A Study of Longitudinal and Transverse Relaxation Rates in Magnetic Resonance Imaging - MRI

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Abstract

Two - second order non-homogenous linear differential equations from the Bloch NMR equations are evolved one as time - dependent and the other as time – independent. The parameters in the equations are equilibrium magnetization M_o, radio frequency $rfB_1(x,t)$ field, gyro-magnetic ratio of blood spin γ, velocity V as well as T_1 and T_2 relaxation times. A general method of solution of each equation under the influence of radio frequency magnetic field ($rfB_1(x,t) \neq 0$) and in the absence of radio frequency magnetic field ($rfB_1(x,t) = 0$) is evolved. However, for the purpose of this study, only T_1 and T_2 relaxation times are varied and analyzed for the measurement of the signals in relation to its effect on human anatomy.

1.0 Introduction

Nuclear Magnetic Resonance (NMR) techniques now referred to as **Magnetic Resonance Imaging (MRI)** is an effective method of studying the anatomy, physiology and pathology of human living tissues. Nuclear Magnetic Resonance, NMR, measures how much electromagnetic radiation of a specific frequency is absorbed by an atomic nucleus that is placed in a strong magnetic field. Its objective is to visualize the atomic and molecular structure of chemical compounds – (Edward 1945). NMR is produced when a radio frequency field is imposed at right angles to a much larger static magnetic field to perturb the orientation of nuclear magnetic moments generated by spinning electrically charged atomic nuclei - (Bloch 2006). The improvement made on NMR towards evolving MRI was on the precision of the radio waves in order to gauge the resonant signals with more accuracy. In addition, means of applying the magnetic fields was enhanced. Mathematical methods by which radio signals could be analyzed and transformed into a useful image that would show precise distinctions between different areas of living tissue was also developed – (Mansfield 2006). The main goal of this study is to establish a methodology of using mathematical techniques so that the accurate measurement of blood flow in human physiological and pathological conditions can be carried out non-invasively and become simple to implement in medical clinics.

2.0 Mathematical Model

2.1 Time – Independent Bloch NMR Flow Equations and Solutions

It is assumed that blood is a Newtonian fluid – (Ayeni 1993). It is magnetized by the static B_0 field to an equilibrium magnetization and that resonance condition exists at Larmor frequency: $f_o = \gamma B - \omega = 0$. The NMR signal is the electro-motive force, e.m.f. induced by the precessing transverse magnetization M_y, of the flowing spins and is dependent on the flow velocity V, T_1 and T_2 relaxation parameters. M_y results from the combined effect of B_o and $rfB_1(x,t)$ on blood spins. For steady flow, blood or fluid flows through a blood vessel of uniform cross section with velocity V. It is also assumed that resonance condition exists within the excitor as well as the detector coils. The x, y, z components of magnetization of fluid flow are given by the Bloch equations below which are fundamental to understanding Magnetic Resonance Imaging:

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2} \qquad 1$$

$$\frac{dM_y}{dt} = \gamma M_z B_1(x) - \frac{M_y}{T_2} \qquad 2$$

$$\frac{dM_z}{dt} = -\gamma M_y B_1(x) - \frac{(M_z - M_o)}{T_1} \qquad 3$$

For blood flow analysis, it is assumed the blood spins to be flowing along the x-direction hence the flow is independent of y and z components. This implies that the flow is constant along y and z directions. Flow against the gravity is made possible by one - way valves, located several centimeters apart in the veins – (Setaro 2006). From the kinetic theory of moving fluids, given a property M of the fluid, then the rate at which this property changes with respect to a point moving along with the fluid be the total derivative:

$$\frac{dM}{dt} = \frac{\partial M}{\partial t} + \frac{\partial M}{\partial x}V_x + \frac{\partial M}{\partial y}V_y + \frac{\partial M}{\partial z}V_z$$

$$\Rightarrow \frac{dM}{dt} = \frac{\partial M}{\partial t} + V.\nabla M$$

Therefore, the three Bloch equations (1 – 3) above become:

$$\frac{dM_x}{dt} = \frac{\partial M_x}{\partial t} + V.\nabla M_x = -\frac{M_x}{T_2} \qquad 4$$

$$\frac{dM_y}{dt} = \frac{\partial M_y}{\partial t} + V.\nabla M_y = \gamma M_z B_1(x) - \frac{M_y}{T_2} \qquad 5$$

$$\frac{dM_z}{dt} = \frac{\partial M_z}{\partial t} + V.\nabla M_z = -\gamma M_y B_1(x) - \frac{M_o - M_z}{T_1} \qquad 6$$

With (5); equation (6) becomes:

$$V^2\frac{\partial^2 M_y}{\partial x^2} + 2V\frac{\partial^2 M_y}{\partial x \partial t} + V(\frac{1}{T_1} + \frac{1}{T_2})\frac{\partial M_y}{\partial x} + (\frac{1}{T_1} + \frac{1}{T_2})\frac{\partial M_y}{\partial t} + \frac{\partial^2 M_y}{\partial t^2} + \{\frac{1}{T_1 T_2} + \gamma^2 B_1^{\,2}(x,t)\}M_y = \frac{\gamma B_1(x,t)M_o}{T_1} \qquad 7$$

Equation 7 is a general second order differential equation which can be applied to any fluid flow – (Awojoyogbe 2002).

To evolve *time-independent flow equation* from equations 4 – 6, all partial derivatives with respect to time can be set to zero (i.e. time independent). This then leads us to:

$$\Rightarrow \frac{d^2 M_y}{dx^2} + \frac{1}{V}(\frac{1}{T_1} + \frac{1}{T_2})\frac{\partial M_y}{\partial x} + \frac{1}{V^2}\{\gamma^2 B_1^2(x) + \frac{1}{T_1 T_2}\}M_y = \frac{M_0 \gamma B_1(x)}{V^2 T_1} \qquad 8$$

Equation 8 can be re-written as

$$\frac{d^2 M_y}{dx^2} + \frac{T_o}{V}\frac{dM_y}{dx} + \frac{1}{V^2}\{\gamma^2 B_1^2 + K\}M_y = \frac{F_0 \gamma B_1}{V^2 T_1} \qquad 9$$

$$T_o = \frac{1}{T_1} + \frac{1}{T_2}; K = \frac{1}{T_1 T_2} and F_o = \frac{M_o}{T_1}$$

Where

Equation 9 is **Time-Independent** Bloch NMR flow equation - (Awojoyogbe 2002).

Case I: $\gamma^2 B_1^{\ 2} << K$ (i.e. radio frequency field $\gamma^2 B_1^{\ 2}$ is negligible).

This is the ground state of the protons, then equation 9 becomes

$$\frac{d^2 M_y}{dx^2} + \frac{T_o}{V}\frac{dM_y}{dx} + \frac{K}{V^2}M_y = \frac{F_o \gamma B_1}{V^2 T_1} \qquad 10$$

Solving 10 using the method of variation of parameters by (Kreyszig 1988) gives -

$$M_y(x) = A_1(x)e^{-\frac{x}{\lambda}} + A_2(x)e^{-\frac{x}{\lambda}} + \frac{F_o \lambda^2 \gamma B_1}{(V^2 - T_o V\lambda + k\lambda^2)} \qquad 11$$

Case II: $Y^2 B^2{}_1 >> K$ (i.e radio-frequency field is introduced), then equation 9 becomes

$$\frac{d^2 M_y}{dx^2} + \frac{T_o}{V}\frac{dM_y}{dx} + \frac{\gamma^2 B_1^2}{V^2}M_y = \frac{F_o \gamma B_1}{V^2 T_1} \qquad 12$$

$$\Rightarrow M_y(x) = C_1 e^{-\frac{x}{\lambda}} + C_2 e^{-\frac{x}{\lambda}} + \frac{F_o \lambda^2 \gamma B_1}{(V^2 - T_o V\lambda + \gamma^2 B_1^2 \lambda^2)} \qquad 13$$

2.2 Time – Dependent Bloch NMR Flow Equations and Solutions

Similarly, a time-dependent equation could be obtained by considering a flow that is independent of the space coordinate, x, implying that the magnetization does not change appreciably over a large x for a very long time, then all partial derivatives with respect to x could be set to zero (time-dependent). Hence equations (4 - 6) become:

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2} \qquad 14$$

$$\frac{dM_y}{dt} = \gamma M_z B_1(t) - \frac{M_y}{T_2} \qquad 15$$

$$\frac{dM_z}{dt} = -\gamma M_y B_1(t) - \frac{(M_z - M_o)}{T_1} \qquad 16$$

From equations 15 and 16, we have

$$\frac{d^2 M_y}{dt^2} + \left(\frac{1}{T_1} + \frac{1}{T_2}\right)\frac{dM_y}{dt} + .\left(\gamma^2 B_1^{\ 2}(t) + \frac{1}{T_1 T_2}\right)M_y = \frac{M_o \gamma B_1(t)}{T_1} \qquad 17$$

Equation 17 is **Time-Dependent** Bloch NMR flow equation – (Awojoyogbe 2004).

