THE ROLE OF MATHEMATICAL SCIENCES IN SUSTAINABLE DEVELOPMENT GOALS

BOOK *of* PROCEEDINGS

International Conference on Contemporary Developments in Mathematical Sciences

in Honour of

Professor Kayode Rufus Adeboye

Department of Mathematics
Federal University of Technology, Minna

70th Birthday & Retirement from Service

Professor Kayode Rufus Adeboye

B.Sc. (Lagos), M.Phil. (Reading), Ph.D. (Ilorin), FMAN, FNMS, FAC

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PROFESSOR Kayode Rufus Adeboye

B.Sc. (Lagos), M.Phil. (Reading), Ph.D. (Ilorin), FMAN, FNMS, FAC

PROFILE

EARLY LIFE AND EDUCATION

Professor Kayode Rufus Adeboye was born in Egbe-Ekiti on April 12th, 1949 to Chief and Mrs. Daniel O. Adeboye. He attended Ekiti Parapo College, Ido-Ekiti, for his Secondary School Education. For his tertiary education, he attended Oluloyo College of Education, Ibadan, Nigeria from 1964 to 1966. He commenced his undergraduate degree programme in 1968 at the University of Lagos and got Western State University scholarship in 1969. He graduated with a B.Sc. with Honours (Second Class Upper Division) in Mathematics in 1971 and following his brilliant performance in his undergraduate course, he was awarded a University of Lagos Postgraduate Scholarship in 1972 and 1973, AFGRAD Scholarship, USA in 1974, Federal Government Postgraduate Scholarship in 1974 and University of Ife fellowship in 1976 to continue his studies at University of Reading, Reading, England. From 1973 to 1975 he worked as a Tutor at the University of Ife now Obafemi Awolowo University.

Professor Kayode Rufus Adeboye left Nigeria in 1975 at the age of 26 on scholarship to pursue postgraduate studies in Mathematics at the University of Reading, Reading, England. On successfully completing this course in 1978, he returned to Nigeria and continued his academic work as Assistant Lecturer (1978 - 1980) and Lecturer II (1980 - 1982) at the University of Ife and Research Officer (1982 - 1989) at the National Teacher Institute, Kaduna, Nigeria. He embarked on his PhD degree in Mathematics at the University of Ilorin, Nigeria in 1988 supervised by Prof. M. A. Ibiejugba of blessed memory.

Kayode Rufus Adeboye was awarded the PhD degree in Mathematics by the University of Ilorin in 1991 at the age of 42.

He later joined the service of Federal University of Technology, Minna in 1989 as Senior Lecturer in the Department of Mathematics/Computer Science and rose to the rank of a Professor of Mathematics in 1997. He taught mathematics at the Federal University of Technology, Minna for more than 30 years.

AT THE FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA

At the Federal University of Technology, Minna, he has been the thesis advisor for twelve Ph.D. students and the joint thesis advisor of many Ph.D. students with Professor N. I. Akinwande, Prof. Y. M. Aiyesimi, Prof. Y. A. Yahaya, Prof. U. Y. Abubakar and Prof. R. O. Olayiwola. He has supervised the studies of 31 M.Tech. Students, 120 postgraduate diploma students between 1994 and 2010 and more than 250 undergraduate students' projects between 1991 and 2019.

The numerous roles he has had at the Federal University of Technology, Minna include but are not limited to the following: School Time-Table Officer (1992 - 1999); Head of Department (1993 - 1999); Member, University Senate (1992 - 2019); Chairman, Computer Centre Service Board (1997 - 2002); Chairman, School Examination Malpractices Committee (1990 - 1999); Member, University Students' Disciplinary and Examination Malpractices Committee (1990 - 2007); Member, School Research Committee (1993 - 2006); Chairman, University Ceremonies Committee (1993 - 2003); Chairman, University Fees Review Committee (1998 - 2003); Member, Appointments and Promotion Committee (1997 - 2000); Chairman, Students' Crisis Investigation Committee, Federal University of Technology, Minna, Nigeria (2001); Chairman, University Staff School Management Board, Federal University of Technology, Minna, Nigeria (1999 - 2003); Chairman, Millenium Bug Committee, Federal University of Technology, Minna, Nigeria (1998 - 2002);Dean, School of Science and Science Education (1999 - 2003); Member, Committee of Deans (1993 - 2003); and Chairman, Senate Committee on Senate Standing Orders (2008).

ACADEMIC AND PROFESSIONAL QUALIFICATIONS

1. Nigeria Certificate in Education (NCE), Oluloyo College of Education, Ibadan, Nigeria. 1967
2. B.Sc. (Hons) Second Class Upper (Mathematics), University of Lagos, Nigeria. 1971
3. M.Phil. (Numerical Analysis), University of Reading, Reading, England. 1979
4. Ph.D. (Mathematics), University of Ilorin, Ilorin, Nigeria. 1991

UNIVERSITY EDUCATION (WITH DATES)

(a) University of Lagos (1968-1971)
(b) University of Reading, Reading, England (1975-1978)
(c) University of Ilorin (1988-1991)

SCHOLARSHIPS AND FELLOWSHIPS AWARDS

(a) Western State University Scholarship (1969)
(b) University of Lagos Postgraduate Scholarship (1972 and 1973)
(c) AFGRAD Scholarship, USA (1974)
(d) Federal Government Postgraduate Scholarship (1975)
(e) University of Ife Fellowship (1976)

AWARDS, HONOURS AND DISTINCTIONS

1. Fellow of Mathematical Association of Nigeria (FMAN), 1999.
2. Fellow of Nigerian Academy of Control (FAC), 2000.

3. Best Practices in University Teaching Project (BESTPUT), Mathematics National University Commission (NUC), Abuja, Nigeria, 2002.
4. IBC's 21st Century Award for Achievement, International Biographical Centre (IBC), Cambridge, UK, 2002.
5. Outstanding Intellectuals of the 21st Century Award, International Biographical Centre (IBC), Cambridge, UK, 2002.
6. Who's Who in the 21st Century Order of Excellence, International Biographical Centre (IBC), Cambridge, UK, 2002.
7. IBC Living Legends Award, International Biographical Centre (IBC), Cambridge, UK, 2003.
8. Contemporary Who's Who Award, American Biographical Institute (ABI), Raleigh, NC, USA, 2003.
9. Man of the Year, American Biographical Institute (ABI), Raleigh, NC, USA, 2003.
10. American Medal of Honour, American Biographical Institute (ABI), Raleigh, NC, USA, 2004.
11. Man of the Year Representing Nigeria, American Biographical Institute (ABI), Raleigh, NC, USA, 2009.
12. Fellow of Nigerian Mathematical Society (FNMS), 2016.

