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Prediction of Weekly Rainfall Both in Discrete and Continuous Time Using Markov Model

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ABSTRACT

In this chapter, a Markov model to study weekly rainfall both in discrete and continuous time is presented. The model predicts and analyzes weekly rainfall pattern of Makurdi, Nigeria using rainfall data of eleven years(2005-2015). After some successful iterations of the discrete time Markov model, its stabilizes to equilibrium probabilities, revealing that in the long-run 22% of the weeks during rainy season in Makurdi, will experience No rainfall, 50% will experience Low rainfall, 25% will experience Moderate rainfall and 2% will experience High rainfall. For the continuous time Markov model, It was observed that, if it is in No rainfall state in a given week, it would take at most 49%, 27% and 16% of the time to make a transition to Low rainfall, Moderate rainfall, and High rainfall respectively in the far future. Thus given the rainfall in a week, it is possible to determine quantitatively the probability of finding weekly rainfall in other states in the following week and in the long run. The model also reveals that, a week of High rainfall cannot be followed by another week of High rainfall, a week of High rainfall cannot be followed by a week of No rainfall, and a week of Moderate rainfall cannot precede a week of High rainfall. With the combined results of the discrete and continuous time Markov model, the rainfall pattern of the study area is better understood. These results are important information to the residents of Markudi and environmental management scientists for effective planning and viable crop production.

Keywords: Markov chain; weekly rainfall; transition probabilities; equilibrium probabilities probability state vector; Makurdi.

1. INTRODUCTION

Rainfall is a type of precipitation that occurs when water vapour in the atmosphere condense into droplets, which can no longer be suspended in the air. The occurrence of rainfall is dependent upon several factors. Factors such as prevailing wind directions, ground elevation, location within a continental mass, and location with respect to mountain ranges, all have a major impact on the possibility of precipitation [1]. Rainfall in Makurdi, Benue state, Nigeria is not ordinarily predictable, its occurrence and amount varies from year to year. The majority of the people living in this part of the country are farmers and rainfall is the major source of water for agricultural activities. This dependence of agricultural production on rainfall variability and quantity, and the unpredictable nature of the rainfall in this part of the country, had over the years led to: improper crop planning and cultivation consequently led poor harvest. The research, is aimed at providing some quantitative information to the farmers and Government that could assists in boosting crop production in the state and the country at large. Rainfall exhibits a strong variability in time and space across the globe. It is well established that rainfall is changing on both the global and the regional scale due to global warming [2]. Rainfall is the principal phenomenon driving many hydrological extremes such as floods, droughts, landslides, debris and mud-flows; its analysis and modeling are typical problems in applied hydrometeorology [3]. Hence, its stochastic modeling is necessary for the prevention of natural disaster. Understanding the rainfall distribution is equally necessary for future planning. This is

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applicable in areas like agriculture, industry, insurance, hydrological studies and the entire planning of a country economy. Agricultural calendar have direct link with the onset and withdrawal of the rainfall which in turn have, direct impact on agricultural productivity [4]. Consequently, information on rainfall probabilities is vital for the design of water supply management, supplementary irrigation schemes and the evaluation of alternative cropping system for effective soil water management plans [3]. Such information can also be beneficial in determining the best adapted plant species and the optimum time of seedling to re-establish vegetation on deteriorated rangelands.

Overwhelming researchers within Nigeria and in around the world, have proposed several methods in attempt to provide information that could enable humanity to make best use of this random phenomenon, either for agricultural purposes or other purposes of fundamental importance to life, such researchers include. Raheem et al., [5] had successfully developed a three-state Markov chain to examine the pattern and distribution of daily rainfall in Uyo metropolis of Nigeria using 15 years (1995-2009) rainfall data. Evaluation of the effect of climate change on daily rainfall using first-order Markov chain model, has been presented by Chulsang et al. [6], the model was applied to Seoul weather station in Korea, the result shows that about 30% of the total change in monthly rainfall amount was due to the change in the number of wet days and the remaining 70% was due to the change in the rainfall intensity. Arumugam and Karthik [7] have investigated the variations of annual rainfall in Tirunelveli district, India based on stochastic method. Rainfall data for 44 years was used, the Markov chain model developed was used to predict annual rainfall for the future years, up to 2025. Katrin et al., [8] had applied Markov chains to analyzed the dynamics and succession of multivariate or compound extreme events. The method was applied to observational data and an ensemble of regional climate simulations for Central Europe. They concluded that, the change in the succession of hot and dry days in summer will probably affect regions in Spain and Bulgaria. The susceptibility to a dynamic change of hot and dry extremes in the Russian region will probably decrease. Mouelhi et al. [9] had proposed a developmental method of stochastic generator of monthly rainfall series. The work was based on the modeling of the occurrence and the quantity of rain in a separate way.

