Analytical Evaluation of Operational Characteristics of a Mid-Plate Adjustable Capacitive Transducer

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Abstract— In this work, the functional principles of capacitive transducer is revealed with attention on the transduction effect of the device using two parallel plates at both ends. A movable plate at the middle of the device is incorporated by which the electrostatic field within the two extreme plates is adjusted to determine the operational consequence of the mid-plate movement. The two existing voltages due to the two end plates and the moving plate are deduced in relation to the existing parameters of the device. A graphical results are obtained and properly evaluated in attempt to ascertain the basic characteristics under which the operational capability of the three – plate capacitive transduction can be improved for adequate operational result of the equipment.

Keywords: capacitive transducer, Circuit Modeling

I. INTRODUCTION

A capacitor can be established when two electrically conductive metallic plates that are separated by an insulating medium (ie a dielectric material) are parallely mounted.

In application of voltage on the metallic plates, electric charges emerge on the plates; and these charges are always equal but opposite in polarity. The charges that appear in every capacitor plates are a mathematical product of the applied voltage and capacitance of the component.

Therefore,

$Q = CV$ ………………………………………………….(1)

C is the capacitance which can be expressed as a ration of the charge, Q to the applied voltage, V.

Thus,

$C = Q/V$ ……………………………………………….(2)

C can be related to the natural variables of the capacitor parameters such as the area, A of the metallic plates, the distance, d that separates the plates and the permittivity constants, $\varepsilon\ (\varepsilon_r\varepsilon_0)$ of the insulator medium.

The variation of the capacitance with respect to the measure and is of optimum implication to the true physical application of the transducer. Thus, acceleration, pressure, vacuum, fluid flow, liquid level , audio sound field , force , relative humidity , displacement ,velocity etc can be accurately determined considering the capacitance of this equipment which can be mathematically represented as

$C = \varepsilon_r\varepsilon_0 A(m - 1) / d$ …………………..…………..(3)

Where:

ε = 8,854.10-12 F/m is the dielectric permittivity of vacuum;
ε = permittivity of the area between the electrodes (for air ε = 1,0005);
A = cross-sectional area of the electrodes.

The capacitance can be influenced by changing the air gap d, the active area of the parallel plates, A and the dielectric properties, ε of the medium between the two plates.

From the equation, capacitance, C is directly proportional to the area of the plates but inversely proportional to the distance, d that separates the two plates.

The expression shows that capacitance of a capacitor can be varied alongside with variation of any of the capacitor variables such as the area of the plates, the distance apart of the two plates and possibly the dielectric material of the insulator medium. This is the basis of the functional characteristics of a capacitance based transducer.

In capacitance transducers with two plate capacitor, varying the capacitance with distance, d gives a hyperbolic and almost a linear response over a minute displacement range. The sensitivity is in inverse proportion to the square of the separating distance, d of the two capacitor plates; and this provides a seemingly high sensitivity of the devices with a little change in d.

A. *Work Format*

In this work the 3- plate capacitive transducer is mathematically modeled to illustrate the electrical impart of the three fundamental parallel plates that compose the device. The third plate is inserted at the mid-point between the two extreme plates labeled A and B. The mid-plate is movable, and a small distance of displacement, x is assumed to be a length that can be covered by the middle plate upon adjustment and within the two extreme capacitor plates of the transducer. These are found in section 2. Section 1 creates a concise view on the general characteristics of capacitors as electronic components and shows how they can obviously operated to achieve a good functional properties for field operation of capacitive transduction.

II. CIRCUIT MODELING OF THE 3 – PLATE CAPACITIVE TRANSDUCER

There is always a linear transduction characteristic when the displacement is large enough; and to achieve this, a mica substrate is placed between the two plates. As well, full scale linearity can be reasonably achieved through a feedback capacitance connected with an amplifier. As an alternative, a 3-plate deferential circuit can be used.

In such arrangement, two fixed plates are usually mounted at the ends and the third one is made movable in-between the plates. As a result, a capacitor building principle emerges by virtue of the two parallel plates that are formed as we take from both end of the fixed plates to the moveable middle plates. This makes it possible to have two capacitances, C_1 and C_2 which are variable given the adjustment of the middle plate.

The differential capacitance pressure transducer which is the composite product of such arrangement does possess circular fixed plates with a movable thin diaghram plate within the space of the circular immovable plates. It is on this variable diaghram that the differential pressure to be measured is directed.

We can illustrate the above device as shown below with A and B as the fixed extreme plates and T, the adjustable diaghram at the mid-distance of A and B.