OTHER ACADEMIC ACTIVITIES AND RESPONSIBILITIES

Professor Kayode Rufus Adeboye was an External Examiner and External Assessor to many Nigeria Universities and Polytechnics. He was also the Chairman and Member, NUC Accreditation Team to many Nigeria Universities. The numerous roles he has had outside the Federal University of Technology, Minna include but are not limited to the following:

(i) Sabbatical leave, Ibrahim Badamasi Babangida University, Lapai, Nigeria (2006 - 2007) and University of Abuja, Abuja (2014 - 2015).

(ii) Course writer, National Teachers' Institute (NTI), Kaduna, Nigeria (1988 -1994).

(iii) Supervisor, NTI-NCE by DLS, Niger state, (1990-1996).

(iv) Supervisor, University of Ibadan External Degree Programme, Niger State, (1990-997).

(v) External Moderator, Niger State College of Education, Minna, (1990- 1998).

(vi) Chief Examiner, JSS (Mathematics), National Examinations Council (NECO), (1999 - To date)

(vii) Chief Examiner, SSCE (Further Mathematics), National Examinations Council (NECO) (2012- To date).

(viii) Resource Person, Postgraduate Course on Computer Science, National Mathematical Centre (NMC), Abuja, Nigeria (1996).

(ix) Coordinator, National Mathematical Centre (NMC), Abuja Internal Conference on Computational Mathematics (1999).

(x) Resource Person, National Mathematical Centre (NMC), Abuja Research programme in Computational Mathematics (1999).

(xi) Chairman, NUC Minimum Academic Standard Curriculum Development for Basic Sciences and Computer Science (1999).

(xii) Coordinator (Mathematics Group), National Open University of Nigeria (NOUN) Course Development Workshop (2002).

(xiii) Member, NUC Minimum Academic Standard Final Year Syllabus for General Studies (2004).
(xiv) Resource Person, National Mathematical Centre (NMC), Abuja Higher Degree Programme (2004, 2007).
(xv) Editor-in-Chief, National Mathematical Centre (NMC), Abuja Seminar Proceedings (2006).
(xvi) Associate Editor, AMSE Journal, France (2005 – 2008)
(xvii) External Examiner, Master's Degree Programme, National Mathematical Centre (NMC), Abuja (2010 - 2016)
(xviii) Associate Editor, Nigerian Mathematical Society (JNMS) (2013 – To date)
(xix) External Assessor, Professorial Appointment, NDA, Kaduna (2019).
(xx) External Assessor for 2 Professorial Appointments, National Mathematical Centre, Abuja (2019)
(xxi) Dean, Faculty of Applied and Natural Sciences, Ibrahim Badamasi Babangida University, Lapai, Nigeria (2006 - 2007).

MEMBERSHIP OF LEARNED SOCIETIES

(i) Member, Nigerian Mathematical Society (NMS).
(ii) Member, Mathematical Association of Nigeria (MAN).
(iii) Member, Nigerian Computer Society (NCS).
(iv) Member, American Mathematical Society (AMS), USA.
(v) Member, International Association of Survey Statistians (IASS), Geneva, Austria.

BOARD MEMBERSHIP

(i) Member, Board of Directors, Internet Exchange Point of Nigeria (IXPN), (2007 – To date).
(ii) Member, Ministerial Committee on Privatization, Federal Ministry of Solid Minerals, Abuja, Nigeria, (2004 - 2005).
(iii) Member, Advisory Committee, American Biographical Institute (ABI), NC, USA, (2004).
(iv) Member, University Governing Council, Federal University of Technology, Minna, Nigeria, (2000 - 2002).
(v) Member, Management Board, National Mathematical Centre (NMC), Abuja, Nigeria, (2000 - 2004).

PUBLICATIONS

Professor Kayode Rufus Adeboye has 2 completed University funded Research projects, Inaugural lecture, 12 Book publications and over 60 Journal publications and these publications are as follows:

Completed University Funded Research Projects:

1. Adeboye, K. R. and Ayeni, R. O. (2000). Hybrid-Collocation-Galerkin Method for Differential Equations with Application to Petroleum Reservoir Mechanics.
2. Adeboye, *K.* R. and Bolarin, O. A., (2009), Application of Numerical Methods to Petroleum Reservoir Mechanics.

Inaugural Lecture:

K. R. Adeboye (2014). Mathematics, Mathematicians and Numerical Analysis: the Bridge and Bridgehead View of Nigeria with Mathematical Prism.

Book Publications:

1. K. R. Adeboye, (2006). Mathematical Methods for Science and Engineering Students, Moonlight publishers, Abuja, Nigeria.

2. K, R. Adeboye, (2006), ICT Policies Development and Applications in Nigerian Educational System, A chapter in a book written to honour Prof Okebukola, former NUC, Abuja Executive Secretary.

3. Adeboye, K. R., (1990), Nigerian Certificate in Education Course Book on Mathematics, Cycle 2, Module 1, Units 1 - 5 (Chapters on Differential Calculus), vol. 1, Published by National Teachers' Institute, Kaduna.

4. Adeboye, K. R., (1990), Nigerian Certificate in Education Course Book on Mathematics, Cycle 2, Module 5, Units 1 - 5 (Chapters on Numerical Analysis), vol. II, 71 - 95, Published by National Teachers' Institute, Kaduna.

5. Adeboye, K. R., (1991), Nigerian Certificate in Education Course Book on Mathematics, Cycle 3, Module 1, Units 6 - 10 (5 Chapters on Introductory Theory of Numbers and Polynomials), vol. 1, 23 - 53, Published by National Teachers' Institute, Kaduna.

6. Adeboye, K. R., (1992), Nigerian Certificate in Education Course Book on Mathematics, Cycle 3, Module 5, Units 6 - 10 (5 Chapters on Real Analysis), vol. II, Published by National Teachers' Institute, Kaduna.

7. Adeboye, K. R., (1993), Nigerian Certificate in Education Course Book on Mathematics, Cycle 4, Module 1, Units 1 - 3 (3 Chapters on Vector Spaces), vol. 1, 1 - 18, Published by National Teachers' Institute, Kaduna.

8. Adeboye, K. R., (1993), Nigerian Certificate in Education Course Book on Mathematics, Cycle 4, Module 2, Units 1 - 5 (5 Chapters on Linear Algebra), vol. I. 71 - 116, Published by National Teachers" Institute, Kaduna.

9. Adeboye, K. R., (1997), CO-Author, MAN Textbook of Mathematics for Science Students in Nigerian Universities, Cornerstone Publications, Ilorin.

10. Adeboye, K. R., (1998), CO-Author, Question and Answer Scries for SSCE by MAN, Cornerstone Publications, Ilorin 11. Adeboye, K. R., (1999), CO-Author, MAN Question and Answer Series for JAMB candidates, Cornerstone Publications, Ilorin.

