$

It follows from this that b_k are the Fourier coefficients obtained when $M_i(r,z)$ is expanded into a Fourier sine series in z (r being kept constant).

Thus $\quad b_k = \frac{2}{1}\int_0^1 M_i(r,z) sin\frac{k\pi z}{L} dz$ (47)

We now find C_{km} from the expansion in equation (3.63). Since b_k is a function of r this is simply the expansion of b_k into a Bessel series.

Consequently, $\quad C_{km} = \frac{2}{J_1^2(\frac{s_m}{a})}\int_0^1 r b_k J_0\left(\frac{s_m}{a}r\right)\, dr$ (48)

Using (46), $\quad C_{km} = \frac{4}{J_1^2(\frac{s_m}{a})}\int_0^1\int_0^1 r M_i(r,z) J_0\left(\frac{s_m}{a}r\right) sin\frac{k\pi z}{L}\, drdz$ (49)

The required solution is

$$M_y\,(a,z,t) = \sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\left\{C_{km}e^{-Dt(\frac{s_m}{a})^2+(\frac{k\pi}{L})^2}\right\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} sin\frac{k\pi z}{L}$$

with C_{km} in (48) as coefficient (50)

With the radio frequency (rf) field, the solution is

$M_y\,(a,z,t) = \sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\left\{C_{km}e^{-Dt(\frac{s_m}{a})^2+(\frac{k\pi}{L})^2}\right\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} sin\frac{k\pi z}{L} + \frac{aF_o}{wT_o}\gamma \sin\,(wt)$ (51)

Assume $M_i(r,z) = \sigma_0$, a constant.

$C_{km} = \frac{4\sigma_0}{J_1^2(\frac{s_m}{a})}\int_0^1\int_0^1 r J_0\left(\frac{s_m}{a}r\right) sin\frac{k\pi z}{L} drdz$ (52)

$C_{km} = \frac{4\sigma_0}{J_1^2(\frac{s_m}{a})}\{\int_0^1 r J_0(\frac{s_m}{a}r)dr\int_0^1 sin\frac{k\pi z}{L}dz$ (53)

$= \frac{4\sigma_0}{J_1^2(\frac{s_m}{a})}\left\{\frac{J_1\left(\frac{s_m}{a}\right)}{\frac{s_m}{a}}\right\}\{\frac{1-cosk\pi}{k\pi}\}$ (54)

$= \frac{4\sigma_0(1-cosk\pi)}{k\pi\frac{s_m}{a}J_1(\frac{s_m}{a})}$ (55)

Substituting for C_{km} in equation 50

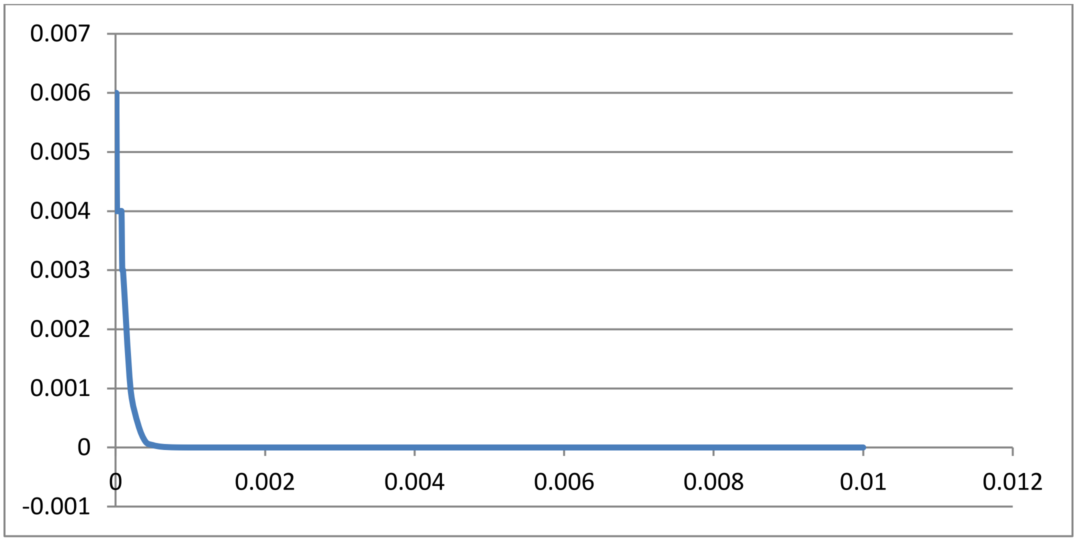
$M_y\,(r,z,t) = \frac{4\sigma_0}{\pi}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{(1-cosk\pi)}{k\pi\frac{s_m}{a}J_1(\frac{s_m}{a})}e^{-Dt(\frac{s_m}{a})^2+(\frac{k\pi}{L})^2)}\right\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} sin\frac{k\pi z}{L}$ (56)