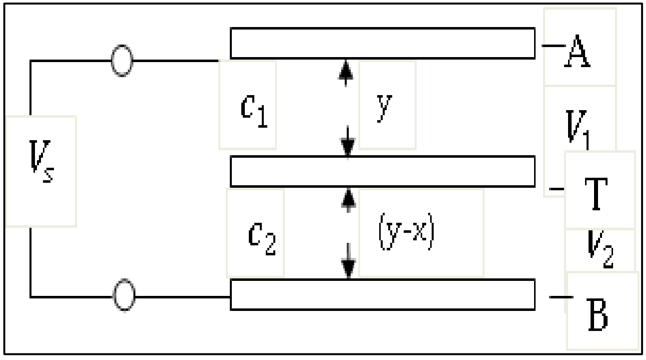
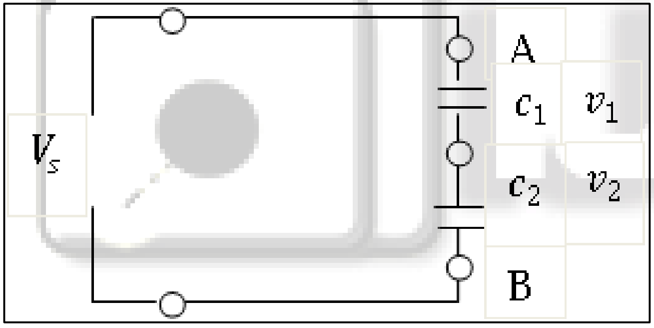


Fig. 1: A schematic diagram of the three plate capacitive transducer

V_s is the source voltage applied at the input terminals of the two fixed capacitor plates, C_1 and C_2 . V_1 and V_2 are the voltages that develop across plates AT and TB respectively. 'd (ie y)' is the distance between AT, and TB when plate T is moved to the middle of the two extreme plates, A and B. X is the distance the diaghram, T can be adjusted when it is moved away from the center, while C_1 and C_2 are the capacitances due to plates AT and TB respectively. The equivalent circuit can be given as shown below:



FFig. 1.1: The equivalent circuit diagram of the three plate capacitive transducer showing the developed voltages v1 and v2 respectively generated by capacitors c1 and c2

Assuming T is moved x distance from the midpoint towards B, then plate B will be (y – x) from T and A will be (y + x) from T. As a result, C_1 and C_2 can be expressed as

$$C_1 = \frac{\varepsilon_0 \varepsilon_r A}{y + x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_r A}{y - x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\text{But, } X_1 = {}^{I}\!/_{J\omega C_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$X_2 = {}^{I}\!/_{J\omega C_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$X_1 = {}^{I}\!/_{J\omega} \left(\frac{E_o E_r A}{y + x}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$X_2 = {}^{I}\!/_{J\omega} \left(\frac{E_o E_r A}{y - x}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Applying the rule of voltage divider principles and mathematical addition of capacitors in series, we will have

$$V_1 = \frac{V_s \cdot X_2}{X_2 + X_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$= V_s \left[{}^{I}\!/_{J\omega} \left(\frac{E_o E_r A}{y - x}\right)\right] / \left[{}^{I}\!/_{J\omega} \left(\frac{E_o E_r A}{y - x}\right) + {}^{I}\!/_{J\omega} \left(\frac{E_o E_r A}{y + x}\right)\right] \ldots (11)$$

$$= \frac{V_s (y - x)}{y + y + x - x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$= \frac{V_s (y - x)}{2y}$$

$$= \frac{V_s y}{2y} - \frac{V_s x}{2y}$$

$$= \frac{V_s}{2} - \frac{V_s x}{2y} = {}^{V_s}\!/_{2} \left(1 - \frac{x}{y}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Solving for V_2, we obtained

$$V_2 = \frac{V_s (y + x)}{2y}$$

$$= {}^{V_s}\!/_{2} \left(1 + {}^{x}\!/_{y}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Taking ΔV as the voltage difference between V_1 and V_2, we can find the expression for the differential output of the two voltages as

$$\Delta V = V_2 - V_1$$

$$= \left[{}^{V_s}\!/_{2} \left(1 + {}^{x}\!/_{y}\right) - {}^{V_s}\!/_{2} \left(1 - {}^{x}\!/_{y}\right)\right]$$

$$= {}^{V_s}\!/_{2y} \left[(y + x) - (y - x)\right]$$

$$= \frac{2x\, V_s}{2y}$$

$$\Delta V = V_s \, {}^{x}\!/_{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Taking a look at the circuit of the device and examining the behavior of the two constituent capacitors at the output of the capacitive transducer, the derived expressions of the capacitors can be evaluated to know how the outputs, V_2 and V_1 depend on the adjustable parameter, x which is the distance, the diaghram, T moves from the center towards any of the end capacitors as it driven within the device.