12. Adeboye, K. R., (2005), Chairman, Editorial Board, Published Conference Proceedings, National Mathematical Centre, Abuja.

Journal Publications:

1. Adeboye, K.R. and Salisu A. (2017). A Re-definition of the Stability Condition for the Parabolic Scheme. *International Journal of Science and Technology (IJST) Uk Publication,* (Accepted for Publication).
2. Adeboye, K.R. and Salisu A. (2017), An H^2-Galerkin Method for the Solution of Parabolic Boundary Value Problems. *Journal of the Nigerian Association of Mathematical Physics,* Vol.41, Pp 453 - 456.
3. Adeboye, K.R., Abiodun A.P. and Salisu A. (2017) A Nonhydrostatic Atmospheric Model for
Numerical Weather Prediction, Using Minna, Nigeria as aCase Study. *NMC Journal* (Presented for Publication).
4. Etuk, Stella Oluyemi and Adeboye, K.R. (2017), Refinements of the Egyptian Fraction Finite Difference Scheme for First and Second Initial Value Problems. *Journal of Science, Technology and Mathematics Education (JOSTMED).* Vol. II
5. K. A Al-Mustapha and Adeboye, K. R. (2017),Variational-Composite Hybrid Fixed Point Iterative Method for the Solution of three-point Boundary Value Problems of Fourth Order Differential Equations. *Journal of the Nigerian Association of Mathematical Physics,* Vol.39, Pp 111-118.
6. Kilicman, Adeboye, K.R. and Wadai, M. (2016), A Variational Fixed Point Iterative Technique for the Solution of Second Order Differential Equations. Malaysian *Journal of Sciences* Vol.35(1), Pp 29 - 36.
7. Adeboye, K.R. and Salisu A. (2016), Super Convergent H^2-Galerkin Method for the Solution of Parabolic Initial Value Problems. *Journal of the Nigerian Association of Mathematical Physics,* Vol.38, Pp 33 - 40.
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46. Adeboye, K. R. (2002). A Super-Convergent H^2 - Galerkin Method for the Solution of Two- point Boundary Value Problems, *NJTR,* 1(1), 36-39.
47. Adeboye, K. R. (2001). A Convergent Explicit One-step Integrator for Initial Value Problems with Singular Solutions, *JOSTMED,*
48. Adeboye, K R. (2000). A Super Convergent H^2 - Galerkin Method for the Solutions of Boundary Value Problems, NMC Proceedings
49. Adeboye, K. R. (1999). A Cubic Order Predictor-Corrector Iterative Method for the Solution of Non-Linear Algebraic Equations. *Journal of Science, Technology and Mathematics Education (JOSTMED),* 2(1), 111-118.
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51. Adeboye, K. R. (1997). H^1 - Galerkin - Collocation and Quasi - iterative Method for Boundary Value Problems, *Abacus,* 26, 490-497.
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England.

CONCLUSION

We'll end this biography quoting from his inaugural lecture about the lack of knowledge of mathematics on the part of students –

Mathematics is needed for the development, maintenance, understanding, quantification and record keeping of our society. Since no society is static and the desire for higher heights in science and technology will continue to increase, the demand on mathematics will ever be on the increase. In view of the universal importance of mathematics to man on earth, it becomes compulsory that those charged with education should find ways of involving more of the younger generation of our days in the study of mathematics. Any nation that cannot get school children involved and be interested in mathematics, will never attain true social, economic, scientific and technological independence. Such a nation will continue to look up to those other nations of the world which through sound mathematics education have become world powers, with sound economic, scientific and technological bases, for her needs, even in matters of political guidance.

The educational system in Nigeria as at now is in a state of comma as a result of neglect. The rule then was "acquire now, neglect or even abandon later". The government acquired all the educational institutions from primary to tertiary level in the late 1970's only to neglect or even abandon them in the early 1980's. Some states established universities only to boost their egos. They never considered the cost of running a university before establishing one. They would want to hire lecturers at the rate of two for a kobo! I wish to advise the NUC to add to the conditions to be satisfied before establishing a university that the minimum conditions of service acceptable are those obtainable at the federal universities and this will put paid to the unbridled histrionics often engaged in by the owners.

REFERENCES

K. R. Adeboye (2014). *Mathematics, Mathematicians and Numerical Analysis: the Bridge and Bridgehead View of Nigeria with Mathematical Prism*. 27^{th} Inaugural Lecture series, Federal University of Technology, Minna, Nigeria.

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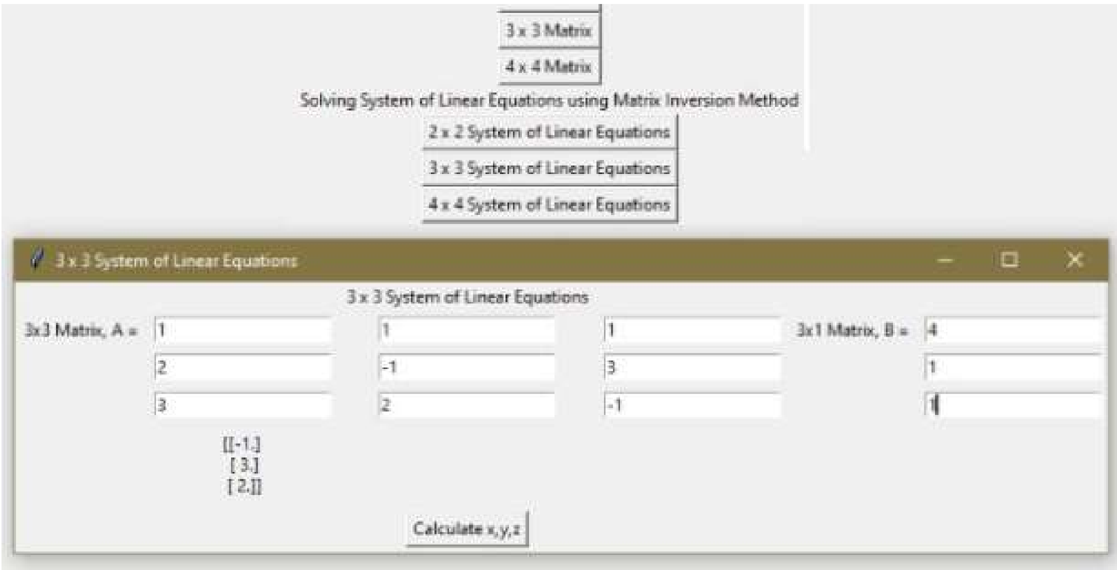

Figure 2. System of Linear Equation

4.3 Discussion of Results

From preminary testing, when the Inverse of a Matrix using Gauss-Jordan elimination method and System of Linear Equation using Matrix Inversion elimination method were solved analytically and the results were compared with the results generated by an automated system, it was observed that the automated system could generate solution to the problem more faster than the analytical method. This shows that the automated system is more efficient and effective in solving inverse of a matrix and system of linear equation.