3.1 Solution of Time-Dependent Bloch Nuclear Magnetic Resonance Flow Equations

Let $k = \frac{1}{T_1 T_2}$; $T_o = \frac{1}{T_1} + \frac{1}{T_2}$; $F_o = \frac{M_o}{T_1}$ and $\gamma\, B_1(t) = \cos wt$ then

equation 10 becomes: $$\frac{dM_y^2}{dt^2} + T_o \frac{dM_y}{dt} + kM_y = F_o \cos wt \qquad 18$$

By an ordinary comparison of equation 18 with

$$my'' + cy' + py = r(t) \qquad 19$$

We can assume: $T_o = \frac{c}{m}$; $k = \frac{p}{m}$; $F_o = \frac{M_o}{T_1} = \frac{1}{m}$ and also let $w = \sqrt{\frac{p}{m}} \Rightarrow w^2 = \frac{p}{m}$

Using the method of undetermined coefficient by (Erwin Kreyszig 1988);

$$my'' + cy' + py = F_o \cos wt \qquad 20$$

Solving the complementary function

$$\Rightarrow y_h(t) = e^{-\alpha t}(a\cos wt + b\sin wt) \qquad 21$$

The *particular integral* is given as

$$y_p(t) = F_o \frac{p - mw^2}{(p - mw^2)^2 + w^2c^2} \cos wt + F_o \frac{wc}{(p - mw^2)^2 + w^2c^2} \sin wt \qquad 22$$

This implies $y(t)$

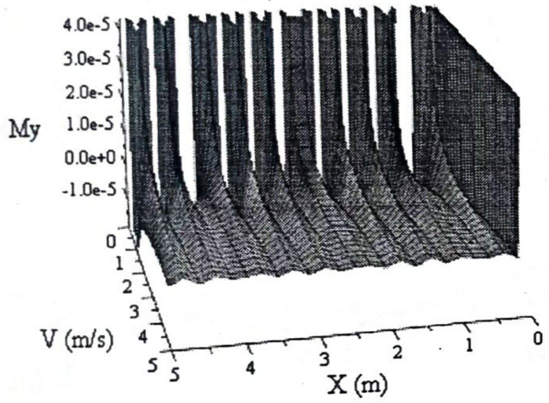
$$= e^{-\alpha t}(A\cos wt + B\sin wt) + \{F_o \frac{k - w^2}{(k - w^2)^2 + (wT_o)^2} \cos wt + F_o \frac{wT_o}{(k - w^2)^2 + (wT_o)^2} \sin wt\} \qquad \text{-23}$$

3.0 Results and Discussions

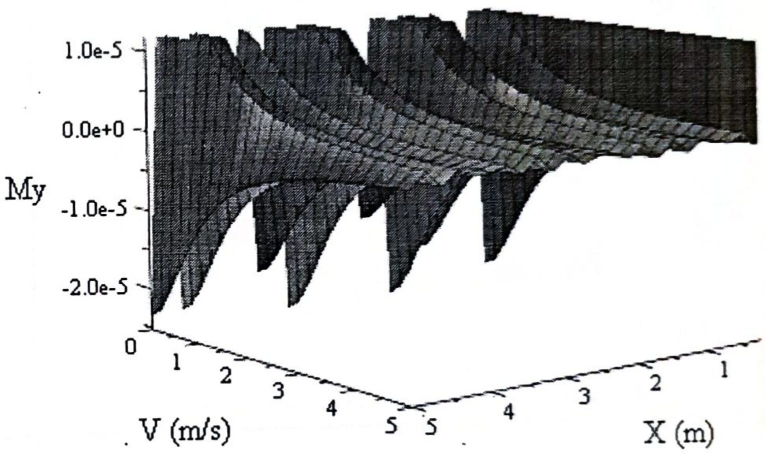
3.1 Plot of a 3-Dimensional Time-Independent Flow Equation

The solution to the equation as solved above is

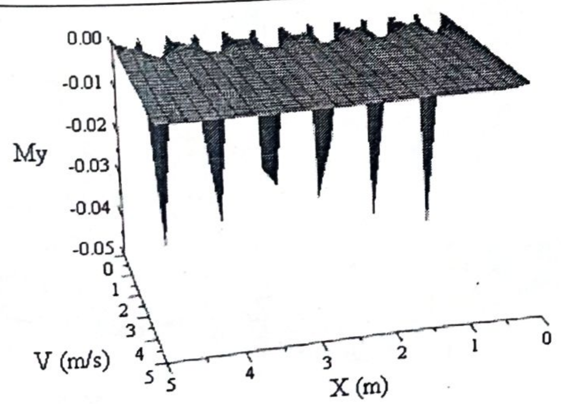
$M_y(x) = A_1(x)e^{-\frac{x}{\lambda}} + A_2(x)e^{-\frac{x}{\lambda}} + \frac{F_o\lambda^2\gamma B_1}{(V^2 - T_oV\lambda + k\lambda^2)}$. Considering ($\gamma^2 B_1^2 << K$) on the left hand column and ($\gamma^2 B_1^2 >> K$) on the right hand column and assuming T1 = 1 is constant and T2 is varied between 0.01 and 0.53; $A_1 = A_2 = 1$; $F_o = 1$ and radio frequency field $\gamma B_1(x) = \cos\frac{x}{\lambda}$, the graphs plotted below would be obtained:



Plot of My against V and X at T2 = 0.01s



Plot of My against V and X at T2 = 0.01s



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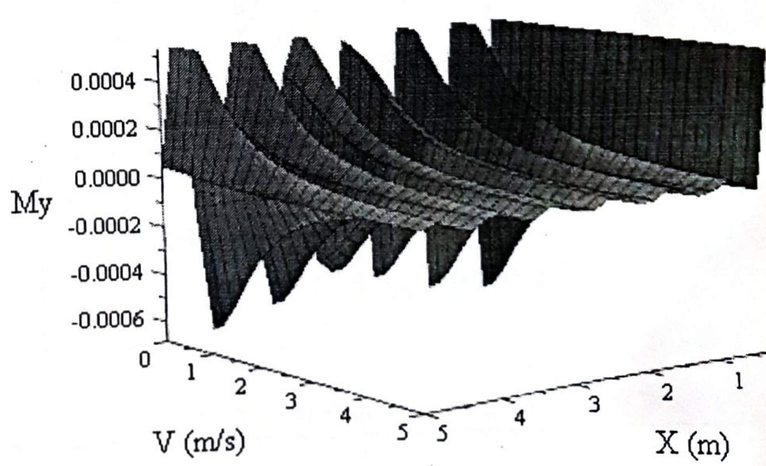
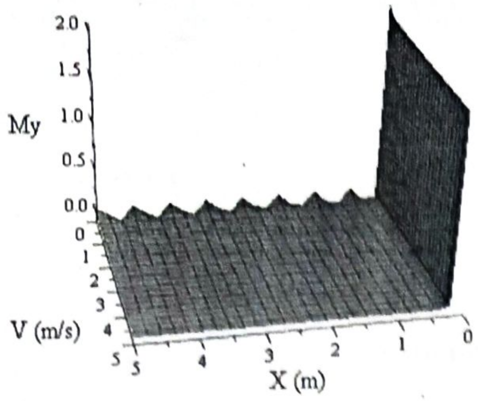
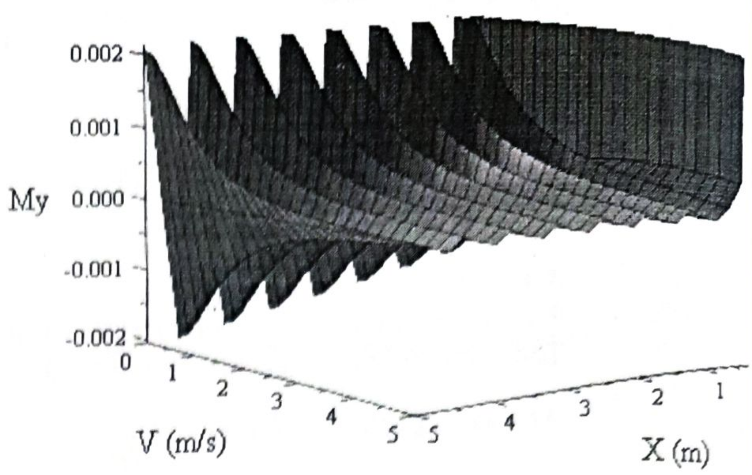


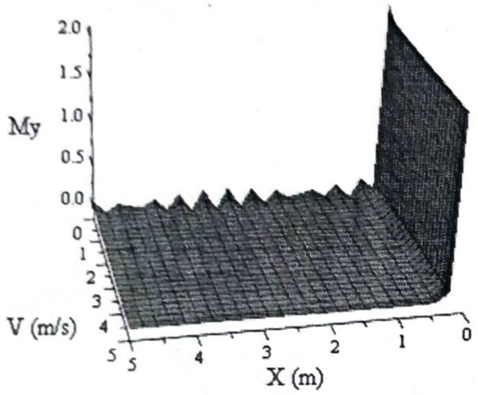
Plot of My against V and X at T2 = 0.05s



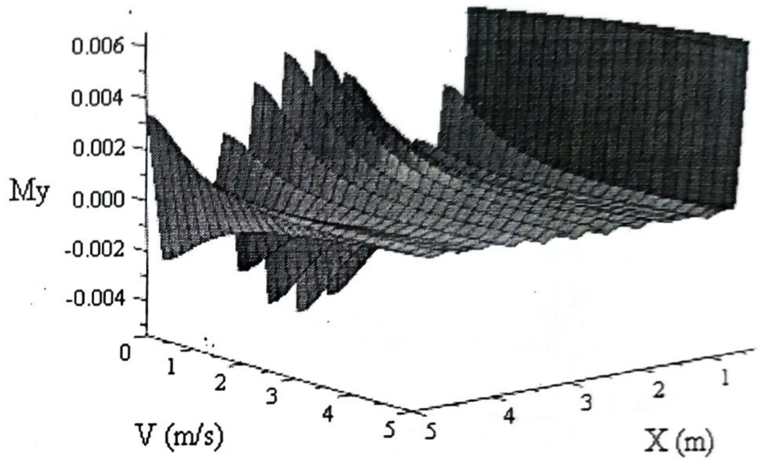
Plot of My agaist V and X at T2 = 0.09s



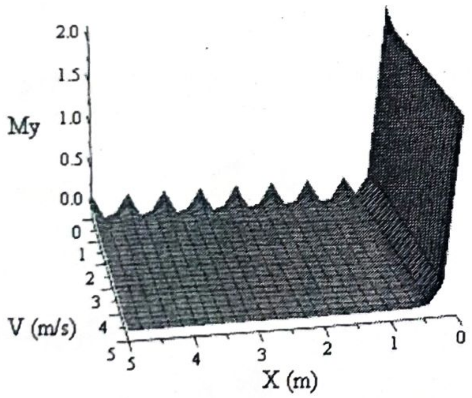
Plot of My against V and X at T2 = 0.09s



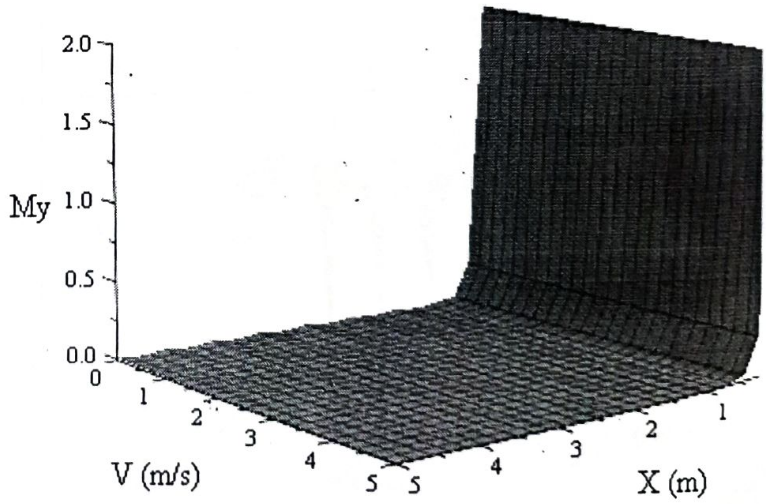
Plot of My against V and X at T2 = 0.13s