2. MATERIALS AND METHODS

2.1 Study Area

The study area of this research is Makurdi. The city is located in central Nigeria along the Benue River , it is located on (latitude 7.7°N, longitude 8.5°), Makurdi is the capital city of Benue state of Nigeria. Benue State is regarded as food basket of the nation because of its rich agricultural produce which include Yam, Rice, Beans, Cassava, Sweet-potato, Maize, Soybean, Sorghum, Millet, Sesame, cocoyam etc. Agriculture is the mainstay of the economy, engaging over 75% of the state farming population. The State also has one of the longest stretches of river systems in the country with great potential for a viable fishing industry, dry season farming through irrigation and for an inland water highway. The vegetation of the State consists of rain forests which have tall trees, tall grasses and oil palm trees that occupy the state's western and southern fringes while the Guinea Savannah is found in the eastern and northern parts with mixed grasses and trees that are generally of average height.

2.2 Sample Collection

The data used in this research work were obtained from the archive of Nigerian Meteorological Agency, Maitama, Abuja. It is the daily rainfall record of Markudi, Benue state for the period of 11 years (2005 to 2015).

2.3 Model Formulation

This research, considers the use of weekly rainfall amount during the rainy season to study weekly rainfall pattern of Makurdi both in discrete and continuous time.

Following Ross [10], we consider a stochastic process $\{X_n, \quad n = 0,1,2,3,........\}$ which takes on a finite or countable number of possible values. Unless otherwise stated, this set of possible values of the process will be denoted by the set of nonnegative integers $\{0,1,2,3,.........\}$ if $X_n = i$ then the process is said to be in state *i* at time *n.*

We suppose that whenever the process is in state *i,* there is a fixed probability P_{ij} that it will next be in state j. That is, we suppose that

$$P\{X_{n+1} = j \mid X_0 = i_0,..., X_{n-2} = i_{n-2}, X_{n-1} = i_{n-1}, X_n = i\} = P_{ij}. \tag{1}$$

for all states $i_0, i_i,........,i_{n-1}, i, j \quad and\ all\ n \geq 0.$ Such a stochastic process is known as a Markov chain. Equation (1), may be interpreted as stating that, for a Markov chain, the conditional distribution of any future state X_{n+1} given the past states $X_0, X_1,.......,X_{n-1}$ and the present state X_n is independent of the past states and depends only on the present state. The value P_{ij} represents the probability that the process will, when in state *i,* next make a transition into state j. Since probabilities are nonnegative and since the process must make a transition into some state, we have that

$$P_{ij} \geq 0, \quad i, j \geq 0; \quad \sum_{j=0}^{\infty} P_{ij} = 1, \quad i = 0,1,2..........$$

Now, suppose that the amount of weekly rainfall in Makurdi in a week during the rainy season is considered as a random variable X , the collection of these random variables over the weeks constitutes a stochastic process $X_n \quad , \quad n = 0,1,2,3..................$

It is assumed that this stochastic process satisfies Markov properties mentioned above. Let the weekly rainfall be modelled by four states, Markov model.

State1: No rainfall
State2: Low rainfall
State3: Moderate rainfall
State4: High rainfall

It is important to mention here that, the classification of states for a Markov model is guided by the purpose in which the model is intended to achieve.

The transition between the states is described by Equation (2) and the transition diagram is represented by Fig. 1, thus we have

2.4 Transition Probability Matrix

$$p = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & 0 \\ 0 & P_{42} & P_{43} & 0 \end{bmatrix} \tag{2}$$