5. Conclusion

The NumPy Standard library aids support for large, multidimensional arrays and matrices, along with a large collection of high-level mathematical functions to operate on these arrays which makes it a valuable exploratory and didactic tool. In this work, an automated system for solving the inverse of a square matrix and finding solution to the system of linear equations using the matrix-inversion method was designed and implemented using Python programming language. The results generated from the automated system were compared with that of the analytical solution. This shows that the automated system is more efficient and effective in solving inverse of a matrix and system of linear equation.

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E15: Reducing Waiting Time of a Network Queuing System Using Open Jackson Network Model

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Abstract:

In Nigeria patients experience prolong waiting time on queues whenever they visit hospitals for treatment. Some of these times wasting are as a result of shortage in man power or unable to assigned the required number of staff at the respective departments of the hospital. This avoidable time wasting usually led to redundancy or deficiency on some departments of the hospital. The Federal Polytechnic Bida school clinic is not left out from this avoidable time wasting. The majority of patients receiving treatments in Federal Polytechnic Bida school clinic are students and staff; this protracted waiting time on queue had often affected the academic performance of the students negatively as a result of missing lectures and worsening of health situation of both the lecturers and students. In this paper, a Network Queuing Model that determines optimal numbers of servers at the nodes of the school clinic network queuing system to reduce waiting time of the patients has been presented. The relevant data was collected for a period four weeks, through direct observations and interviews. The number of arrivals and departures were also obtained. The total expected waiting time of the patient in the current system before modification was 50minutes with total number of 10 servers in all the nodes, while the total new expected waiting time of patient in the system after modification was reduced to 19 minutes with total number of 17 servers in all the nodes. The study has determined optimal number of servers at the nodes of the school clinic network system. Results from this study are important information to the management of the school clinic for proper planning and better service delivery.

Key words: Network Queuing System, Nodes, Servers, School Clinic.

1. Introduction

Queuing theory is considered to be branch of operations research. It constitutes a powerful tool in modeling and performance analysis of many complex systems, such as computer networks, hospital network system, telecommunication systems, call centre, flexible manufacturing systems and service systems. Recently, the queuing theory including queuing systems and networks arouse mathematicians', engineers' and economics interests. It is clear that the queuing system in the school clinic is a network type because mostly patients move from one service facilities to another within the clinic in order complete treatment. Network of queues are used to model possible conflict of queuing when a set of resources are shared. It is a model in which jobs departing from one queue arrive at another, it describes a situation where the input from one queue is the output of one or more queues. Examples of where queuing network can be applied are machine shops, communication network, hospital system, movement of memos within an organization just to mention a few.

Literature

On Queuing Systems, Danish Engineer, Erlang in 1913 first analyzed queue or queuing theory or waiting lines in the context of telephone facilities. He started with the problem of the congestion of telephone traffic and later on, extended to business application and waiting lines. The ideas have since seen applications in telecommunication, traffic engineering, computing and the design of factories, shops, offices and hospitals (Schelechter, 2009). Lawal *et al*., (2019) have studied the waiting and service costs of a multi-server queuing system at National Health Insurance scheme (NHIS) unit of General Hospital Minna, Niger state, Nigeria. The two conflicting costs were balanced and the optimal performance for the queuing system was then determined. It was concluded that for the morning session, the average queuing length, waiting time of patients as well as over utilization of doctors at the unit could be reduced at an optimal sever level of 3 doctors at a minimum total cost of ₦6219.98 per hour as against the present server level of 2 doctors with high total cost of ₦26025.12 per hour which include waiting and service costs and for the evening session the present sever level of 1 doctor should be maintained. The researchers recommend that the result of the research is an important information to the management of NHIS unit of the General Hospital Minna to provide better service to the patients at a minimum cost. A successful application of queuing system in hospital management has also been reported in (John, 2010; Olaniyi, 2004; Kebe and Onah, (2012); Lakshmi, and Sivakumar, 2013; Adaji, 2018). Also, researchers such as (Damondhar and shastraka, 2018), (Muniratet *al.*, 2015), (Nityangini and Pravin, 2016), (Shastrakar and Pokley, 2017) and (Sushil*et al*., 2017) had successfully applied queuing theory in other fields.

2. Materials and Methods

The type of queuing system adopted by a organizations solely dependent on the type of service being provided. The Federal Polytechnic school clinic practice network type of queuing system. Queuing network is composed of several random queue systems, mostly limited and single queue systems. Diverse types of patients go by through the network in many ways and are served by the service nodes within the network system. A queuing network system has a set of nodes (i). Each node has a number of servers (s) and a single node can be regarded as a queuing system. Patients can have access to the queuing network from any node. The arrival rate from the outside is λ and the arrival rate of Node i is λ_i. After the patients queues and gets the service at a node (the service rate of Node i is μ_i), he can leave the network system or go to another node, or even return to the former node.

2.1 Model Formulation

The Federal Polytechnic Bida is a Federal owned tertiary institution, located in Bida Local Government of Niger state, Nigeria. The school clinic is within the campus and is made up of six different departments. In this study, each department is regarded as node of the network system. The data used in this research were gathered from the six departments of the Federal Polytechnic Bida school clinic and they obtained based on the arrival and departure rate as well time spent at each node. The method of data gathering was direct observation and personal interview by the research team. The data gathering was for a period of four weeks. This data gathering was carried from Monday to Sunday. In a day, gathering of the data was for a total of six (6) hours at different times of the day. For each node, the number of arrivals and departures together with service times were taken at intervals of 4 minutes arrivals of patients into a node, while the departure rate was obtained also by the average number of four (4) minutes departures of patients at that particular node. Each of the nodes was observed for a period of one (1) hour daily. All the nodes under consideration have at least a server.

3.2 *Model Assumptions*

We make the following assumptions for Federal Polytechnic School clinic.

1. The Federal Polytechnic Bida school clinic network queuing system is considered as an independent queuing system.
2. Queuing discipline in Federal Polytechnic Bida school clinic is first come first served.
3. The external arrival pattern follows a Poisson arrival process.
4. Each node in the Federal Polytechnic school clinic has at least a server with Exponential services time.
5. The service rate in the school clinic depends on the number of patients at each node Of the system.
6. In the school clinic the medical personnel's are regarded as servers.
7. The entire medical service provider in the school clinic must working in full capacity.
8. Service rate is independent of line length.

The study of the school clinic queuing system is based on Jackson open network model, the Federal polytechnic school clinic is made up of the following departments, the Registration, nursing, Consultation, Pharmacy, Laboratory and Accounting. We assumed in this research that patients who come into the Federal Polytechnic school clinic for services will commence by going first to the registration unit to register and then proceed to the nursing unit, from there, patients move to see the doctor at the consultation unit, this procedure continue until the patients depart from the clinic. At each department in school clinic, there is only one queue and one service at a time.

