DETECTION OF BLOCKAGE IN A RADIALLY SYMMETRIC CYLINDRICAL PIPE USING DIFFUSION MAGNETIC RESONANCE EQUATION

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Abstract
In this study, Magnetic Resonance Imaging (MRI) is used to detect partial and total blockage of a cylindrical pipe by unused engine oil. Diffusion Magnetic Resonance (DMR) equation is solved analytically for flow of fluid in a radially symmetric cylindrical pipe. Appropriate boundary conditions were imposed and the radial axis varied to depict partial and total blockage in the pipe. The results show that for free flow, the magnetization is between 0.004 and 0.005. For partial blockage, the magnetization reduces (signal loss) in value to 0.00001, and for total blockage it is zero (0). This method is a viable alternative for detecting blockage in fluid pipelines in oil and gas industry due to its non invasive analysis of flow in fluid. The MRI model also registers signal in its first few seconds or micro-seconds. The analysis is also applicable to process industries where different network of pipes are used, or machines use cylindrical pipes or tubes in transporting materials especially when there is a partial or total blockage at any point in the network.

Keywords: Bloch NMR Equations, DMR equation, Cylindrical pipe, Magnetization.

Introduction
Diffusion Magnetic Resonance Imaging (DMRI) is one of the most rapidly evolving techniques in the MRI field. It provides accurate assessment of the individual component or multi-component systems in a matter of minutes whereas traditional radioactive tracer techniques may take weeks for each component (Awojoyogbe *et al*., 2011). Diffusion and flow can be measured very delicately and accurately using Magnetic Resonance Imaging (Hazlewood *et al*., 1974). Diffusion coefficient of a substance, defined as the amount of material that diffuses in a certain time plays a vital role in the detection of blockage in a pipe using MRI. Random diffusion motion of water molecules has intriguing properties depending on the physiological and anatomical environment of the organisms being studied. This is the principle being exploited by the method of DMRI. Though not widely known, it has been noted for long that nuclear magnetic resonance is capable of quantifying diffusion movement of molecules as a result of uniqueness in relaxation rates - T_1 and T_2 (Yusuf *et al*., 2010).

Some attempts have been made in the past to detect blockage. Yuan used time splitting algorithms and Godunov mixed format to simulate the pulse propagation in the blocked pipelines (Yuan *et al.,* 2014). Another technique used by Sattar is by the system frequency response. This is a technique whereby the frequency response is used in the detection of partial blockages in a pipeline (Sattar *et al*., 2008). Similar to this is the method adopted by Mohapatra for the detection of partial blockages in single pipelines by the frequency response method (Mohapatra *et al.,* 2006). Wang also investigated analytically the effects of a partial blockage on pipeline transients. A partial blockage is simulated using an orifice equation, and the influence of the blockage on the unsteady pipe flow is considered in the equation using a Dirac delta function (Wang *et al.,* 2005).

In this work, the principle of Magnetic Resonance is applied to a cylindrical oil pipe under the influence of radiofrequency field as a probe to perturb the molecules of the unused engine oil. This causes the nuclei to absorb energy from the applied electromagnetic (EM) pulse(s) and radiate this energy at a specific resonance frequency which depends on the strength of the magnetic field and other factors. This allows the observation of specific magnetic properties of an atomic nucleus. A Radio Frequency (RF) transmitter is needed to transmit energy into the fluid under consideration in the cylinder in order to "activate" the nuclei so that they emit a signal (Waldo & Arnold, 1983). The relaxation process itself is referred to as the free induction decay (FID). It is the observable NMR signal generated by non-equilibrium nuclear spin magnetization precessing about the magnetic field conventionally along z direction (Hopf *et al.,* 1973). This time-domain signal is typically digitized and then Fourier transformed in order to obtain a frequency spectrum of the NMR signal i.e. the NMR spectrum (Duer, 2004). The study of the behaviour of diffusion or flow of engine oil at the point of partial and total blockage is here now undertaken.