Finally, the solution is

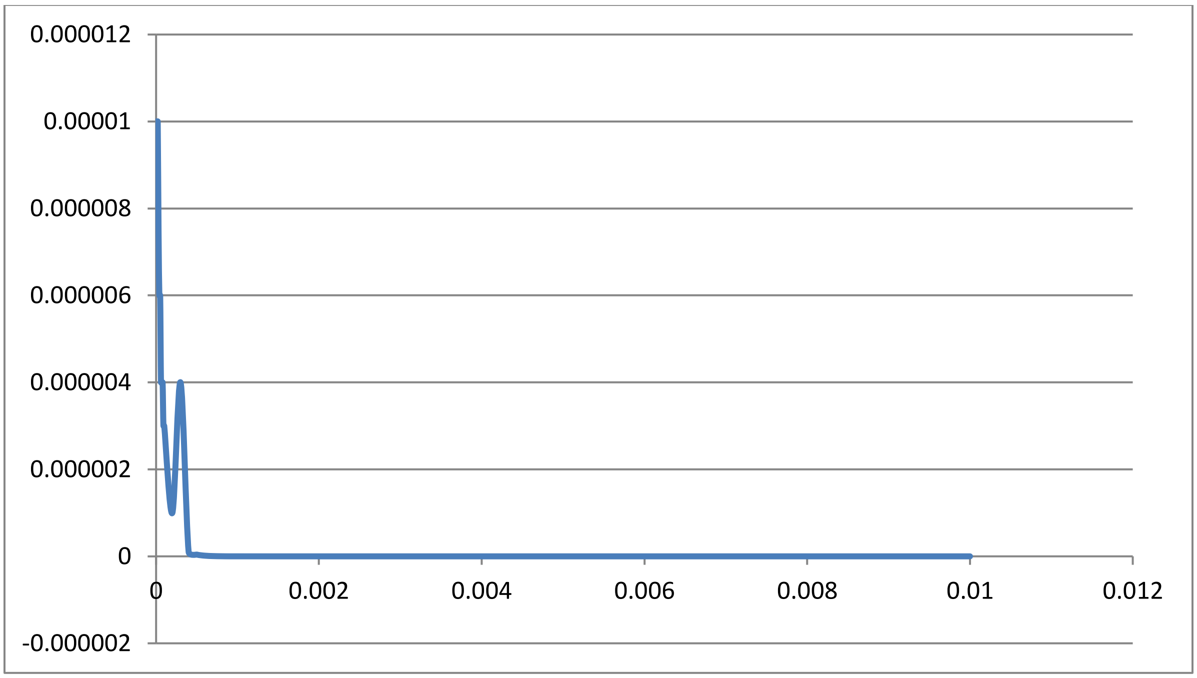
$$M_y\,(r,z,t) = \frac{4\sigma_0}{\pi}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\left\{\frac{(1-cosk\pi)}{k\frac{s_m}{a}J_1(\frac{s_m}{a})}e^{-Dt(\frac{s_m}{a})^2+(\frac{k\pi}{L})^2}\right\}\left\{J_0\left(\frac{s_m}{a}r\right)\right\} sin\frac{k\pi z}{L} + \frac{bF_o}{wT_o}\gamma \sin\,(wt)$$