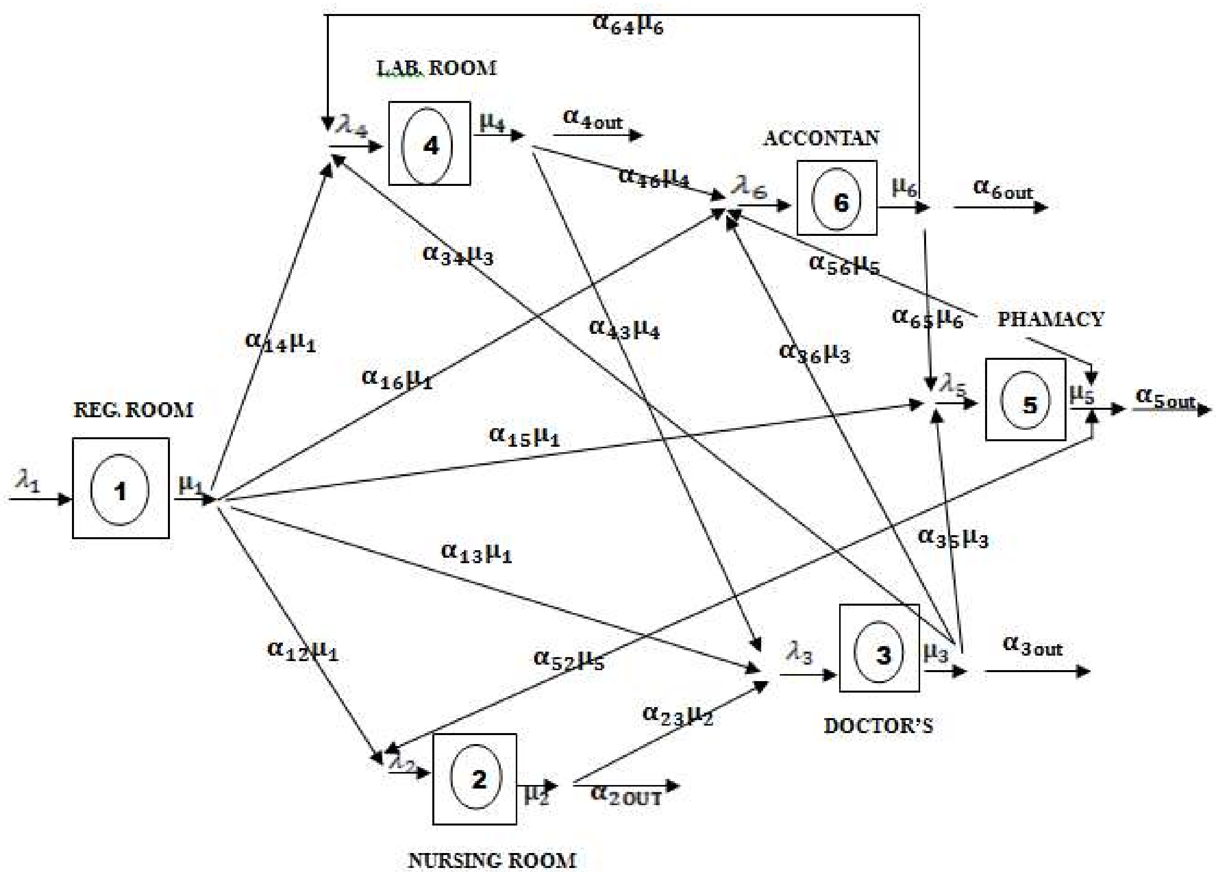

Figure 3.1. Schematic Diagram of Federal Polytechnic School Clinic Network Queuing System.

Where: λ_i is the arrival rate of the patients, for i = 1, 2…6

μ_i is the departure rate of the patients, for i = 1,2…6

α_{ij} are the weight of moving from node i to node j. for i = 1,2…6.

The notations used for the presentation of the data in the school clinic are node 1, node 2, node 3, node 4, node 5 and node 6, all through the period of 4 weeks, which includes the number of arrivals and number of departures with an interval of 4 minutes.

They are denoted as follow:

Registration point denoted by node1; Nursing station point denoted by node2.

Doctors' Room is denoted by node3; Laboratory unit is denoted by node4.

Pharmacy unit denoted by node5; Accountant's denoted by node6

The arrival rate $\lambda_i = \frac{1}{mean\ number\ of\ arrival}$, for i = 2….6. (1)

The departure rate $\mu_i = \frac{1}{mean\ number\ of\ departure}$, for i= 1,2,…….6 (2)

$$\rho = \frac{\lambda_i}{\mu_i},\ for\ i = 1,2, \dots \dots 6 \qquad (3)$$

The expected number of patients on the queue is given as

$$l_q = \frac{\rho}{m-\rho}, \qquad (4)$$

Where m stands for the number of servers at the node

The expected waiting time of the patients on the queue is given as:

$$w_{qi} = \frac{l_q}{\lambda_i}, \qquad (5)$$

The expected number of in the system is given as

$$l_s = l_q + \rho \qquad (6)$$

The expected waiting time of the patients in the system for node 1-6 is given as

$$w_i = \frac{l_s}{\lambda_i}, \quad \text{for } i = 1,2,\dots 6. \qquad (7)$$

3 .3 Model Equations

From Figure1, we obtained the following model equations.

$$\lambda_2 = \alpha_{12}\mu_1 + \alpha_{52}\mu_5 \qquad (8)$$

$$\lambda_3 = \alpha_{13}\mu_1 + \alpha_{23}\mu_2 + \alpha_{43}\mu_4 \qquad (9)$$

$$\lambda_4 = \alpha_{14}\mu_1 + \alpha_{34}\mu_3 + \alpha_{64}\mu_6 \qquad (10)$$

$$\lambda_5 = \alpha_{15}\mu_1 + \alpha_{35}\mu_3 + \alpha_{65}\mu_6 \tag{11}$$

$$\lambda_6 = \alpha_{16}\mu_1 + \alpha_{36}\mu_3 + \alpha_{46}\mu_4 + \alpha_{56}\mu_5 \tag{12}$$