The Bloch NMR Equations

The x, y, z components of magnetization of fluid flow are given by the Bloch equations which are fundamental to understanding Magnetic Resonance Images:

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2} \tag{1}$$

$$\frac{dM_y}{dt} = \gamma M_z B_1(x) - \frac{M_y}{T_2} \tag{2}$$

$$\frac{dM_z}{dt} = -\gamma M_y B_1(x) - \frac{M_o - M_z}{T_1} \tag{3}$$

where

M_o = equilibrium magnetization

M_x = component of transverse magnetization along the x-axis

M_y = component of transverse magnetization along y-axis

M_z = component of magnetization along the field (z -axis)

γ = gyro-magnetic ratio of fluid spins

B_o = static magnetic field

$B_1(x,t)$ = radio-frequency (RF) magnetic field

T_1 = Longitudinal or spin lattice relaxation time

T_2 = Transverse or spin-spin relaxation time

V = the flow velocity

From the fundamental Bloch equations (1) - (3), the diffusion equation was evolved with the diffusion coefficient D evolving intrinsically without any additional term as done by Torrey (1956). The NMR diffusion equation as derived by Awojoyogbe *et al.* (2011) is given as:

$$\frac{\partial M_y}{\partial t} = D\frac{\partial^2 M_y}{\partial r^2} + \frac{F_o}{T_o}\gamma B_1(r,t) \tag{4}$$

where the diffusion coefficient [illegible] was accurately defined in terms of MRI flow parameters fluid velocity, V , T_1 and T_2 relaxation rates (as $T_0 = \frac{1}{T_1} + \frac{1}{T_2}$) and $F_o = \frac{M_o}{T_1}$.

The above diffusion equation with [illegible], called diffusion coefficient was evolved as an intrinsic part of the Bloch Nuclear Magnetic Resonance (NMR) equations.

Solution of the Diffusion Equation in Radially Symmetric Cylinder

Since the cylinder under consideration is radially symmetric, then it is independent of θ. Therefore M_{xy} can be expressed as

$$M_{xy} = M_{xy}(r, z, t) \quad (5)$$

where M_{xy} is the transverse magnetization.

In cylindrical coordinates, Equation (4) transforms to

$$\frac{\partial M_{xy}}{\partial t} = D\left(\frac{\partial^2 M_{xy}}{\partial r^2} + \frac{1}{r}\frac{\partial M_{xy}}{\partial r} + \frac{\partial^2 M_{xy}}{\partial z^2}\right) + [illegible] \quad (6)$$

Consequently, the solutions to the Equation (6) are:

$$U(t) = [illegible] \quad (7)$$

$$F(r) = C_1 J_0(\mu r) + C_2 Y_0(\mu r) \quad (8)$$

$$Z(z) = [illegible] \quad (9)$$

Combining the solution to the diffusion equation (6), this gives the product of the quantities in (7), (8) and (9) plus [illegible] i.e.

$$M_{xy} = [illegible] \quad (10)$$

$$M_{xy}(r, z, t) = [illegible] \quad (11)$$

The last function on the right hand side is the radio-frequency field applied to perturb the molecules of the fluid. Therefore for the solution of

$$[illegible] \quad (12)$$

The radio frequency field (rf) field is defined as

$$[illegible] \quad (13)$$

which implies

$$[illegible] \quad (14)$$

$$[illegible] \quad (15)$$

Consequently,

$$M_{xy}(r, z, t) = [illegible] \quad (16)$$

Solution using the Initial and Boundary Conditions

We shall examine the behaviour of diffusion or flow of engine oil at the point of free flow, partial and total blockage as shown in Figures 1a, 1b and 1c below:

(a)

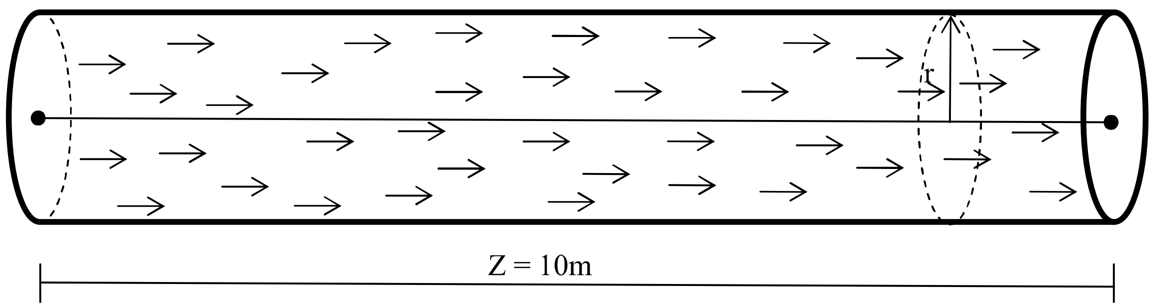

Figure 1(a): Free flow of Unused Engine Oil in the cylinder

(b)