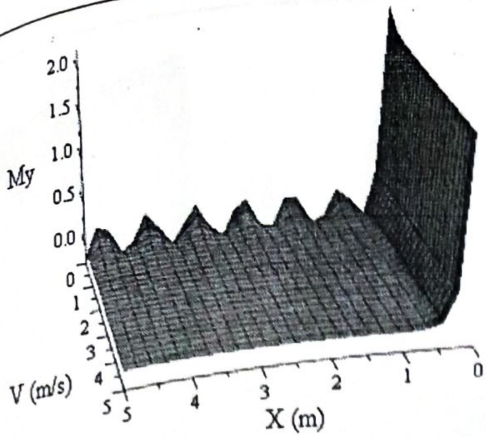
Plot of My against V and X at T2 = 0.13s



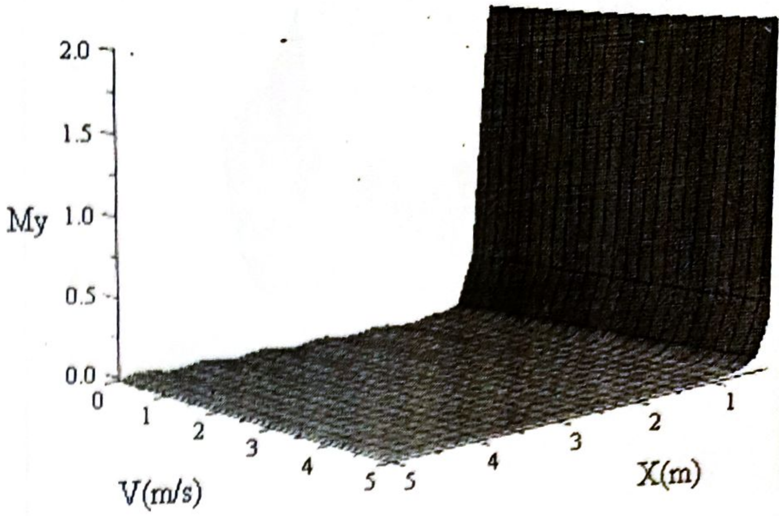
Plot of My against V and X at T2 = 0.17s



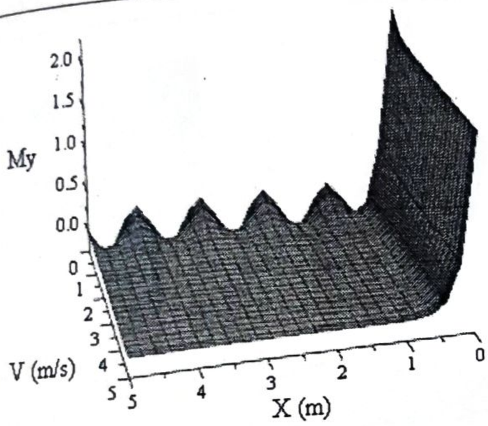
Plot of My against V and X at T2 = 0.17s



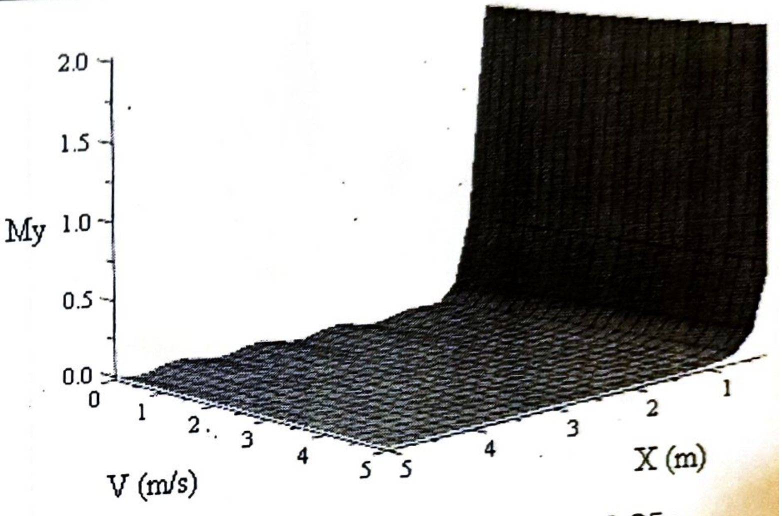
Plot of My against V and X at T2 = 0.21s



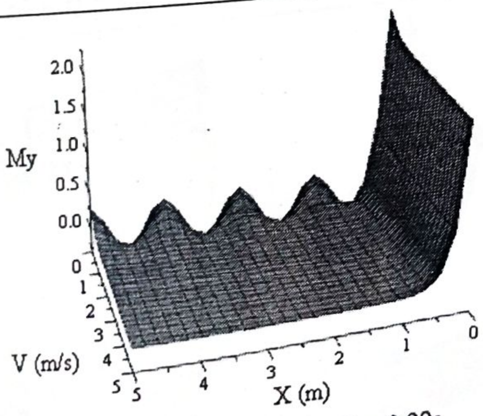
Plot of My against V and x at T2 = 0.21s



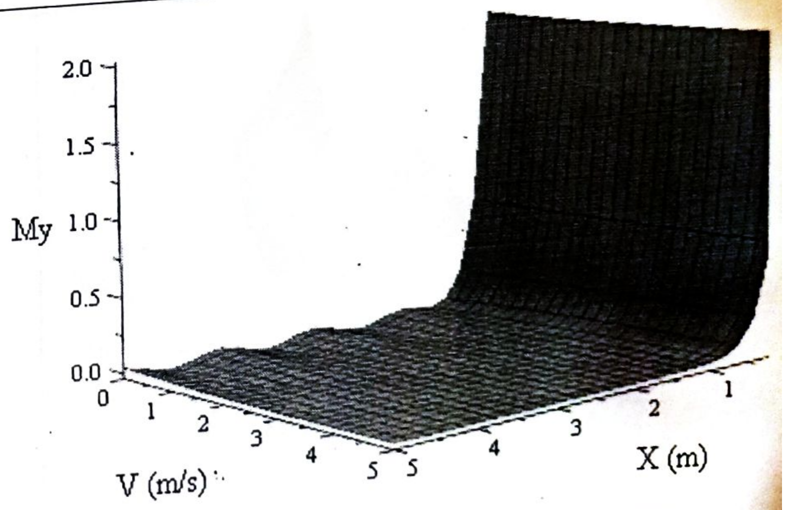
Plot of My against V and X at T2 = 0.25s



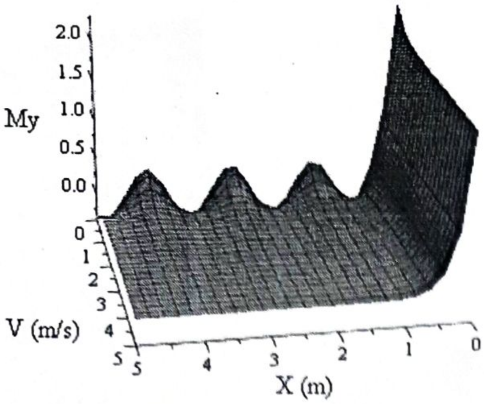
Plot of My against V and X at T2 = 0.25s



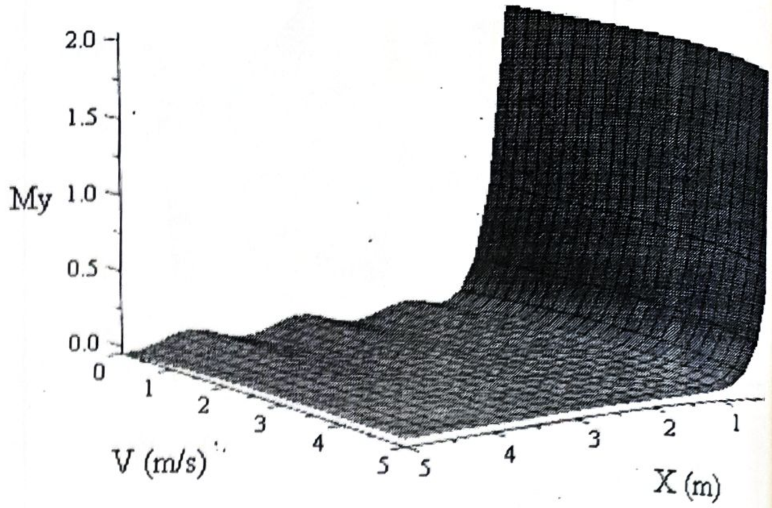
Plot of My against V and X at T2 = 0.29s



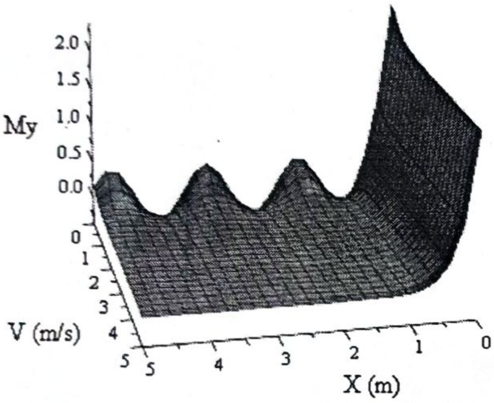
Plot of My against V and X at T2 = 0.29s



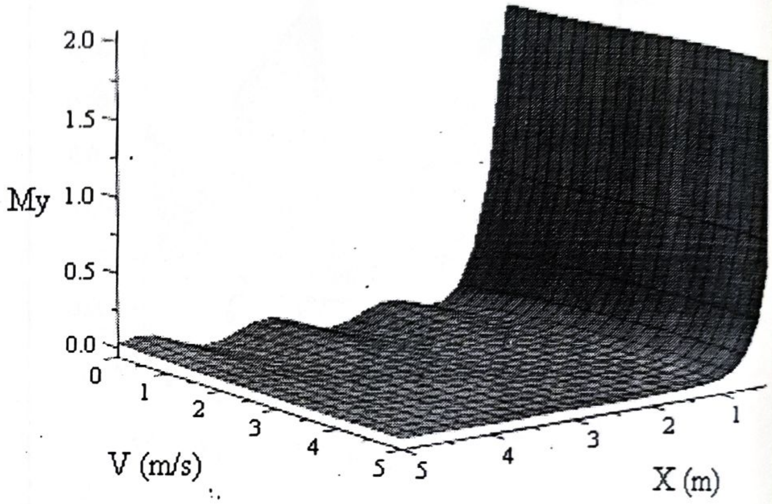
Plot of My against V and X at T2 = 0.33s