Also,

$$\mu_1 = \alpha_{12}\mu_1 + \alpha_{13}\mu_1 + \alpha_{14}\mu_1 + \alpha_{15}\mu_1 + \alpha_{16}\mu_1 \tag{13}$$

$$\mu_2 = \alpha_{23}\mu_2 + \alpha_{2out}\mu_2 \tag{14}$$

$$\mu_3 = \alpha_{34}\mu_3 + \alpha_{35}\mu_3 + \alpha_{36}\mu_6 + \alpha_{3out}\mu_3 \tag{15}$$

$$\mu_4 = \alpha_{43}\mu_4 + \alpha_{46}\mu_4 + \alpha_{4out}\mu_4 \tag{16}$$

$$\mu_5 = \alpha_{52}\mu_5 + \alpha_{56}\mu_5 + \alpha_{5out}\mu_5 \tag{17}$$

$$\mu_6 = \alpha_{64}\mu_6 + \alpha_{65}\mu_6 + \alpha_{6out}\mu_6 \tag{18}$$

Where; $\alpha_{12,}\alpha_{13,}\alpha_{14,}\alpha_{15,}\alpha_{16,}\alpha_{23,}\alpha_{2out,}\alpha_{34,}\alpha_{35,}\alpha_{36},\alpha_{3out},\alpha_{43},\alpha_{46,}\alpha_{4out,}$

4

$\alpha_{52,}\alpha_{56,}\alpha_{5out,}\alpha_{64,}\alpha_{6out,}$ *are to determined*

Model equations (8-18) could be expressed as:

$$\lambda_2 = \mu_1\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + 0\alpha_{15} + 0\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + \mu_5\alpha_{52} + 0\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{64} + 0\alpha_{65} + 0\alpha_{6out} \tag{19}$$

$$\lambda_3 = 0\alpha_{12} + \mu_1\alpha_{13} + 0\alpha_{14} + 0\alpha_{15} + 0\alpha_{16} + \mu_2\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + \mu_4\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + 0\alpha_{56} + 0\alpha_{5out}$$
$$0\alpha_{64} + 0\alpha_{65} + 0\alpha_{6out} \tag{20}$$

$$\lambda_4 = 0\alpha_{12} + 0\alpha_{13} + \mu_1\alpha_{14} + 0\alpha_{15} + 0\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + \mu_3\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + 0\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{56} + 0\alpha_{5out} + \mu_6\alpha_{64} + 0\alpha_{65} + 0\alpha_{6out} \tag{21}$$

$$\lambda_5 = 0\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + \mu_1\alpha_{15} + 0\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + \mu_3\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + \mu_5\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{64} + \mu_6\alpha_{65} + 0\alpha_{6out} \tag{22}$$

$$\lambda_6 = 0\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + 0\alpha_{15} + \mu_1\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+\mu_3\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + \mu_4\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + \mu_5\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{64} + +0\alpha_{65} + 0\alpha_{6out} \quad (23)$$

Also,

$$\mu_1 = \mu_1\alpha_{12} + \mu_1\alpha_{13} + \mu_1\alpha_{14} + \mu_1\alpha_{15} + \mu_1\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + 0\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{64} + 0\alpha_{65} + 0\alpha_{5out} \quad (24)$$

$$\mu_2 = 0\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + \mu_1\alpha_{15} + 0\alpha_{16} + \mu_2\alpha_{23} + \mu_2\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + \mu_5\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{64} + 0\alpha_{65} + 0\alpha_{6out} \quad (25)$$

$$\mu_3 = 0\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + \mu_1\alpha_{15} + 0\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + \mu_3\alpha_{34} + \mu_3\alpha_{35}$$
$$+\mu_3\alpha_{36} + \mu_3\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + 0\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{64} + 0\alpha_{65} + 0\alpha_{6out} \quad (26)$$

$$\mu_4 = 0\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + \mu_1\alpha_{15} + 0\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + \mu_4\alpha_{43} + \mu_4\alpha_{46} + \mu_4\alpha_{4out} + 0\alpha_{52} + 0\alpha_{56} + 0\alpha_{5out}$$
$$+0\alpha_{64} + 0\alpha_{65} + 0\alpha_{6out} \quad (27)$$

$$\mu_5 = 0\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + \mu_1\alpha_{15} + 0\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + \mu_5\alpha_{52} + \mu_5\alpha_{56} + \mu_5\alpha_{5out}$$
$$+0\alpha_{64} + 0\alpha_{65} + 0\alpha_{6out} \quad (28)$$

$$\mu_6 = 0\alpha_{12} + 0\alpha_{13} + 0\alpha_{14} + 0\alpha_{15} + 0\alpha_{16} + 0\alpha_{23} + 0\alpha_{2out} + 0\alpha_{34} + 0\alpha_{35}$$
$$+0\alpha_{36} + 0\alpha_{3out} + 0\alpha_{43} + 0\alpha_{46} + 0\alpha_{4out} + 0\alpha_{52} + 0\alpha_{56} + 0\alpha_{5out}$$
$$+\mu_6\alpha_{64} + \mu_6\alpha_{65} + \mu_6\alpha_{6out} \tag{29}$$

Model equations (19-29) above can be represented in the matrix form as:

$$\begin{pmatrix}
\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_5 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu_1 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_6 & 0 & 0 \\
0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_6 & 0 \\
0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & \mu_4 & 0 & 0 & \mu_5 & 0 & 0 & 0 & 0 \\
\mu_1 & \mu_1 & \mu_1 & \mu_1 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mu_2 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & \mu_3 & \mu_3 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & \mu_4 & \mu_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_5 & \mu_5 & \mu_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_6 & \mu_6 & \mu_6
\end{pmatrix}
\begin{pmatrix}
\alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \alpha_{15} \\ \alpha_{16} \\ \alpha_{23} \\ \alpha_{2out} \\ \alpha_{34} \\ \alpha_{35} \\ \alpha_{36} \\ \alpha_{3out} \\ \alpha_{43} \\ \alpha_{46} \\ \alpha_{4out} \\ \alpha_{52} \\ \alpha_{56} \\ \alpha_{5out} \\ \alpha_{64} \\ \alpha_{65} \\ \alpha_{6out}
\end{pmatrix}
=
\begin{pmatrix}
\lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6
\end{pmatrix} \tag{30}$$

3.4 Mathematical Formulation of New Departure Rate

For us to be able to minimize the waiting time, which is the main aim of this research work, there is need to introduce a new departure rate for each node.