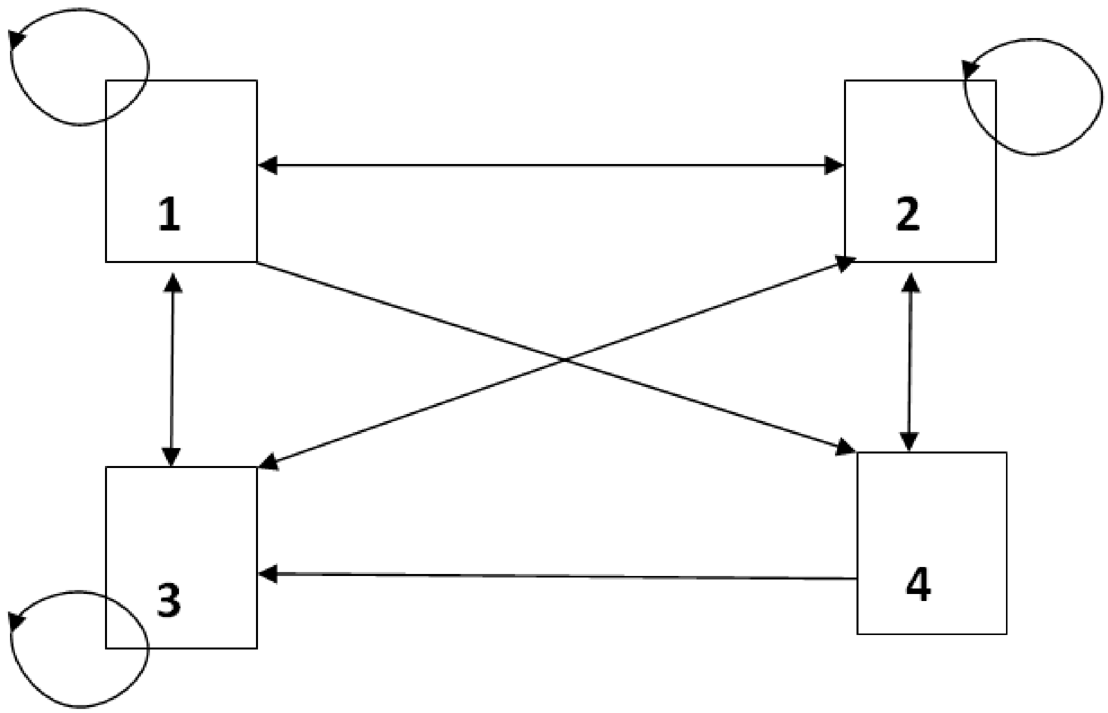

Fig. 1. Transition diagram of the weekly rainfall model

The transition probability matrix and the transition diagram were obtained based on data collected and the rainfall states adopted for the study area.

2.5 The Discrete Time Markov Model for the Weekly Rainfall

Markov model that is discrete state and discrete time is considered in this section as a result,

we Let $P^{(n)}$ represent the probability state vectors of the Markov chain,

Where n can take values from zero to infinity (n = 0, 1, 2, 3,4,....................)

and $P_i^{(n)}$ is the probability that the weekly rainfall is in the i^{th} state at the n^{th} week. $P^{(0)}$ is the initial state vector of the Markov chain and $P^{(n)}$is the state vector at the n^{th} week

Then we can write $$p^{(n+1)} = P^{(n)}P. \quad (3)$$

Where P is our transition probability matrix and $p^{(n+1)}$ is the state vector at the $(n+1)^{th}$ week.

on iteration, we have

$$p^{(n)} = p^{(0)}P^n \quad (4)$$

this mean that the initial state vector $P^{(0)}$ and the transition matrix P determine the state vector $P^{(n)}$ at the n^{th} week

If we now, let $p^{(n)} = \left[p_1^n \quad p_2^n \quad p_3^n \quad p_4^n\right]$ (5)

denote the probabilities of finding the weekly rainfall amount in any of the four states at the n^{th} week and also let $p^{(0)} = \left[p_1^o \quad p_2^o \quad p_3^o \quad p_4^o\right]$ (6)

Denotes the initial state vector, then our First order Markov Chain Model for weekly rainfall amount pattern prediction in the long run, in Makurdi can be represented by

$$\left[p_1^n \quad p_2^n \quad p_3^n \quad p_4^n\right] = \left[p_1^o \quad p_2^o \quad p_3^o \quad p_4^o\right] \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & {}_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & 0 \\ 0 & p_{42} & p_{43} & 0 \end{bmatrix}^n \quad (7)$$

2.6 Limiting State Probabilities

The state occupation probabilities is independent of the starting state of the process, if number of the time the state transition is large thus the process reaches a steady state after a sufficiently large period of time. This is equilibrium distribution

$$\pi = (\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)$$

If we let $n \to \infty$ in equation 4, we have

$$\pi = \pi p \quad (8)$$

and also $\pi = \sum_{i=1}^{4} \pi_i = 1$ (9)

these equations will be use to find the limiting state probabilities for our model

2.7 The Continuous Time Markov Model for the Weekly Rainfall

Following [11], we consider a stochastic process that is discrete state and continuous time. A continuous time stochastic process $\{X(t)\}$ is an infinite family of random variables indexed by the continuous real variable that is for any fixed t, X(t) is a random variable and the collection of all these (for all t) is the stochastic process we ordinarily think of t as time, so we may expect $X(t_1)$, the random variable at time t_1, to be dependent on $X(t_o)$, where $t_0 < t_1$, but not upon $X(t_2)$, where $t_2 > t_1$. The value of $X(t_1)$ is the state of the process at time t_1.

let $p_{ij}(t),$ denotes transition probability function which is discrete state and continuous time , where

$$p_{ij}(t) = pr\{X(t) = j / X(0) = i\} \tag{10}$$

Now, a fair amount can be known about these $p_{ij}(t)$ functions just as a consequence of the fact that they are, for all t, probabilities. For example, they are non-negative bounded functions because a probability must lie between 0 and 1. The values of the functions at $t = 0$ can be deduced because $p_{ij}(0) = pr\{X(0) = j / X(0) = i\}, \quad clearly \quad for \quad i \neq j$