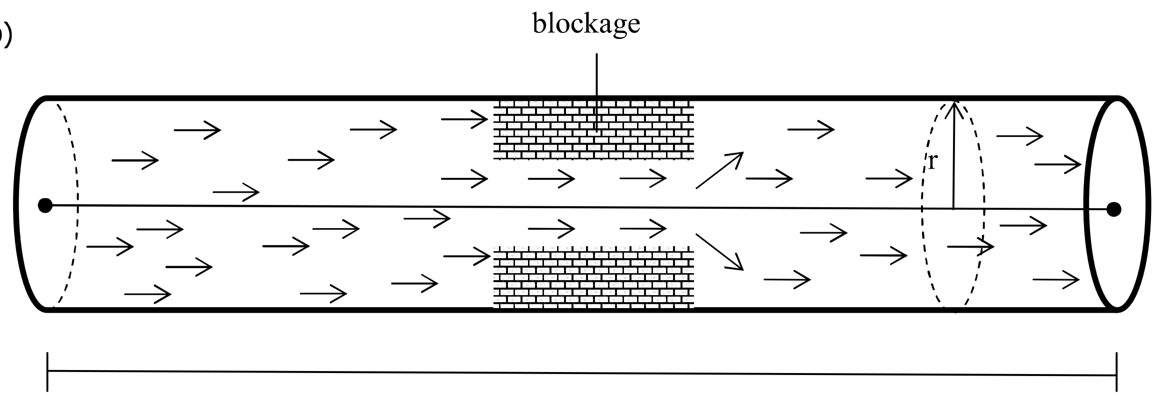

Figure 1(b): Partial blockage of Unused Engine Oil in the cylinder

(c)

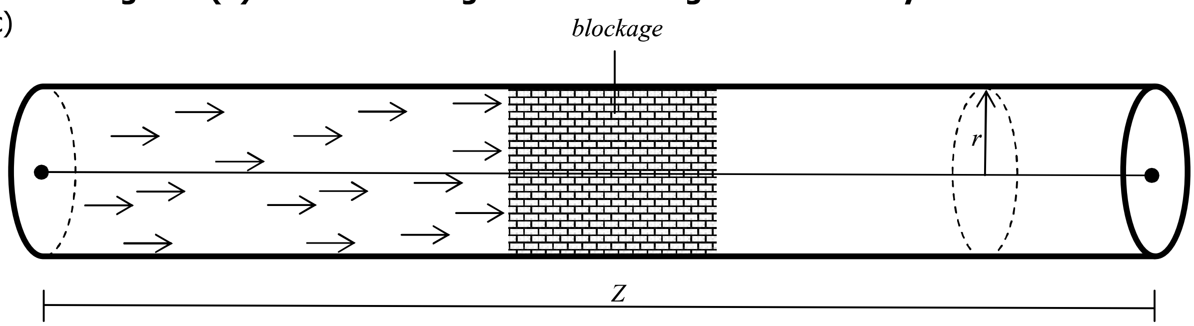

Figure 1(c): Total blockage of Unused Engine Oil in the cylinder

The following conditions shall be imposed:

i) [illegible]

ii) [illegible]

iii) [illegible]

iv) [illegible]

v) [illegible] (17)

where r *is the space depicting the blockage* and t is the direction of flow and both are defined as follows -

$$[illegible] \quad (18)$$

Finally, the solution for the magnetization of any molecule of the fluid at any point r and time t is given as:

$M_y(r,z,t) =$ [illegible] (19)

Results and Discussion

The fluid under consideration is unused engine oil and its diffusion coefficient is [illegible]. T_1 and T_2 values of unused engine oil were used and the following substitution made for free flow, partial and total blockages of the pipe:

[illegible] (20)

Based on our computational algorithm, Figures 1a, 1b and 1c were obtained for free flow, partial and total blockages respectively.