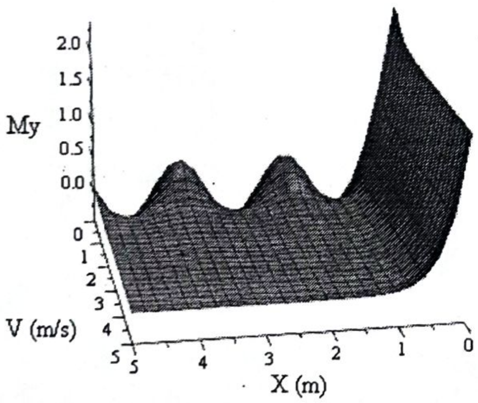
Plot of My against V and X at T2 = 0.33s



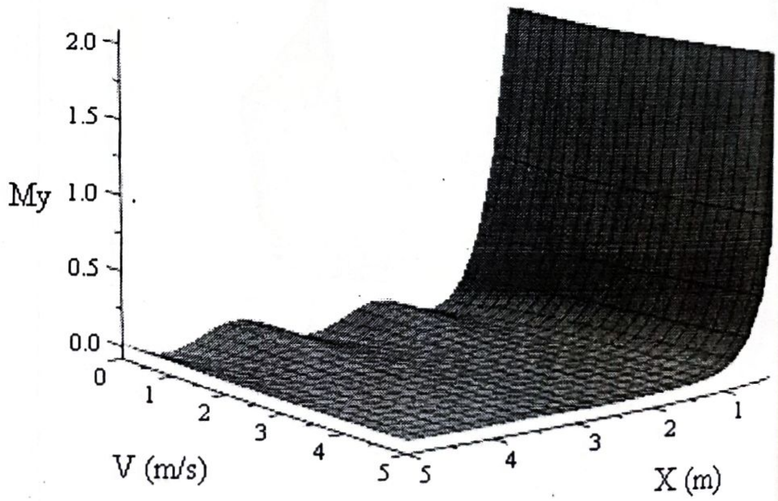
Plot of My against V and X at T2 = 0.37s



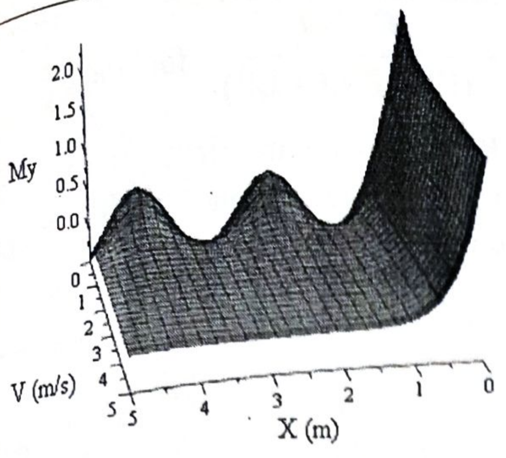
Plot of My against V and X at T2 = 0.37s



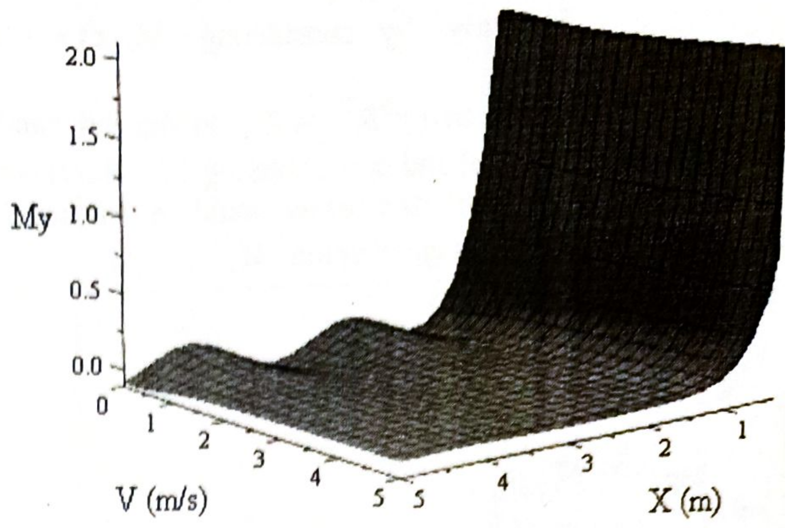
Plot of My against V and X at T2 = 0.41s



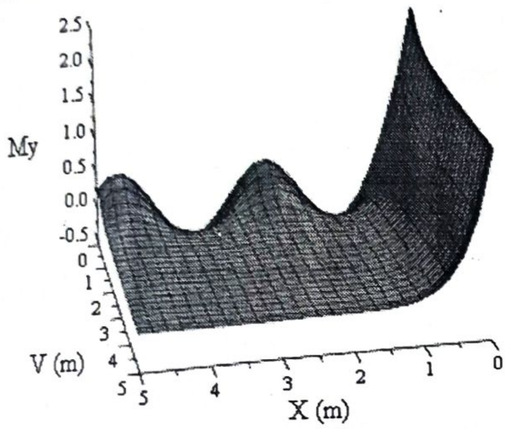
Plot of My against V and X at T2 = 0.41s



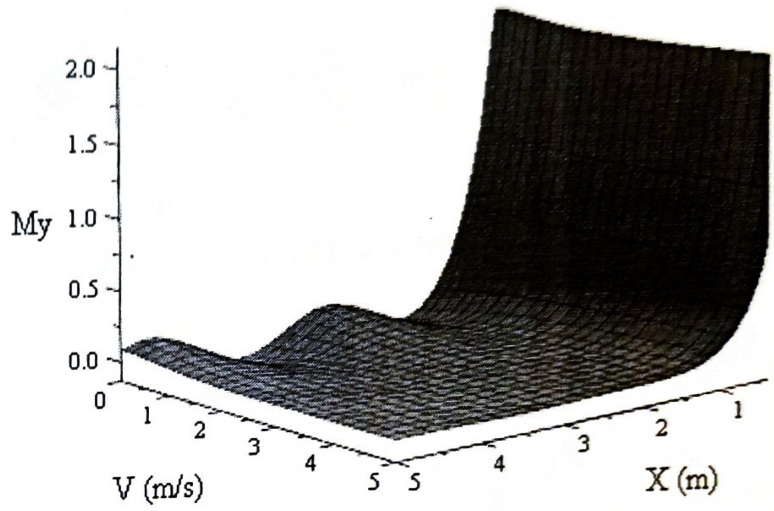
Plot of My against V and X at T2 = 0.45s



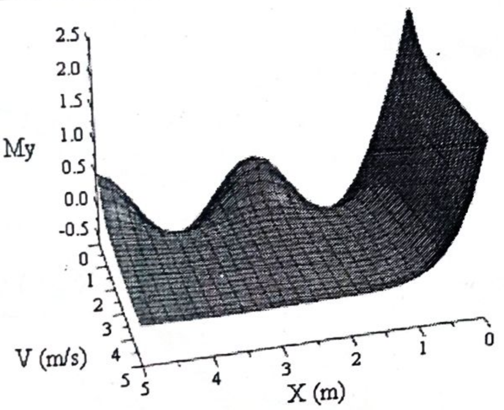
plot of My against V and X at T2 = 0.45s



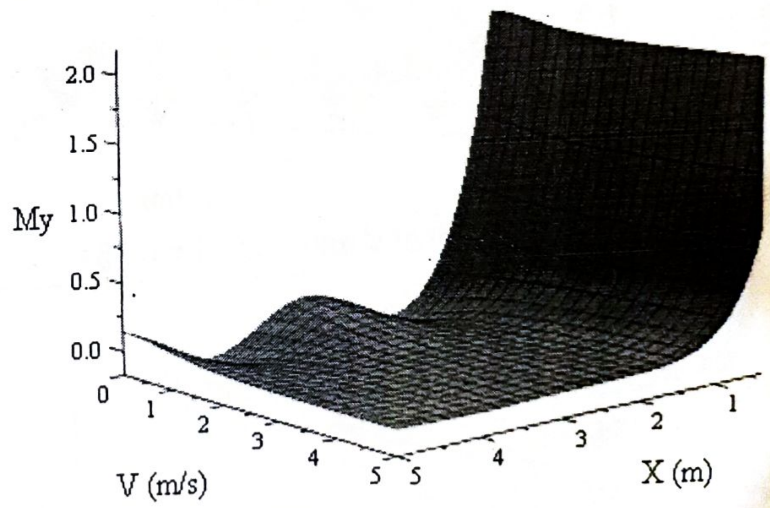
Plot of My against V and X at T2 = 0.49s



Plot of My against V and X at T2 = 0.49s



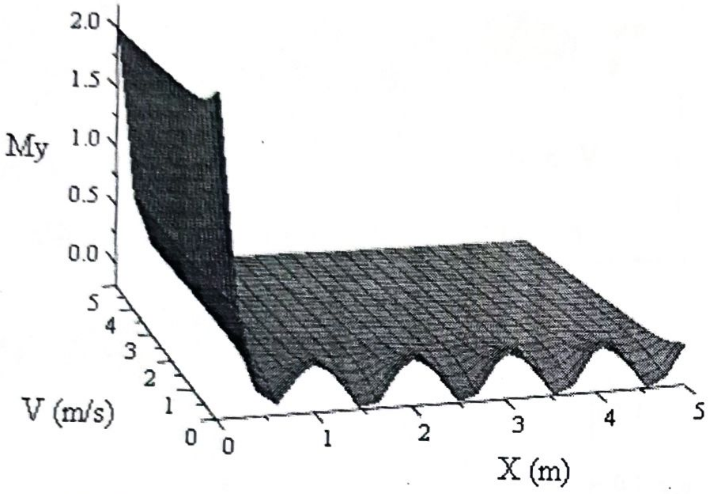
Plot of My against V and X at T2 = 0.53s