$p_{ij}(0) = 0 \quad and \quad for \quad i = j \quad p_{jj}(0) = 1$

If we fix i and vary j over all states, the sum of the $p_{ij}(t)$ must equal 1 (for all t)

Now, under the assumption that the $p_{ij}(t)$ are continuous functions then we may express p_{ij} for small δt by the use of Maclaurin's series, thus

$$p_{ij}(\delta t) = p_{ij}(0) + p'_{ij}(0)\delta t + o(\delta t^2) \tag{11}$$

Where $o(\delta t^2)$ represent all terms of the order of the $(\delta t)^2$ or higher. If we consider this expression for $i \neq j,$ and let $q_{ij} = p'_{ij}(0)$ we obtain

$$p_{ij}(\delta t) = q_{ij}\delta t + o(\delta t^2) \tag{12}$$

This is a linear approximation to $p_{ij}(t)$ which is good approximation as long as δt is small. q_{ij} is called the transition rate from $i \quad to \quad j$

For $i = j,$ the Maclaurin's series expansion yields

$$\begin{aligned} &p_{jj}(\delta t) = 1 + p'_{jj}(0)\delta t + o(\delta t^2) \\ &let \quad q_{jj} = p'_{jj}(0) \end{aligned} \tag{13}$$

We get the linear approximation

$$p_{jj}(\delta t) = 1 + q_{jj}\delta t + o(\delta t^2) \tag{14}$$

If we now, consider the forward Chapman-Kolmogorov equation

Which is a equation for studying stationary Markov processes thus:

$$p_{ij}(t + \delta t) = \sum_k p_{ik}(t) p_{kj}(\delta t)$$

for small δt , and substitute our linear approximation, we get

$$p_{ij}(t+\delta t) = p_{ij}(t)[1+q_{jj}\delta t + o(\delta t^2)] + \sum_{k\neq j} p_{ik}(t)\left[q_{kj}\delta t + o(\delta t^2)\right]$$

$$\frac{p_{ij}(t+\delta t) - p_{ij}(t)}{\delta t} = p_{ij}(t)q_{jj} + \frac{p_{ij}(t)o(\delta t^2)}{\delta t} + \sum_{k\neq j}\left[p_{ik}(t)q_{kj} + \frac{p_{ik}(t)o(\delta t^2)}{\delta t}\right]$$

$$= \sum_k p_{ik}(t)q_{kj} + \sum_k \frac{p_{ik}(t)o(\delta t^2)}{\delta t}$$

Taking limit as $\delta t \to 0$

$$\frac{dp_{ij}(t)}{dt} = \sum_k p_{ik}(t)q_{kj} \quad (15)$$

In matrix form we have $\frac{dp(t)}{dt} = p(t)Q$ (16)

Where $\frac{dp(t)}{dt}$ is the matrix whose $(i,j)^{th}$ element is $\frac{dp_{ij}(t)}{dt}$,

$p(t)$ is the matrix whose $(i,j)^{th}$ element is $p_{ij}(t)$, and Q is the matrix whose $(i,j)^{th}$ element is q_{ij}

The elements of Q may be further related by extending the properties of P(t)

In particular, since for each i

$$\sum_j p_{ij}(t) = 1$$

$$\frac{d}{dt}\left[\sum_j p_{ij}(t)\right]_{t=0} = \left[\frac{d}{dt}(1)\right]_{t=0}$$

$$\sum_j \frac{d}{dt} p_{ij}(t)\Big|_{t=0} = 0$$

$$\sum_j q_{ij} = 0$$

In words, each row of Q must sum to zero. Since off-diagonal element in non negative, the diagonal element q_{ii}, must be equal in magnitude and opposite in sign to the sum of others in the same rows. That is

$$q_{ii} = -\sum_{i \neq j} q_{ij} \tag{17}$$

To obtain the solution to equation(16) , the initial condition $P_i(0), \quad i = 1,2,3,4;$ must be specified
Taking the Laplace transform of equation(16), we obtained

$$P(s) = P(0)(SI - Q)^{-1} \tag{18}$$

Thus p(t) is obtained as the inverse transform of P(s) [12]

3. RESULTS AND DISCUSSION

The data used in this research work were obtained from the archive of Nigerian Meteorological Agency, Maitama, Abuja. It is the daily rainfall record of Markudi, Benue state for the period of 11 years (2005 to 2015). The summary is presented in Table 1 below.