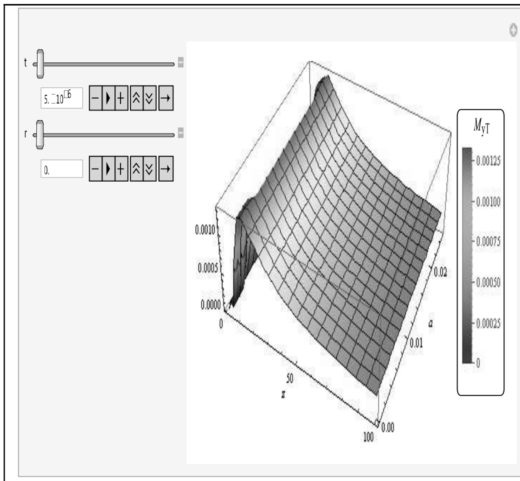

Figure 2a: Magnetization (My) against time (t = 0.000005 sec.) for unused engine oil when there is no blockage (r = 0)

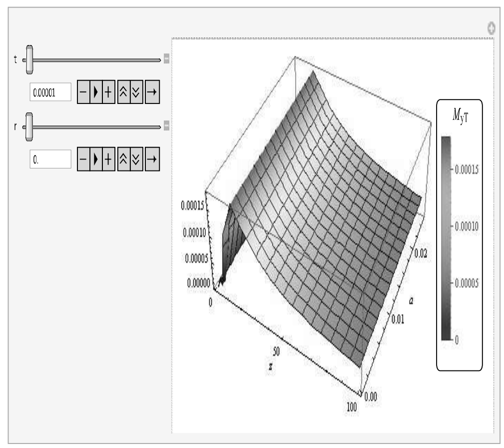

Figure 2b: Magnetization (My) against time (t = 0.00001 sec.) for unused engine oil when there is no blockage (r = 0)

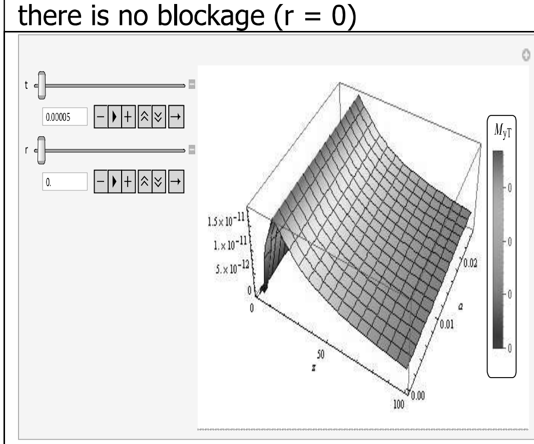

Figure 2c: Magnetization (My) against time (t =0.00005 sec.) for engine oil when there is no blockage (r = 0)

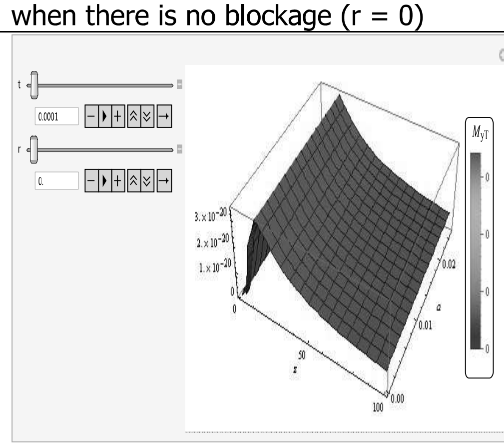

Figure 2d: Magnetization (My) against time (t =0.0001 sec.) for unused engine oil when there is no blockage (r = 0)

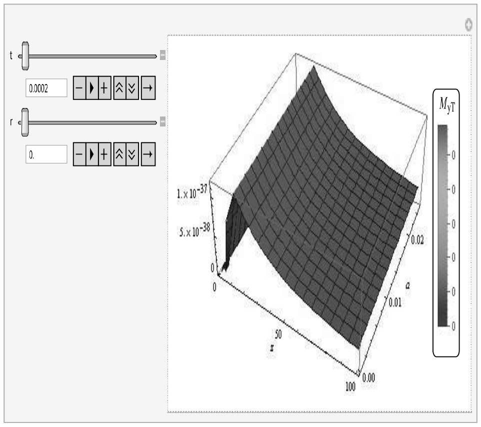

Figure 2e: Magnetization (My) against time (t =0.0002 sec.) for unused engine oil when there is no blockage (r = 0)