Therefore, model equations (8- 12) will generate equations (31 - 35) below;

$$\lambda_2 = \alpha_{12}\mu_1 + 0\mu_2 + 0\mu_3 + 0\mu_4 + \alpha_{52}\mu_5 + 0\mu_6 \tag{31}$$

$$\lambda_3 = \alpha_{13}\mu_1 + \alpha_{23}\mu_2 + 0\mu_3 + \alpha_{43}\mu_4 + 0\mu_5 + 0\mu_6 \tag{32}$$

$$\lambda_4 = \alpha_{14}\mu_1 + 0\mu_2 + \alpha_{34}\mu_3 + 0\mu_4 + 0\mu_5 + \alpha_{64}\mu_6 \tag{33}$$

$$\lambda_5 = \alpha_{15}\mu_1 + 0\mu_2 + \alpha_{35}\mu_3 + 0\mu_4 + 0\mu_5 + \alpha_{65}\mu_6 \tag{34}$$

$$\lambda_4 = \alpha_{16}\mu_1 + 0\mu_2 + \alpha_{36}\mu_3 + \alpha_{46}\mu_4 + \alpha_{56}\mu_5 + 0\mu_6 \tag{35}$$

Equations (31 – 35) are represented by equation (36)

$$\begin{pmatrix} \alpha_{12} & 0 & 0 & 0 & \alpha_{52} & 0 \\ \alpha_{13} & \alpha_{23} & 0 & \alpha_{43} & 0 & 0 \\ \alpha_{14} & 0 & \alpha_{34} & 0 & 0 & \alpha_{64} \\ \alpha_{15} & 0 & \alpha_{35} & 0 & 0 & \alpha_{65} \\ \alpha_{16} & 0 & \alpha_{36} & \alpha_{46} & \alpha_{56} & 0 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{pmatrix} = \begin{pmatrix} \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{36}$$

3. Results and Discussion

We begin this section with computation of mean arrival and departure time for all the nodes in the network system, the results of the computation is presented in Table1 below.

Table 4.1: The Mean arrival and Mean departure time obtained for each of the nodes from the raw data collected at the Clinic.

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Mean arrival	1.809	1	1.819	1.632	1.676	1.791
Mean departure	1.767	1	1.265	1.468	1.491	1.432

To obtain solution for equation (30), we use the mean arrival and mean departure time obtained from the raw data collected as presented in Table 1 above to calculate the arrival rate and departure rate using equation (1) and (2) for $\lambda_i, i = 2,....,6$ *and* $\mu_i, i = 1,...,6)$ then substitute them into equation (30). Hence, we obtained the solution for equation (30) as presented below

$$\begin{pmatrix} \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \alpha_{15} \\ \alpha_{16} \\ \alpha_{23} \\ \alpha_{2out} \\ \alpha_{34} \\ \alpha_{35} \\ \alpha_{36} \\ \alpha_{3out} \\ \alpha_{43} \\ \alpha_{46} \\ \alpha_{4out} \\ \alpha_{52} \\ \alpha_{56} \\ \alpha_{5out} \\ \alpha_{64} \\ \alpha_{65} \\ \alpha_{6out} \end{pmatrix} = \begin{pmatrix} 0.6313043478 \\ 0.0678260896 \\ 0.1234782609 \\ 0.12000000000 \\ 0.05739130435 \\ 0.42800000000 \\ 0.57200000000 \\ 0.3256743257 \\ 0.3196803197 \\ 219780219800 \\ 0.1348651349 \\ 0.2437562438 \\ 0.4145854146 \\ 0.3416583417 \\ 0.8741258741 \\ 0.0989010989 \\ 0.02697302697 \\ 0.38971315530 \\ 0.3847675569 \\ 0.2255192878 \end{pmatrix} \quad (37)$$

From equation (37), based on the result, the following deductions are made:

From equation (37), based on the result, the following deductions are made:

At node 1(Registration unit), the weights $\alpha_{12}, \alpha_{13}, \alpha_{14}, \alpha_{15}$ and α_{16} are 0.6313043478, 0.0678260896, 0.1234782609, 0.12000000000,and 0.05739130435 respectively shows that there is a high probability of a patient leaving node 1 (registration) to join the queue for service at node 2 (Nurse unit) than any other node. The least probability is that a patient leaves node 1 to node 6.

At node2 (Nurses unit), the weights α_{23} and α_{2out} which are 0.42800000000 and 0.57200000000 which shows that there is a high probability that a patient leaves node 2 and goes directly to node 3 (Doctor unit).

At node3 (Doctors point), the weights $\alpha_{34}, \alpha_{35}, \alpha_{36}$ and α_{3out} which are 0.3256743257, 0.3196803197, 219780219800, 0.1348651349 respectively shows that there is a high probability that a patient leaves node 3 (Doctor unit) to join the queue for service either at node 4 or node 5. The least probability is that a patient leaves node 3 (Doctor unit) and moves out of the system.

At node 4 (Laboratory point), the weights α_{43}, α_{46} and α_{4out} which are 0.2437562438, 0.4145854146, and 0.3416583417 respectively shows that of 2 weight, there is a higher probability that a patient leaves node 4 and go back to node 3 (i.e to see Doctor) than out of the system.

At node5 (Pharmacy point), the weights $\alpha_{52}, \alpha_{56},$ and α_{5out} which are 0.8741258741, 0.0989010989 and 0.02697302697 respectively shows that there is a high probability that a patients leaves node 5 to join the queue for service either at node 2 or node 6. The least probability here is that a patient leaves node 5 and goes out of the system.

At node6 (Account point), the weights α_{64}, α_{65} *and* α_{60ut} which are 0.38971315530, 0.3847675569 and 0.2255192878 respectively shows that there is a high probability that a patient leaves node 6 (Account unit) to join the queue for service either at node 4 or node 5. The least probability here is that a patient's leaves node 6 and goes out of the system

From the information in above Table 4.1, we have calculated the arrival and departure rates for each node, expected number of patient in the system, expected number of patient in the queue, expected waiting time in the system and expected waiting time in the queue.

For node 1, the arrival rate $\lambda_1 = \frac{1}{meannumberofarrival} = \frac{1}{1.809} = 0.55$ person per minute.

The departure rate for node 1 is given as:

$$\mu_1 = \frac{1}{meannumberofdeparture} = \frac{1}{1.767} = 0.566 .$$

Hence, we followed the same procedure to calculate for node 2- to node 6, the values obtained are presented in Table 2. Other values of the performance measure for all the nodes are computed and presented in Table 2.

$$\rho = \frac{\lambda_1}{\mu_1} = \frac{0.55}{0.566} = 0.97$$

The expected number of patients in the queue is given as

$l_q = \frac{\rho}{m-\rho} = \frac{0.97}{4-0.97} = 0.32$ Patient.

Where m stands for the number of servers at the node1 (Registration point).

The expected waiting time of the patients on the queue is given as $w_{q1} = \frac{l_q}{\lambda_1} = \frac{32}{0.55} = 0.59$ minute.

The expected number of patients in the system is given as:

$l_s = l_q + \rho = 0.32 + 0.97 = 1.29 \approx 1.3$ Patients

The expected waiting time in the system for node 1 is given as $w_1 = \frac{l_s}{\lambda_1} = \frac{1.3}{0.55} = 2.5$ minutes. These parameters calculated for node1 can be done for nodes 2 – 6.

The expected waiting time in node 1 through node 6 were calculated as $w_1 = 5$ minutes for node 1, $w_2 = 2$ minutes for node 2, $w_3 = 5.5$ minutes, for node 3, $w_4 = 16.4$ minutes for node 4, $w_5 = 15$ minutes for node 5 and $w_6 = 8.9$ minutes for node 6.