Table 1. A summary of weekly rainfall amount in Makurdi between 2005-2015 and states distribution

Weekly rainfall in mm	Frequency	State
0	68	No Rainfall
Rainfall amount <61	254	Low Rainfall
61-120	51	Moderate Rainfall
Rainfall amount >120	9	High Rainfall

3.1 Application of the Discrete Time Markov Model

The transition count matrix is shown below and is obtained from Table 1

$$M = \begin{bmatrix} 29 & 25 & 2 & 2 \\ 26 & 161 & 47 & 7 \\ 4 & 4 & 10 & 0 \\ 0 & 6 & 2 & 0 \end{bmatrix} \tag{19}$$

From equation10, using the maximum likelihood estimator i.e

$$p_{ij} = \frac{f_{ij}}{\sum_{j=1}^{4} f_{ij}} \quad ij = 1,2,3,4 \tag{20}$$

Where f_{ij} is the historical frequency of transition from state i to state j, we to obtained the transition probability matrix given below

$$P = \begin{bmatrix} 0.5 & 0.431 & 0.034 & 0.034 \\ 0.108 & 0.668 & 0.195 & 0.029 \\ 0.222 & 0.222 & 0.555 & 0 \\ 0 & 0.75 & 0.25 & 0 \end{bmatrix} \qquad (21)$$

The state of the process at time n, Xn is related to the process at time n+1 through what is known as the transition probabilities. If the process is in state i at time n, at next time step n+1, it will either stay in state i or move or transfer to another state j .The probabilities for these changes in state are defined by equation (1), called One-Step transition probabilities.

3.2 n-Step Transition Probability

The probability of moving from state i to state j in n times steps is called n-step transition probability and is defined by $p_{ij}^{(n)} = prob\{X_n = j / X_0 = i\}$ Linda [13]

Calculating P^n, we have, on iteration

$$P^2 = \begin{bmatrix} 0.304 & 0.536 & 0.128 & 0.029 \\ 0.169 & 0.558 & 0.249 & 0.023 \\ 0.258 & 0.367 & 0.359 & 0.014 \\ 0.137 & 0.556 & 0.285 & 0.022 \end{bmatrix} \qquad (22)$$

$$P^4 = \begin{bmatrix} 0.221 & 0.526 & 0.227 & 0.024 \\ 0.214 & 0.506 & 0.257 & 0.022 \\ 0.235 & 0.483 & 0.258 & 0.021 \\ 0.212 & 0.500 & 0.265 & 0.021 \end{bmatrix} \qquad (23)$$

$$P^6 = \begin{bmatrix} 0.218 & 0.508 & 0.248 & 0.022 \\ 0.220 & 0.504 & 0.252 & 0.022 \\ 0.223 & 0.502 & 0.249 & 0.022 \\ 0.221 & 0.502 & 0.253 & 0.022 \end{bmatrix} \qquad (24)$$

$$P^8 = \begin{bmatrix} 0.220 & 0.504 & 0.250 & 0.022 \\ 0.220 & 0.504 & 0.251 & 0.022 \\ 0.220 & 0.503 & 0.250 & 0.022 \\ 0.221 & 0.503 & 0.251 & 0.022 \end{bmatrix} \qquad (25)$$

$$P^{10} = \begin{bmatrix} 0.219 & 0.503 & 0.249 & 0.022 \\ 0.220 & 0.504 & 0.250 & 0.022 \\ 0.220 & 0.503 & 0.250 & 0.022 \\ 0.220 & 0.504 & 0.250 & 0.022 \end{bmatrix} \tag{26}$$

$$P^{16} = \begin{bmatrix} 0.219 & 0.502 & 0.249 & 0.022 \\ 0.219 & 0.502 & 0.249 & 0.022 \\ 0.219 & 0.501 & 0.249 & 0.022 \\ 0.219 & 0.502 & 0.249 & 0.022 \end{bmatrix}. \tag{27}$$

$$P^{18} = \begin{bmatrix} 0.219 & 0.501 & 0.249 & 0.022 \\ 0.219 & 0.501 & 0.249 & 0.022 \\ 0.219 & 0.501 & 0.249 & 0.022 \\ 0.219 & 0.502 & 0.249 & 0.022 \end{bmatrix} \tag{28}$$

$$P^{18} = \begin{bmatrix} 0.22 & 0.50 & 0.25 & 0.02 \\ 0.22 & 0.50 & 0.25 & 0.02 \\ 0.22 & 0.50 & 0.25 & 0.02 \\ 0.22 & 0.50 & 0.25 & 0.02 \end{bmatrix}. \quad corrected \quad to \quad 2 \quad decimal \quad places$$

3.3 Limiting State Probabilities

The state occupation probabilities is independent of the starting state of the process, if number of the time the state transition is large thus the process reaches a steady state after a sufficiently large period of time. Thus