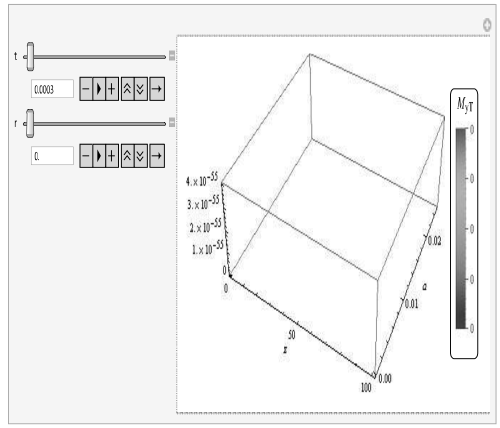

Figure 2f: Magnetization (My) against time (t =0.0003 sec.) for unused engine oil when there is no blockage (r = 0)

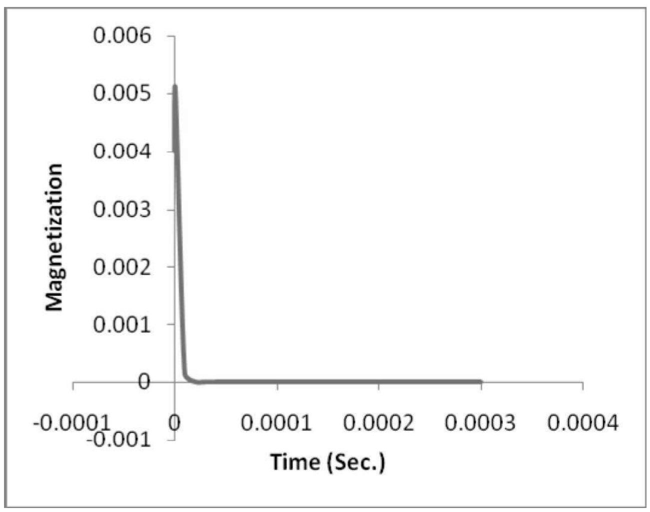

Figure 2g: Flow in a Cylinder of Unused Engine Oil when there is No Blockage (Free Flow)

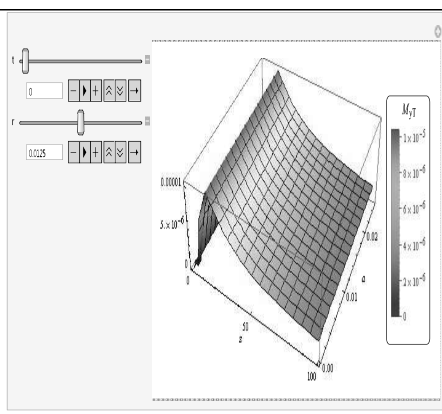

Figure 3a: Magnetization (My) against time (t = 0 sec.) when blockage is half the radius (r = 0.0125m)

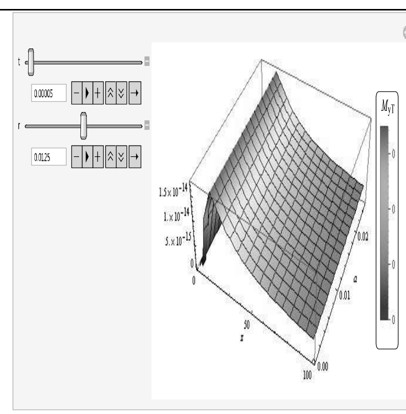

Figure 3b: Magnetization (My) against time (t = 0.00005 sec.) when blockage is half the radius (r = 0.0125m)