5.0 Results and Discussion

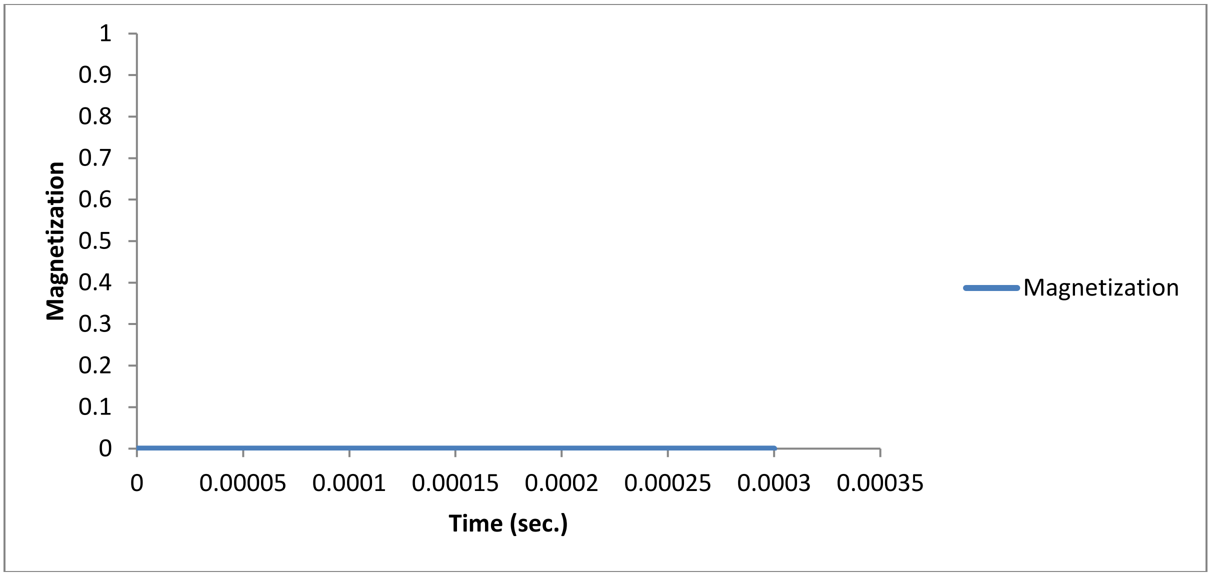
In discussing the results, the three conditions of free flow, partial and total blockages were considered:



5.1 Plot of magnetization against time for a free flow fluid (no blockage) inside the cylinder



5.2 Plot of magnetization against time for partial blockage inside the cylinder



5.3 Plot of magnetization against time for a total blockage inside the cylinder

6.0 Conclusion

It is obvious from graphs 5.1 to 5.3 that in a free flow condition, the free induction decay (FID) is demonstrated signifying no blockage and the magnetization is between 0.0 – 0.006 whereas under partial blockage, the magnetization reduces in value (0.0 – 0.00001) and there is an irregularity between 0.0 and 0.000004 .

Blockage of oil pipes is a common feature in our environment which sometimes leads to leakages and oil spill. Unfortunately, it takes an appreciably long time before the problems are discovered. In process industries where different network of pipes are being put to use or machines make use of cylindrical pipes or tubes in transporting materials or in functioning properly, problems are encountered whenever there is a partial or total blockage at any point in the network. This may not be discovered in good time and may also lead to a breakdown of activities and long down time in operation whenever the fault is eventually discovered. Quick detection of such problems as well as prompt provision of solution underscores the importance of diffusion magnetic resonance imaging. This will prevent long down time in process industries where pipes are being used in transporting materials or fluids.

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