As n increases, P^n gets closer and closer to equation (28), that is $n \geq 18$ the transition probabilities stabilises to equation(28), and from equation (4), with the initial state probability vector $\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}$ we have

$$P^n = \begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix} \begin{bmatrix} 0.219 & 0.501 & 0.249 & 0.022 \\ 0.219 & 0.501 & 0.249 & 0.022 \\ 0.219 & 0.501 & 0.249 & 0.022 \\ 0.219 & 0.501 & 0.249 & 0.022 \end{bmatrix}. = \begin{pmatrix}0.219 & 0.501 & 0.249 & 0.022\end{pmatrix}$$

$$= \begin{pmatrix}0.22 & 0.5 & 0.25 & 0.02\end{pmatrix} \quad corrected \quad to \quad 2 \quad decimal \quad places$$

This is the probability of finding the weekly rainfall amount fall in any of the four states for large $n(i.e \quad n \geq 18)$

From equation (8), the limiting state probability vector is given by

$$\pi = \pi P = \begin{pmatrix}0.22 & 0.50 & 0.25 & 0.02\end{pmatrix}$$

Where π correspond to equation (28)

This shows that in the long-run, 22% of the weeks during rainy season in Markudi will experience No rainfall, 50% will experience Low rainfall, 25% will experience Moderate rainfall, 2% will experience High rainfall. It is important to mention here that the result does not sum up to 100% because 1% was lost during approximation process in the computations.

In this section, we have applied the principle of Markov in discrete time to study weekly rainfall amount in Markudi. In the transition probability matrix, we have P_{11}, P_{12}, P_{13}, and P_{14} = 0.50, 0.431, 0.034 and 0.034 respectively for the first year, after some iteration; these values stabilised to 0.219, 0.501, 0.249 and 0.022 respectively at the 18 steps . These are the equilibrium probabilities. That is, the probabilities of having weekly rainfall in state1, state2, state3 and state4 respectively in the long run. For instance, the value of P_{13} increases steadily from 0.034 for the first year and stabilised to 0.249 at the 18steps. This is probabilities of having no rainfall week during the rainy season making a transition to moderate rainfall in the following week or subsequent weeks cannot be more than 0.249. Similar interpretation is given to P_{12}, and P_{14} as well as to the other transition probabilities. With this Markov chain model, it is easy to make a prediction of what the rainfall (state) may be in the following week and in the long run if we know the present rainfall state on a discrete time scale.

From equation(2), it can be observed that P_{41}, P_{34}, P_{44} = 0, these mean that, there are not transition between these states. For example for the P_{44} = 0, it means that, it is not possible for a week of high rainfall in Makurdi to be followed by another week of high rainfall , and also for the $P_{41,}$ it is not possible to for a week of high rainfall to be followed by a week of no rainfall. Similarly, for P_{34}, a week of moderate rainfall cannot precede a week high Rainfall.

3.4 Application of the Continuous Time Markov Model

Unlike the model considered in the previous section where results were expressed at discrete point in time only, this model will provide result at any point in the time scale. To achieve this, we need to consider the transition count matrix of the weekly rainfall in equation (19)

Normalizing Equation (19) using Equation (17), we have equation (29) below

$$A = \begin{bmatrix} -29 & 25 & 2 & 2 \\ 26 & -80 & 47 & 7 \\ 4 & 4 & -8 & 0 \\ 0 & 6 & 2 & -8 \end{bmatrix} \tag{29}$$

Thus, the matrix A can be interpreted as the reciprocal of the *mean times* of the negative exponentially distributed random variable having the cumulative distribution

$1 - \ell^{-\lambda t}$ and mean value $\frac{1}{\lambda}$.

The above matrix indicates that if the rainfall is no rainfall state, the time it takes to make a transition to low rainfall state is exponentially distributed with mean 25 weeks. That is, if the rainfall is in No rainfall state, it has a probability $\frac{1}{25}\delta t$ of making transition to low rainfall state, a probability $\frac{1}{2}\delta t$ of making transition to moderate rainfall state and a probability $\frac{1}{2}\delta t$ of making transition to high rainfall state in the time interval $(t, \quad t + \delta t)$. Also if the rainfall is in low rainfall state it has a probability

$\frac{1}{26}\delta t$ of making transition to no rainfall state, a probability $\frac{1}{47}\delta t$ of making transition to moderate rainfall state and a probability $\frac{1}{7}\delta t$ of making transition to high rainfall state in the time interval $(t, \quad t+\delta t)$. Similarly, if the rainfall is in moderate rainfall state it has a probability $\frac{1}{4}\delta t$ of making transition to both no rainfall state and low rainfall state and a probability of 0 to high rainfall state, in the time interval $(t, \quad t+\delta t)$. Similar interpretation is given to being in high rainfall state making transition to no rainfall state, low rainfall, and moderate rainfall with probabilities, 0, $\frac{1}{6}\delta t$, and $\frac{1}{2}\delta t$ respectively in the time interval $(t, \quad t+\delta t)$.