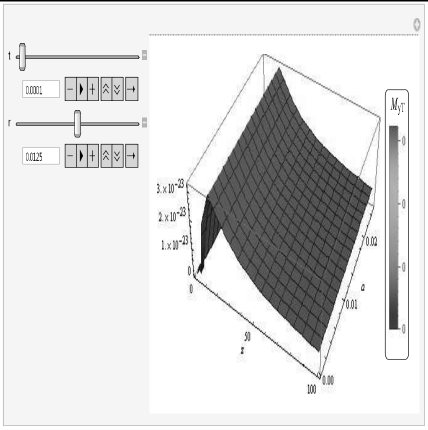

Figure 3c: Magnetization (My) against time (t = 0.0001 sec.) when blockage is half the radius (r = 0.0125m)

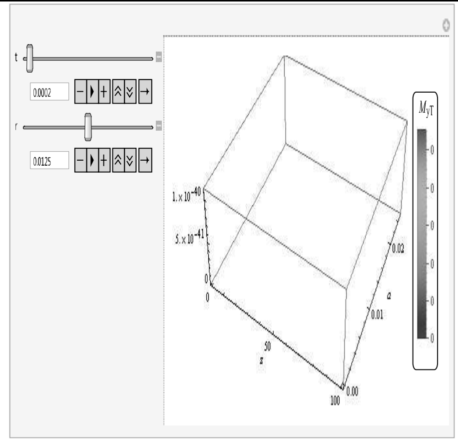

Figure 3d: Magnetization (My) against time (t = 0.0002 sec.) when blockage is half the radius (r = 0.0125m)

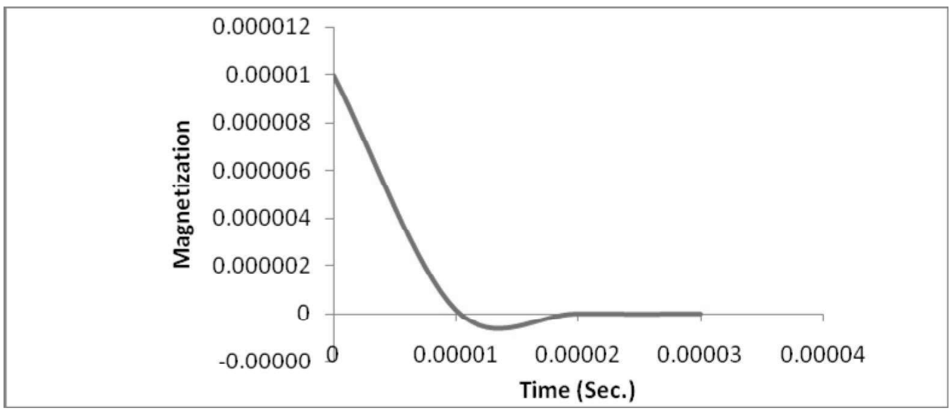

Figure 3e: Flow in a Cylinder of Unused Engine Oil When the Blockage is half its Radius (Partial Blockage)

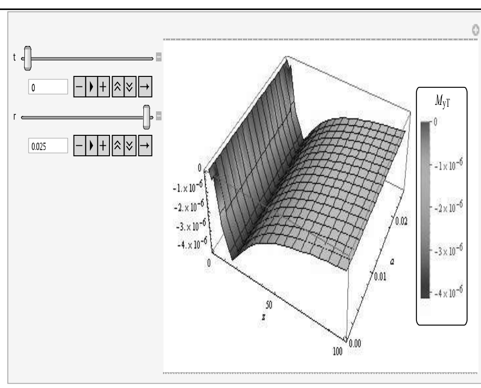

Figure 4a: Magnetization (My) against time (t = 0sec.) when there is total blockage (r = 0.025m)

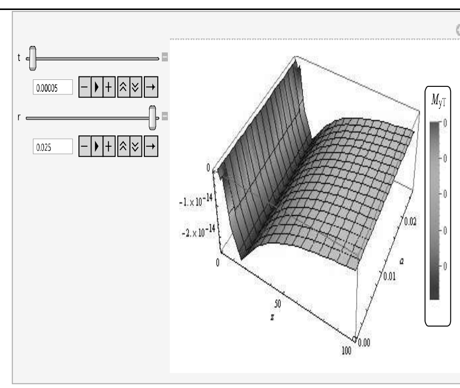

Figure 4b: Magnetization (My) against time (t = 0.00005 sec.) when there is total blockage (r = 0.025m)