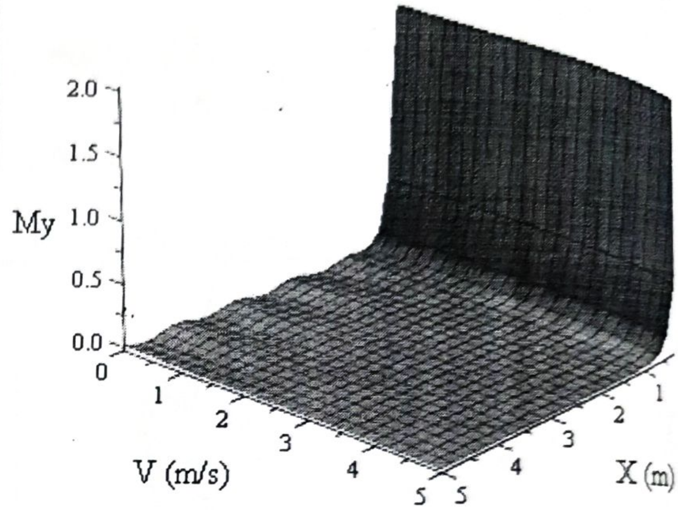
Plot of My against V and X at T2 = 0.53s

Fig.1:- *3-Dimensional Time-Independent Flow Equation with T1 constant and T2 varied.*

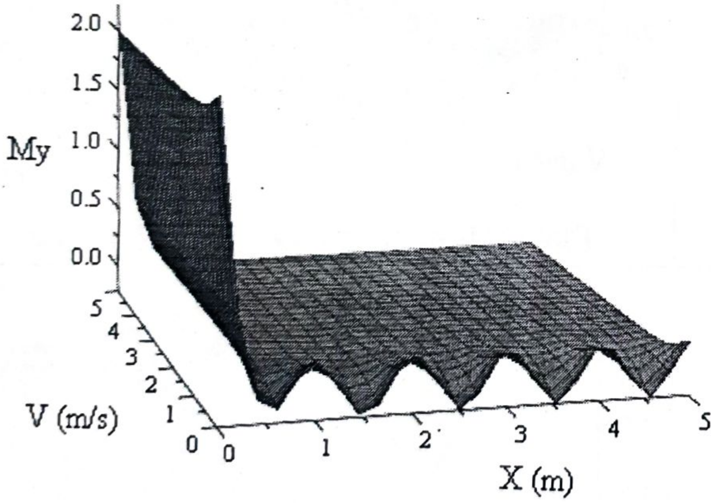
Similarly, by considering $M_y(x) = A_1(x)e^{-\frac{x}{\lambda}} + A_2(x)e^{-\frac{x}{\lambda}} + \frac{F_o\lambda^2\gamma B_1}{(V^2 - T_oV\lambda + k\lambda^2)}$ for the condition ($\gamma^2 B_1^{\ 2} << K$) on the left hand colum and ($\gamma^2 B_1^{\ 2} >> K$) on the right hand column) and now assuming T2 = 0.3 is constant and T1 is varied between 0.8 and 1.3; the graphs plotted below would be obtained. This is to examine if varying T1 has any effect on the magnetization M_y.



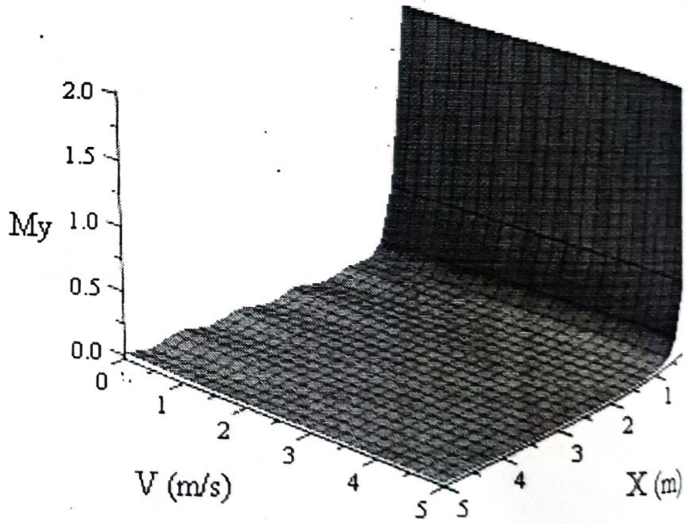
Plot of My against V and X at T1 = 0.8s.



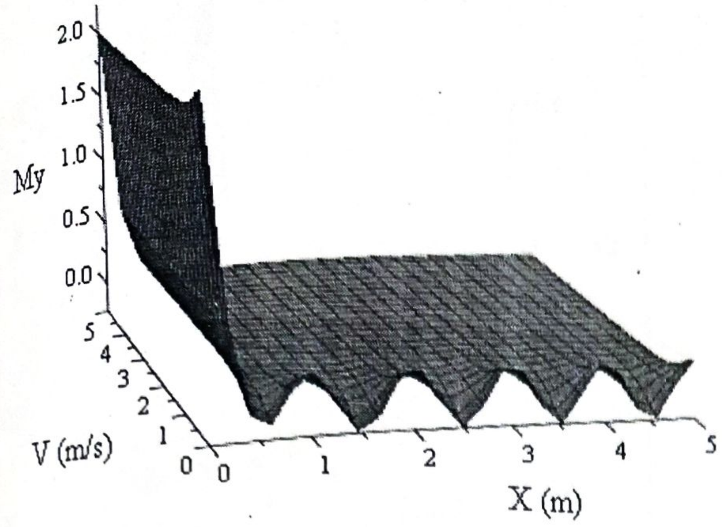
Plot of My against V and X at T1 = 0.



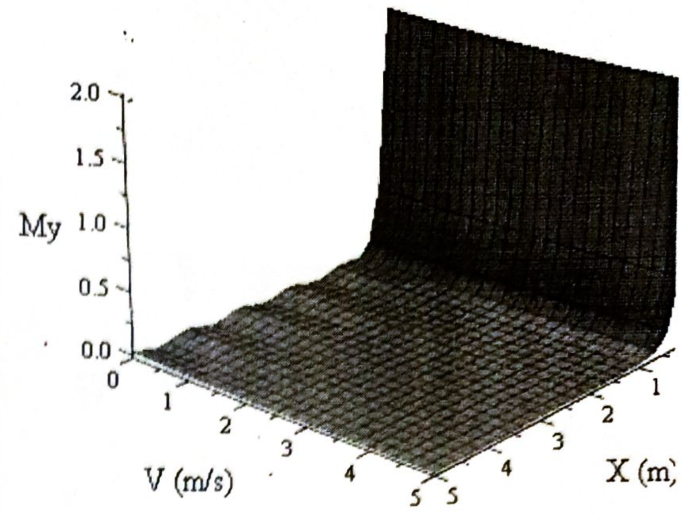
Plot of My against V and X at T1 = 0.85s