The Matrix A can be expressed as the expected value of the exponential distribution thus

$$Q = \begin{bmatrix} -1.04 & 0.04 & 0.5 & 0.5 \\ 0.04 & -0.2 & 0.02 & 0.14 \\ 0.25 & 0.25 & -0.5 & 0 \\ 0 & 0.17 & 0.5 & -0.67 \end{bmatrix} \tag{30}$$

Now , to obtained solution for equation (18), we have that

$$IS - Q = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -2.50 & 0.5 & 1 & 1 \\ 0 & -1.17 & 0.17 & 1 \\ 0.33 & 0.25 & -0.58 & 0 \\ 1 & 0.5 & 0 & -1.5 \end{bmatrix} \tag{31}$$

$$= \begin{bmatrix} s+1.04 & -0.04 & -0.5 & -0.5 \\ 0.04 & s+0.2 & -0.02 & -0.14 \\ -0.25 & -0.25 & s+0.5 & 0 \\ 0 & -0.17 & -0.5 & s+0.67 \end{bmatrix} \tag{32}$$

We suppose that the initial state of the process is $P(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$

Thus

$$P(s) = P(0)(SI - Q)^{-1}$$

$$P(s) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s+1.04 & -0.04 & -0.5 & 0.5 \\ -0.04 & s+0.2 & -0.02 & -0.14 \\ -0.25 & -0.25 & s+0.5 & 0 \\ 0 & -0.17 & 0.5 & s+0.67 \end{bmatrix}^{-1} \quad (33)$$

Solving equation (33), using maple software we have the following equations to compute

$$P_{12}(t) = 0.488 - 0.679\ell^{-0.401t} + 1.06\times10^{-25}(1.80\times10^{24}\cos(0.153t) - 2.48\times10^{24}\sin(0.153t))\ell^{-1.004t}$$

$$P_{13}(t) = 0.266 + 0.545\ell^{-0.401t} - 2.129\times10^{-25}\ell^{-1004t}(3.806\times10^{24}\cos(0.153t) + 2.922\times10^{24}\sin(0.153t))$$

$$P_{14}(t) = 0.164 - 0.036\ell^{-0.401t} + 2.662\times10^{-24}\ell^{-1.006t}(-4.79\times10^{22}\cos(0.153t) + 8.77\times10^{23}\sin(0.153t))$$

$P_{12}(t)$ is the conditional probability that the weekly rainfall will be in low rainfall state at time t given that the weekly rainfall was in no rainfall state at time zero

$P_{13}(t)$ is the conditional probability that the weekly rainfall will be in moderate rainfall state at time t given that the weekly rainfall was in no rainfall state at time zero

$P_{14}(t)$ is the conditional probability that the weekly rainfall will be in high rainfall state at time t given that the weekly rainfall was in no rainfall state at time zero

$P_{11}(t)$ is the conditional probability that the weekly rainfall will be in no rainfall state at time t given that the weekly rainfall was in no rainfall state at time zero. This the compliment of $P_{12}(t)$, $P_{13}(t)$ and $P_{14}(t)$ this can be calculated at any point in time using $\sum_j p_{ij} = 1, \quad ij = 1,2,3,4$,

to obtain $P_{11}(t)$,one may use $P_{11}(t) = 1 - \left(p_{12}(t) + P_{13}(t) + P_{14}(t)\right)$

The values of the equations evaluated for t = 0 to 25 are tabulated in Table 2 and are illustrated graphically in Fig. 2.

Table 2. The transition probabilities

t	$P_{12}(t)$	$P_{13}(t)$	$P_{14}(t)$
0	0.000000	0.000000	0.000000
1	0.087732	0.302213	0.223710
2	0.197351	0.380988	0.225555
3	0.286862	0.379895	0.198041
4	0.351632	0.356771	0.178654
5	0.396334	0.332300	0.168832
6	0.426582	0.312415	0.164674
7	0.446882	0.297773	0.163215
8	0.460466	0.287480	0.162874
9	0.469547	0.280411	0.162928
10	0.475619	0.275616	0.163083
11	0.479679	0.272385	0.163235
12	0.482395	0.270215	0.163356
13	0.484212	0.268761	0.163444

t	$P_{12}(t)$	$P_{13}(t)$	$P_{14}(t)$
14	0.485428	0.267786	0.163506
15	0.486786	0.267134	0.163548
16	0.487395	0.266697	0.163577
17	0.487559	0.266405	0.163596
18	0.487668	0.266209	0.163609
19	0.487741	0.266078	0.163618
20	0.487789	0.265991	0.163624
21	0.487823	0.265932	0.163627
22	0.487845	0.265893	0.163630
23	0.487859	0.265866	0.163632
24	0.487845	0.265849	0.163633
25	0.487859	0.265837	0.163634