Therefore, the total expected waiting time of the patients in the current system is computed as:

$$W = w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 5 + 2 + 5.5 + 16.4 + 15 + 8.9 = 50.3 \approx 50\text{minutes}$$

4.2. The values of these performance measures of the network system are presented in the Table 2 below:

Table 4.2: Performance Measure for the Current Network System at the Clinic

Nodes i	Number of Servers	ρ_i	Lq	Ls	W_q	W_S
1	4	0.97	0.3	1.3	0.59	2.5
2	2	1.00	1.0	2.0	1.00	2.0
3	1	0.70	2.3	3.0	4.18	5.5
4	1	0.90	9.0	10	14.75	16.4
5	1	0.89	8.0	9.0	13.30	15
6	1	0.80	4.0	5.0	7.14	8.9
Total	10	5.26	24.6	30.3	40.96	50.3

*4.1. **Solution For New Departure Rate***

To obtain the new departure rate for the network system, we solve equation(41), this is done by substituting the values of arrival rate obtained in the previous section into equation(41) then we obtain equation(43)

$$\begin{pmatrix} 0.63 & 0 & 0 & 0 & 0.09 & 0 \\ 0.07 & 0.43 & 0 & 0.024 & 0 & 0 \\ 0.12 & 0 & 0.33 & 0 & 0 & 0.39 \\ 0.12 & 0 & 0.32 & 0 & 0 & 0.38 \\ 0.06 & 0 & 0.22 & 0.41 & 0.09 & 0 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{pmatrix} = \begin{pmatrix} 1.00 \\ 0.55 \\ 0.61 \\ 0.60 \\ 0.56 \end{pmatrix} \qquad (38)$$

Solving Equation (43), we obtained equation (44) below

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{pmatrix} = \begin{pmatrix} 11.82000 \\ 11.92000 \\ 11.85000 \\ 11.75000 \\ 9.82000 \\ 12.23000 \end{pmatrix} \qquad (38)$$

From equation (38), in order to serve patients with the space interval of 4 minutes. We have to divide the values in equation (44) by 4.

The recommended number of servers therefore for each node is given in equation (39)

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{pmatrix} = \begin{pmatrix} 2.95 \\ 2.89 \\ 2.96 \\ 2.93 \\ 2.45 \\ 3.08 \end{pmatrix} \approx \begin{pmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 2 \\ 3 \end{pmatrix} \qquad (39)$$

The values in equation (39) indicates that in order for us to reduce the waiting time in the system to an acceptable level, node one will require a total of three servers, node two will require a total of three servers, node three will require a total of three servers, node four will require a total of three servers, node five will require a total of two servers and node six will require a total of three servers.

With the new estimated number of servers at each node given by equation (39). The arrival rates for each are assumed to remain the same since we do not have control over it. A new departure rates and the expected waiting time for each node is estimated.

The new departure rates for nodes are as follows:

$$\mu_1 = recommended\ departure\ rate\ per\ 4\ mins = \frac{3}{4} = 0.6\ person\ per\ minute \quad (40)$$

$$\mu_2 = recommended\ departure\ rate\ per\ 4\ mins = \frac{3}{4} = 0.6\ person\ per\ minute \quad (41)$$

$$\mu_3 = recommended\ departure\ rate\ per\ 4\ mins = \frac{3}{4} = 0.6\ person\ per\ minute \quad (42)$$

$$\mu_4 = recommended\ departure\ rate\ per\ 4\ mins = \frac{3}{4} = 0.6\ persons\ per\ minute \quad (43)$$

$$\mu_5 = recommended\ departure\ rate\ per\ 4\ mins = \frac{2}{4} = 0.5\ person\ per\ minute \quad (44)$$

$$\mu_6 = recommended\ departure\ rate\ per\ 4\ mins = \frac{3}{4} = 0.6\ person\ per\ minute \quad (45)$$

4.2. Computations of New Expected Waiting Time In The System

For Node 1:

$$\rho = \frac{\lambda_1}{\mu_1} = \frac{0.55}{0.6} = 0.9$$

The expected number of patients in the queue is given as

$$l_q = \frac{\rho}{m-\rho} = \frac{0.9}{3-0.9} = 0.4 \text{ patients}$$

The expected waiting time in the queue is given as $w_{qi} = \frac{l_q}{\lambda_1} = \frac{0.4}{0.55} = 0.7$

The expected number of patients in the system is given as

$$l_s = l_q + \rho = 0.4 + 0.9 \text{ patients.}$$

The expected waiting time of the patients in the system for node 1 is given as

$$w_1 = \frac{l_s}{\lambda_1} = \frac{0.4}{0.55} = 2.4 \text{ minutes.}$$

These parameters calculated for node1 can be done for nodes 2 – 6.

The expected waiting time in node 1 through node 6 were calculated as $w_1 = 2.4$ minutes for node 1, $w_2 = 3$ minutes for node 2, $w_3 = 1.8$ minutes, for node 3, $w_4 = 2.4$ minutes for node 4,

$w_5 = 2.5$ minutes for node 5 and $w6 = 6.8$ minutes for node 6.

Therefore, the total expected waiting time in the system after modification is

Therefore, the total expected waiting time in the system is given as

$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 =$ 2.4+3+1.8+2.4+2.5+6.2 = 18.8 ≈ 19 minutes.

4.3. It can be observed from the table that the total number of serves is 17; this is optimal number of servers that can reduce waiting time to 18.8 minutes, approximately 19.minutes. as it also be seen from the table. These values are presented in Table 4.3 below:

Table 4.3: Showing all the Results Obtained after Modification

Nodes i	Number of Servers (m_i)	ρ_i	Lq	Ls	W_q	W_S
1	3	0.9	0.4	1.3	0.7	2.4
2	3	1.7	1.3	3.0	1.3	3.0
3	3	0.9	0.4	1.3	0.7	2.4
4	3	1.0	0.5	1.5	0.8	2.5
5	2	1.2	1.5	3.7	2.5	6.2
6	3	0.9	0.4	1.3	0.7	2.3
Total	17	6.6	2.3	12.1	6.7	18.8

Table 4.4 Showing the comparison between current number of servers and optimal number of servers obtained

Nodes i	Current number of servers	Optimal number of servers obtained
1	4	3
2	2	3
3	1	3
4	1	3
5	1	2
6	1	3
Total	10	17

4. Conclusion

The network queuing system of Federal Polytechnic Bida school clinic has been effectively investigated and studied. The study has determined optimal number of servers at the nodes of the school clinic network queuing system to reduce waiting time of the patients. Results from the studyis an important information to the management of Federal Polytechnic Bida school clinic for proper planning and efficient service delivery.

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