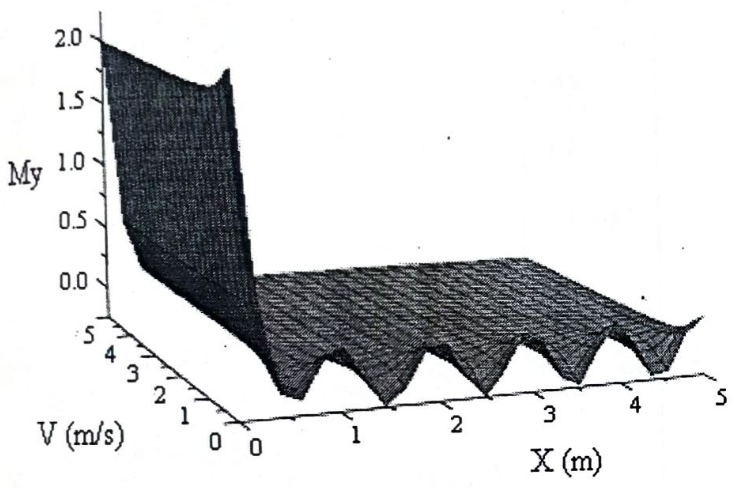
Plot of My against V and X at T1 0.8



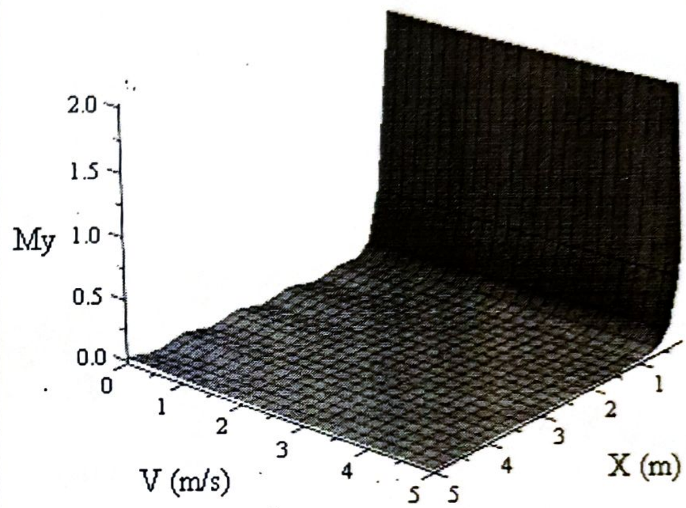
Plot of My against V and X at T1 = 0.9s



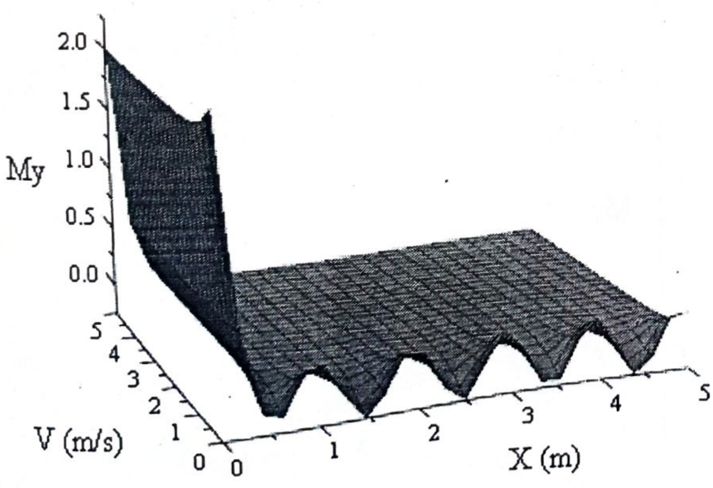
Plot of My against V and X at T1 = 0.9



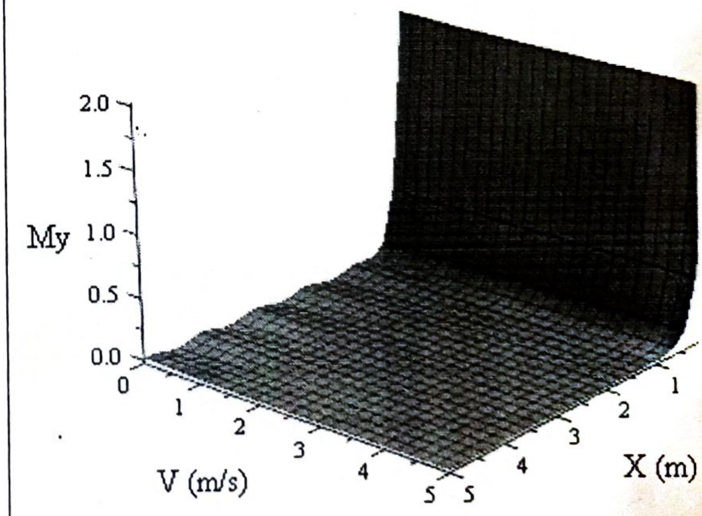
Plot of My against V and X at T1 =0.95s



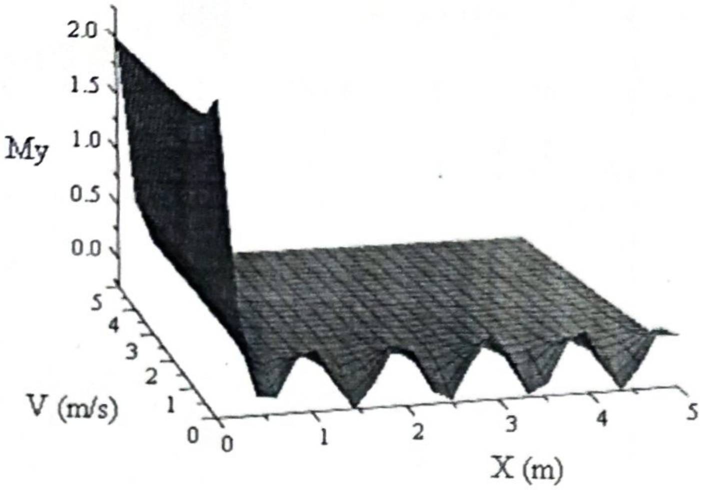
Plot of My against V and X at T1 = 0.9



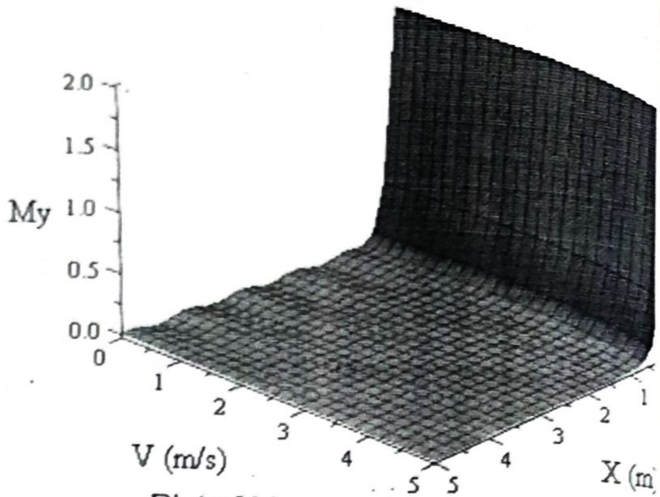
Plot of My against V and X at T1 = 1.0s



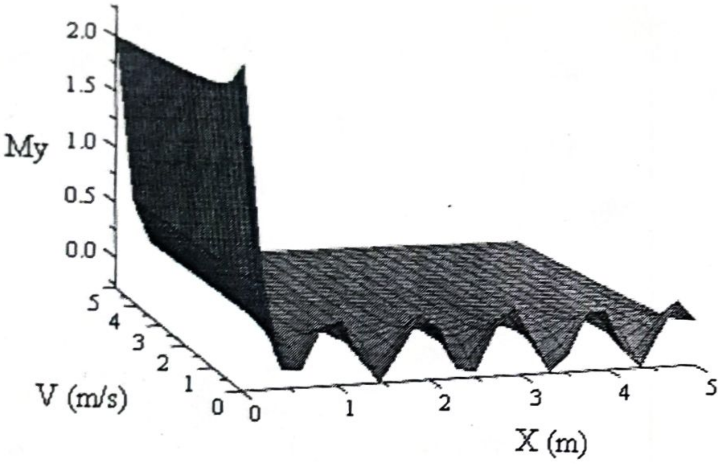
Plot of My against V and X at T1 = 1.(



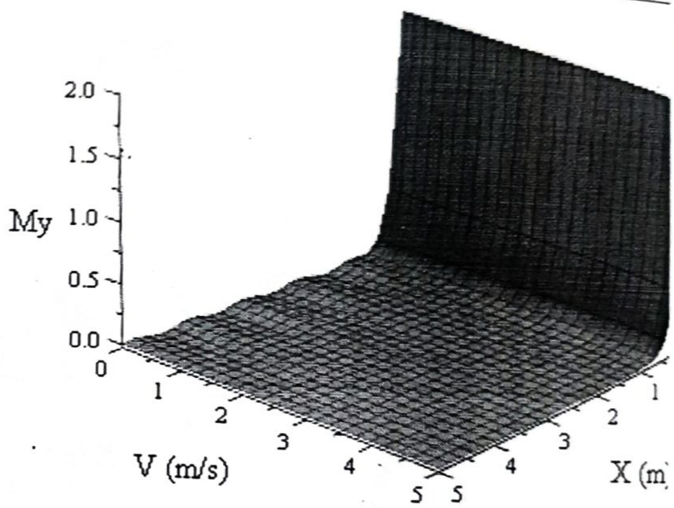
Plot My against V and X at T1 =1.05s



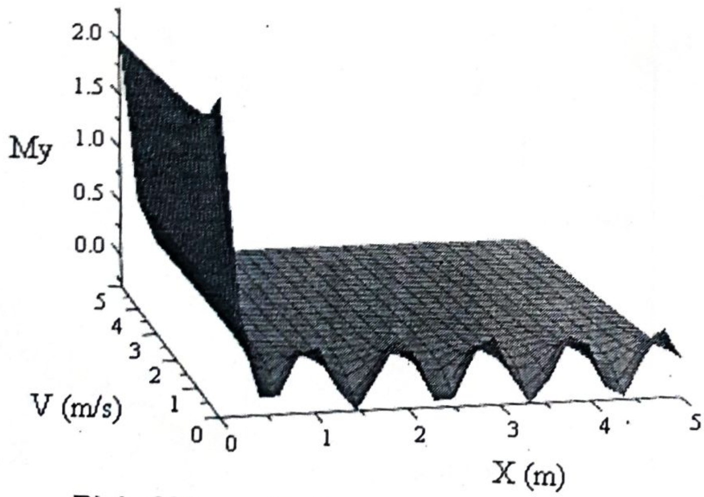
Plot of My against V and X at T1 = 1.(



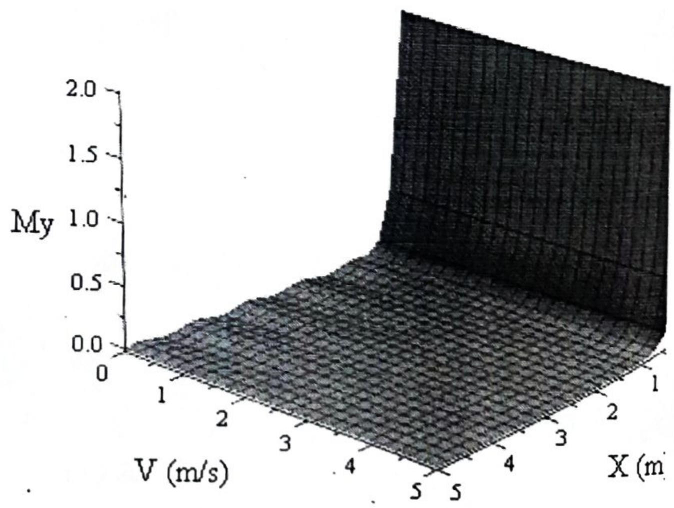
Plot of My against V and X at T1 = 1.1s



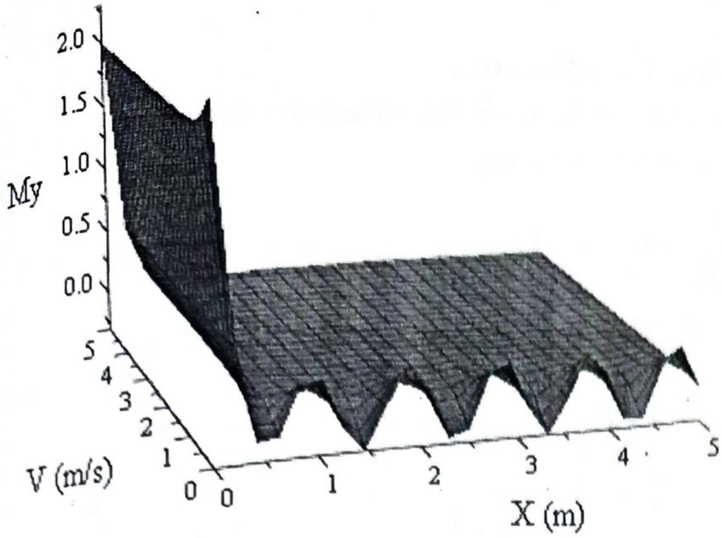
Plot of My against V and X at T1 = 1.