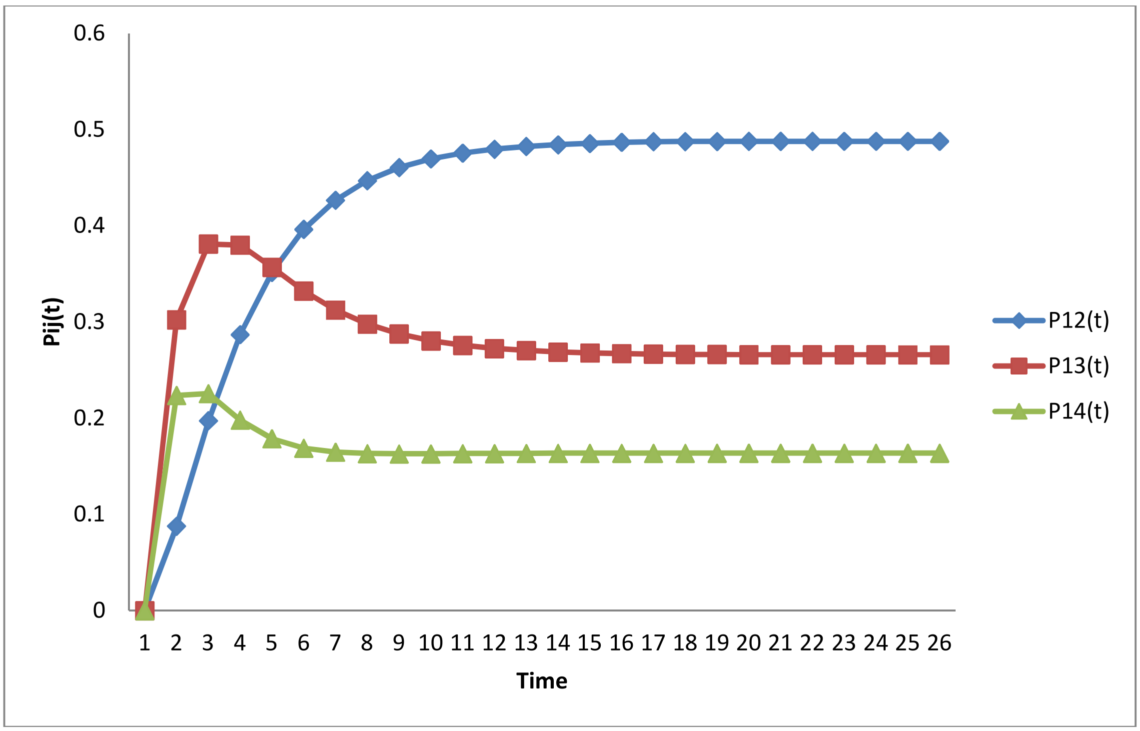

Fig. 2. The graph of Transition Probabilities

We observe that, the limit of each function as t goes to infinity or fairly large is immediately apparent, both in the functions themselves and in the graphs of the functions. The convergence is smooth and monotonic, as opposed to discontinuous, oscillating or both.

The result is presented in Table 2 and illustrated graphically in Fig. 2. The model enables us to determine the values of $P_{12}(t)$, $P_{13}(t)$, $P_{14}(t)$ respectively at any time t. It is observed from the table that P_{12} rose steadily and stabilizes to 0.49 as t tends to infinity, also P_{13} rose, later dropped and stabilized to 0.27 as t tends to infinity, similarly P_{14} rose and drop sharply and later stabilized to 0.16 as t tends to infinity. These are the equilibrium transition probabilities. For instance, if the process is No rainfall state in a given week, it would take at most 59%, 27% and16% of the time to make transition to Low rainfall state, moderate rainfall, and high rainfall state respectively in the long run. Thus given the rainfall in a week it is possible to determine quantitatively the probability of finding rainfall in other states in the following week(s) and in the long run.

4. CONCLUSIONS

A stochastic model to analyze and predict weekly rainfall pattern both in discrete and continuous has been presented. With the combined results of both the discrete and continuous time Markov model, the rainfall pattern of the study area is better understood. The results from the model are an important information, that could assists the residents to better understand the dynamics of weekly rainfall of the study area which may be helpful for effective planning and viable crop production.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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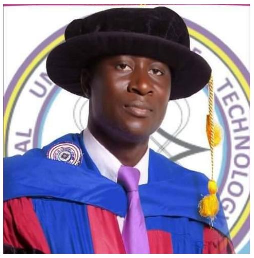

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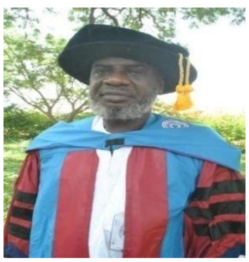

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