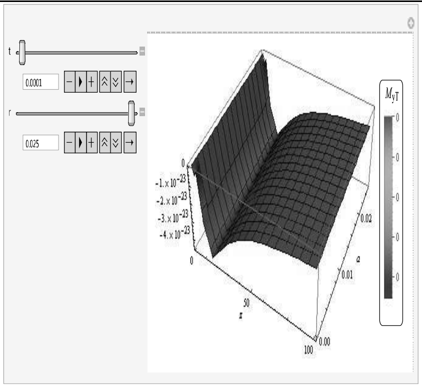

Figure 4c: Magnetization (My) against time (t = 0.0001 sec.) when there is total blockage (r = 0.025m)

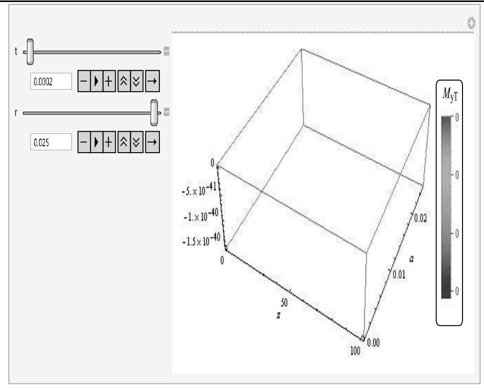

Figure 4d: Magnetization (My) against time (t = 0.0002 sec.) when there is total blockage (r = 0.025m)

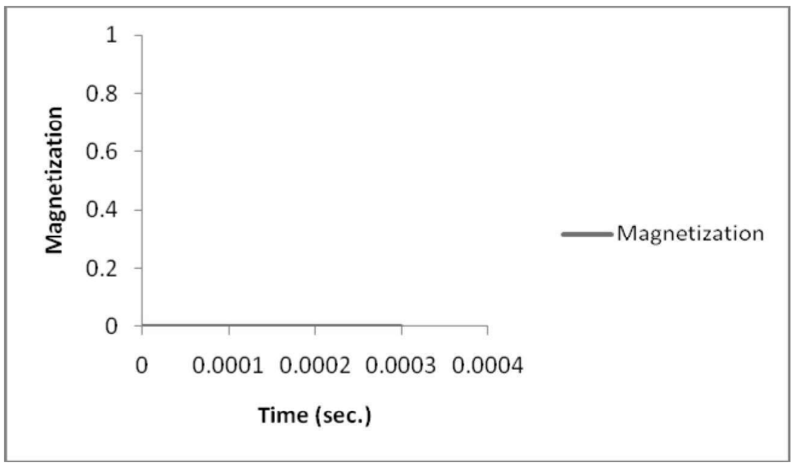

Figure 4e: Flow in a cylinder of unused engine oil when there is total blockage

Conclusion

We have developed a mathematical procedure for detailed analysis of fluid flow in a cylindrical pipe based on the analytical solution of the Bloch NMR diffusion equation with appropriate boundary conditions. The simple analytical solution obtained contains very important magnetic resonance flow parameters which can be useful for the non invasive analysis of flow in fluid leakages and oil spill. For example, the free induction decay (FID) is demonstrated signifying no blockage and the magnetization is between 0.004 - 0.005. Under partial blockage, the magnetization reduces (signal loss) in value to 0.00001 and for total blockage it is 0.0. This may lead to quick identification of problems whenever it arises so as to elicit immediate control and solution before much damage is done. The analysis can also be useful in process industries where different network of pipes are used or machines that use cylindrical pipes or tubes in transporting materials especially when there is a partial or total blockage at any point in the network.

It is noted with keen interest that this study focused on the assessment of degree of blockage in a cylindrical pipe. However, an anomaly may occur that blockage or plaque size may not always be the only determining factor in the assessment of degree of blockage; plaque composition can be a more significant determinant than plaque size. Thus, risk may correlate more closely with plaque morphology and surface features than with size. This possibility emphasizes the importance of developing non-invasive imaging methods for characterizing plaque morphology and composition in addition to determining area and wall thickness. This possibility will be studied in details in our next investigation especially as applied to gas and oil industry.

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