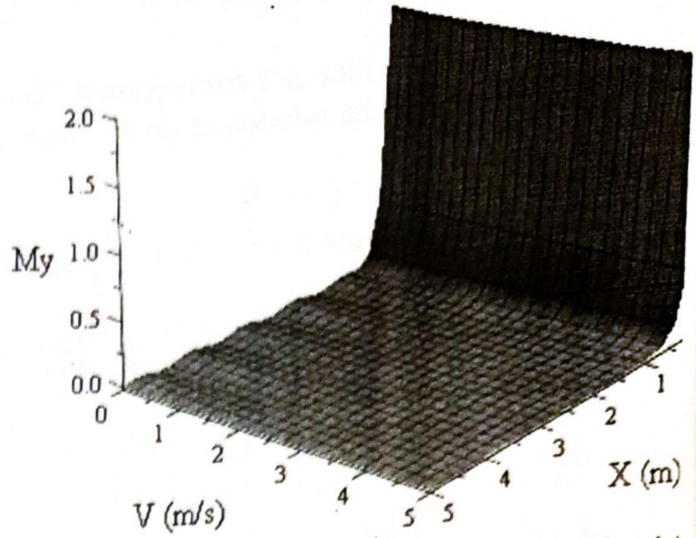
Plot of My against V and X at T1 = 1.15s



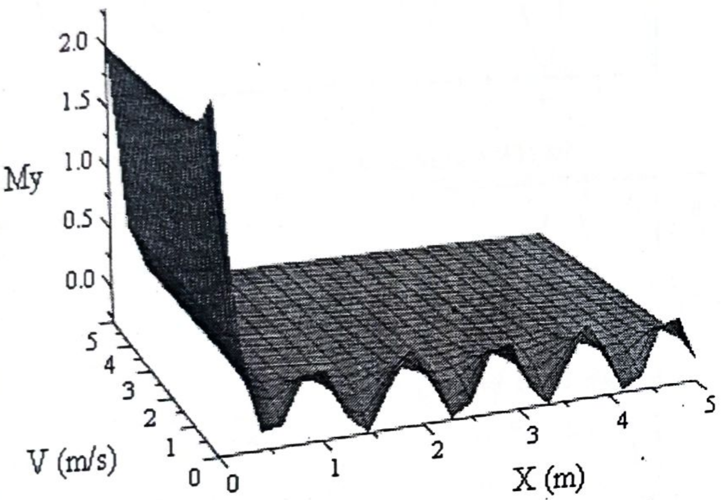
Plot of My against V and x at T1 = 1.1



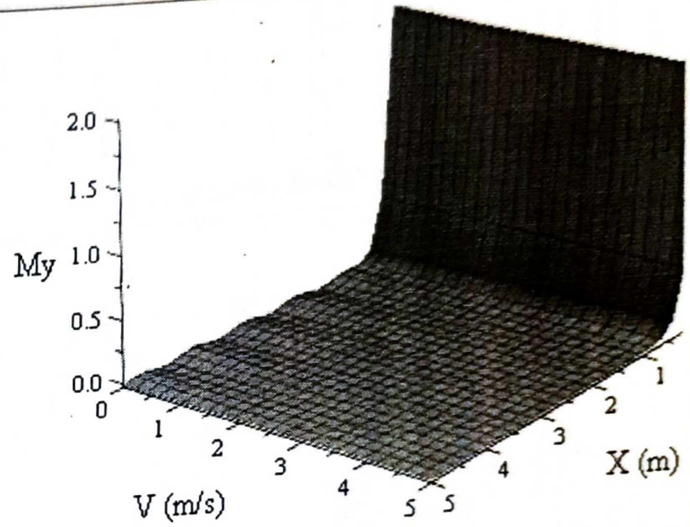
Plot of My against V and X at T1 = 1.2s



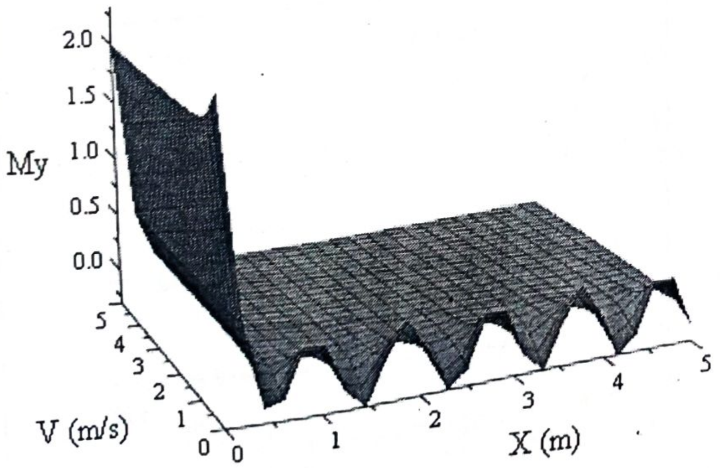
Plot of My against V and X at T1 = 1.



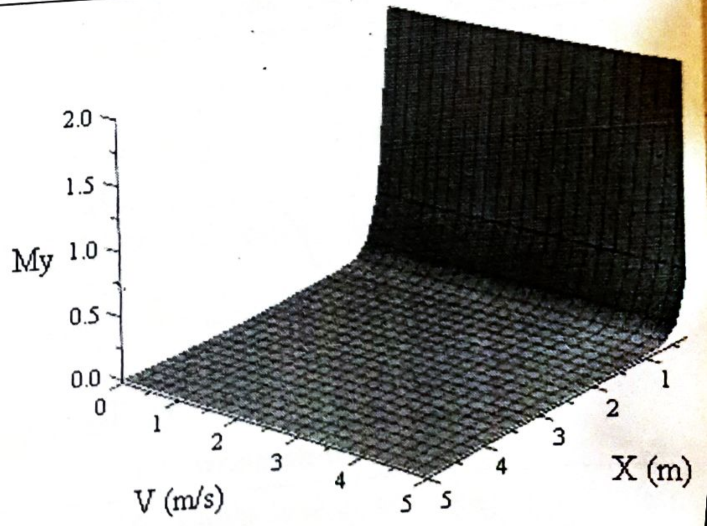
Plot of My agaist V and X at T1 = 1.25s



Plot of My against V and X at T1 = 1.2



Plot of My against V and X at T1 =1.3s



Plot of My against V and X at T1 = 1.3

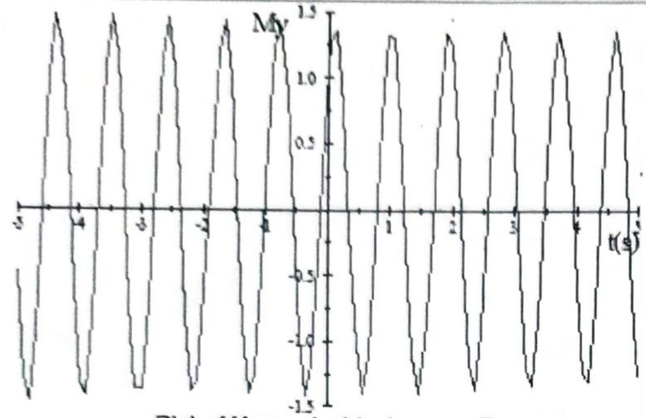
***Fig.2:-** 3-Dimensional Time-Independent Flow Equation with T2 constant and T1 varied.*

3.2 Plot of 2-Dimensional Time-Dependent Flow Equation

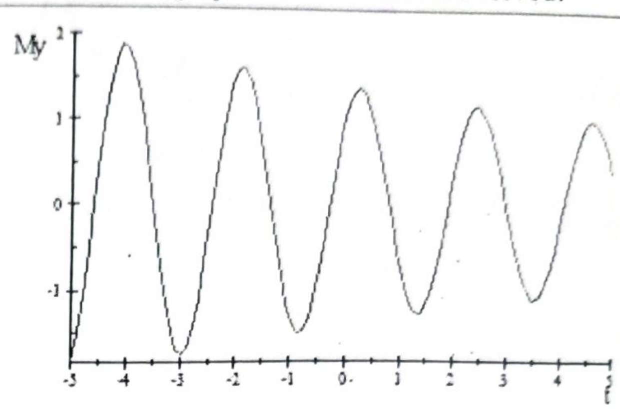
Recall the solution of time – dependent flow equation from 23, the complementary part is

$$y_h(t) = e^{-\alpha t}(a\cos wt + b\sin wt)$$

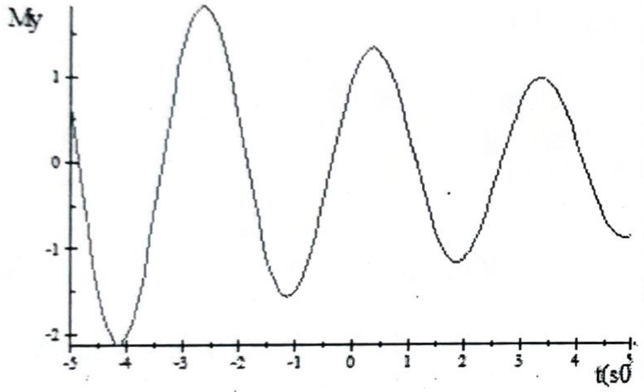
Assume $a = b = 1$; $\alpha = \frac{T_o}{2}$ and $w = \sqrt{\frac{1}{T_1 T_2}}$ and by keeping $T_1 = 1$ (constant) and varying T_2 between 0.01 and 0.53 at regular intervals, the graph below would be evolved:



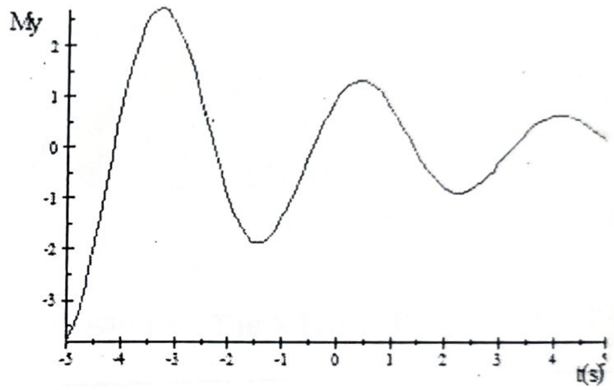
a. Plot of My against t when w = 7



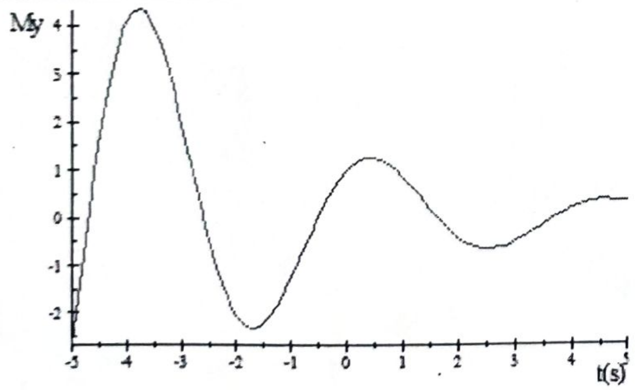
b. Plot of My against t when w = 2.9



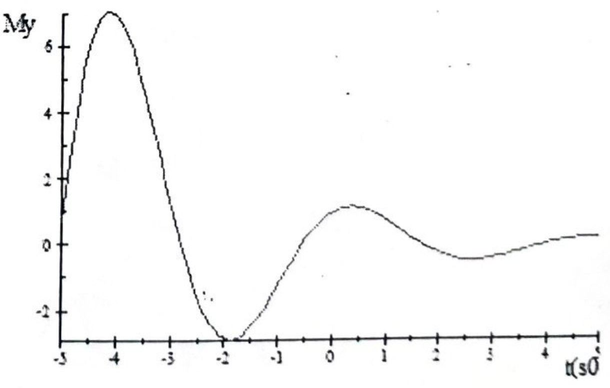
c. Plot of My against t when w = 2.1



d. Plot of My against t when w = 1.7



e. Plot of My against t when w = 1.5



f. Plot of My against t when w = 1.4

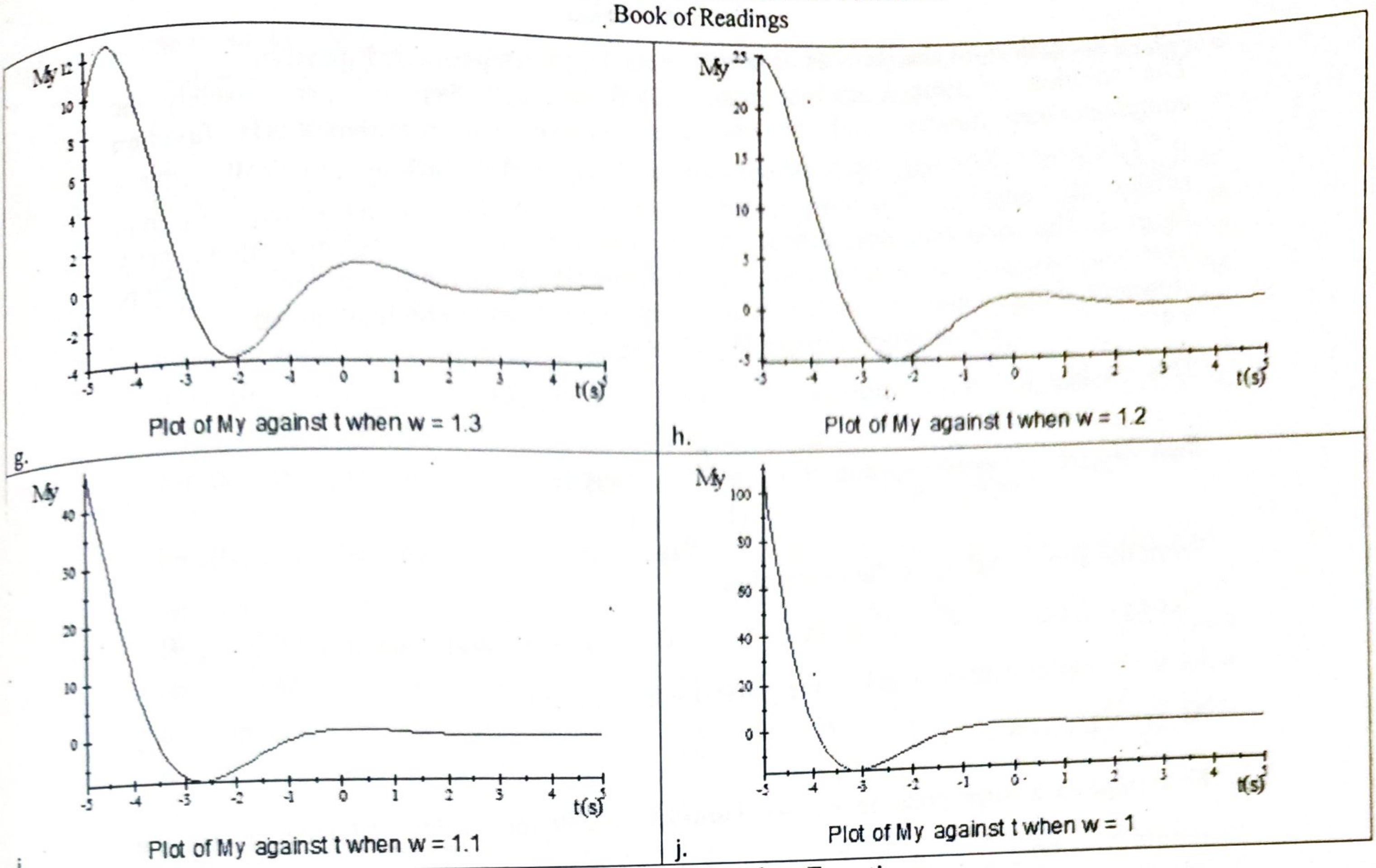


Fig.3:- *Plot of 2-Dimensional Time-Dependent Flow Equation*

3.3 Discussion on the Results obtained from Time - Independent Equation

From figure 1, where radio frequency is negligible i.e. ($\gamma^2 B_1^2 << K$) on the left hand column, the effect is that between 0.01s and 0.05s both the transverse magnetization M_y and the velocity V, show appreciable difference. The implication is that the strength of the signal is high with high velocity as there is rapid interaction between the molecules and its environment. However, the response of the magnetization and velocity do not show significant difference between 0.09s and 0.53s implying the strength of the signal is low. Similarly in the right hand column, *where radio frequency is applied* i.e. ($\gamma^2 B_1^2 >> K$) the effect is that between 0.01s and 0.13s both the transverse magnetization M_y and the velocity V, show appreciable difference *with an extended duration due to the radio frequency field.* However, the response of the magnetization and velocity do not show significant difference thereafter.

From the various graphs plotted above for M_y with respect to the separate variation of T_1 and T_2, it is generally obvious that the NMR system as could be seen mathematically is more sensitive to change in the spin – spin relaxation time (T_2) than the spin – lattice relaxation time (T_1). This is evident from figure 2 where T_2 is constant when T_1 was varied for both conditions $\gamma^2 B_1^2 << K$ and $\gamma^2 B_1^2 >> K$. The graphs here do not show significant difference from one another. This also confirms that only T_2 has effect on MRI when distinguishing between various organs of the human body. Note that transverse magnetization M_y carry information about the atoms and their environment.

3.4 Discussion on the Results obtained from Time - Dependent Equation

The solution of time-dependent Bloch NMR equation has two parts namely the *complementary function* and the *particular integral*. The **complementary function** $e^{-\alpha t}(A\cos wt + B\sin wt)$ approaches zero as t approaches infinity practically after a sufficiently long time. This complementary part represents the transient solution as seen in figure 3. The complementary function can lead to three different types of motion namely over-damping, critical damping and under damping which are not in focus in this write up. However, damped and un-damped forced oscillation will be briefly highlighted.

3.4.1 Un-damped Forced Oscillation

This is when $T_o = 0$. Assuming $w^2 \neq w_o^2$ where $w_o^2 = \frac{k}{m}$

then $M_{yp}(t) = \frac{F_o}{m(w_o^2 - w^2)}\cos wt = \frac{F_o}{k[(1-\frac{w}{w_o})^2]}\cos wt$

From the above and using the expression-

$A\cos x + B\sin x = \sqrt{A^2+B^2}\cos(x \pm \delta)$; $C = \sqrt{A^2+B^2}$ and $\tan\delta = \frac{\sin\delta}{\cos\delta} = \mp\frac{B}{A}$

for the complementary solution, the general solution will be -

$M_y(t) = C\cos(w_o t - \delta) + \frac{F_o}{m(w_o^2 - w^2)}\cos wt$

This implies a superposition of two harmonic oscillations whose frequencies are the natural frequency $\frac{w_o}{2\pi}$ i.e. the frequency of the free un-damped motion of the system and the frequency $\frac{w}{2\pi}$ of the output. This phenomenon is called *resonance* and it forms the basis of operation in Nuclear Magnetic Resonance – NMR. However, if w is close to w_o, the particular solution is $M_{yp}(t) = \frac{F_o}{m(w_o^2 - w^2)}(\cos wt - \cos w_o t)$ corresponding to the initial condition $y(0) = 0$ and $y'(0) = 0$. By using the relation

$$\cos v - \cos u = 2\sin\frac{u+v}{2}\sin\frac{u-v}{2},$$

Equation 22 may be re-written as $M_{yp}(t) = \frac{2F_o}{m(w_o^2 - w^2)}\sin\frac{w_o + w}{2}t\sin\frac{w_o - w}{2}t$

This results in beats as the difference between the input and natural frequencies is small.

3.4.2 Damped Forced Oscillation

The second case is when $T_o > 0$. This implies there is damping. From solution, $M_y(t)$

$= e^{-\alpha t}(A\cos wt + B\sin wt) + \{F_o\frac{k - w^2}{(k-w^2)^2 + (wT_o)^2}\cos wt + F_o\frac{wT_o}{(k-w^2)^2 + (wT_o)^2}\sin wt\}$

The first part which is the complementary function $e^{-\alpha t}(A\cos wt + B\sin wt)$ approaches zero as t approaches infinity and practically after a sufficiently long time and the general solution $M_y(t)$ now represents the transient solution which tends to the steady state solution $M_{yp}(t)$. Hence, after a sufficiently long time, the output corresponding to a purely sinusoidal input will practically be a harmonic solution whose frequency is that of the input. In this case, the amplitude will always be *finite* as against the situation when it is un-damped in which case the amplitude is *infinite* as w approaches w_o.

5.0 Discussion of Results

In this study, the analytic solution of Nuclear Magnetic Resonance (now called Magnetic Resonance Imaging) has been presented under two different headings- (a) time independent and (b) time dependent. In each case, two instances were considered- the free precision of the nuclei or negligible field and the presence of an exciting field or driving force. In each of these instances, T_1 longitudinal or spin – lattice relaxation time was kept constant while T_2 transverse or spin – spin relaxation time varied and the process reversed. Graphs of these different stages were plotted and the results obtained analyzed. It has further been discovered that for blood, while spin – lattice relaxation time (T_1) is constant, the spin – spin relaxation time (T_2) shows remarkable difference.

Finally, from the various graphs plotted above for Magnetization M_y with respect to the separate variations of T_1 and T_2, It is generally obvious from the plots in Time-Independent equations (Figures 1 and 2) that the NMR (now MRI) system as developed mathematically is more sensitive to change in T_2 (transverse or spin – spin relaxation time) than T_1 (longitudinal or spin – lattice relaxation time). This is evident in all the cases with T_2- constant (figures 2), where the curves for each set of the varying T_1 do not show significant difference from one another. For time-dependent (figure 3), the **complementary function** $e^{-\alpha t}(A\cos wt + B\sin wt)$ approaches zero as t approaches infinity practically after a sufficiently long time. This complementary function represents the transient solution. On the whole, it can be concluded that the particular integral is the only part contributing to the transverse magnetization.

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