Therefore, as we earlier obtained, for capacitor C_2

$$V_2 = \frac{V_S}{2}\left(1 + \frac{x}{y}\right)$$

Where

V_2 = voltage a cross capacitor C_2
V_s = source voltage for the capacitive transducer
y = the distance from each of the extreme end capacitors when the mid plate (ie the diaghram, T) of the transducer is located at the center. This is always a fixed distance in the transducer.
x = the adjustable distance to which the mid plat can be varied at any time.

Therefore, from our V_2 equation, we assigned values to the constants in the expression in order to obtain the two variables of the equation in order to see how they depend on each other. As such, 10V can be allocated to V_s which is the source voltage while 4mm to distance y, in the equation, as shown below:

$$V_2 = \frac{V_s}{2}\left(1 + \frac{X}{y}\right)$$

$$yV_2 = \frac{V_s}{2} y + \frac{V_s}{2} X$$

But, y = 4mm and V_s = 10V

$yV_2 = 5 \, x \, 4 + 5x$

$4V_2 = 20 + 5x$

$V_2 = 5 + \frac{5X}{4}$ and this means that

$V_2 = 5 + \frac{5X}{4}$

Repeating the diagram for analysis as mid-plate is adjusted, we have

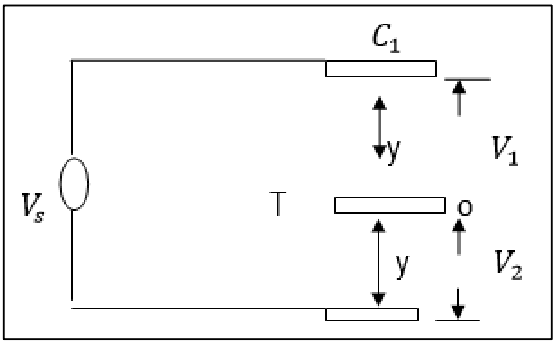


Fig. 1.2: schematic diagram showing the central position of the mid-plate of the capacitive transducer

As the mid- plate, T moves x_1 distance towards C_1, the circuit takes the form below.

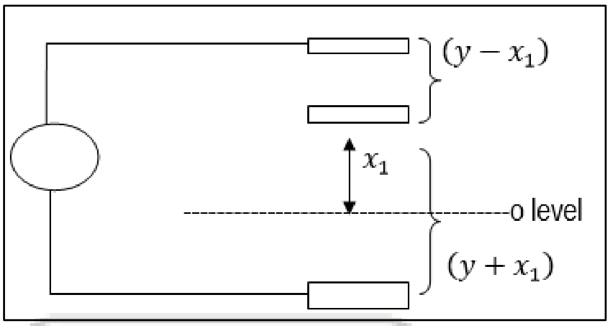


Fig. 1.3: schematic diagram showing the mid-plate of the capacitive transducer adjusted x_1 distance from the center towards plate A

As the plate, T (diagram) is shifted x_2 distance from the centre toward $c_{2,}$ we obtain the diagram as shown

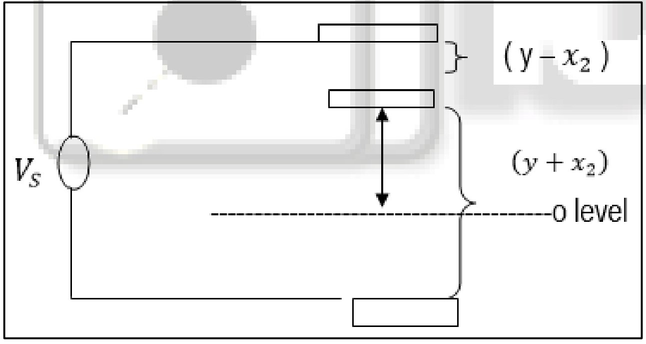


Fig. 1.3: schematic diagram showing the mid-plate of the capacitive transducer adjusted $(y + x_2)$ from the center towards plate A

Now the distance between $c_{2,}$ and T is (y – x_2). Then, if we continue this way, a time will be reached when the surface of the diagram, T (i.e the middle plate) and the surface of the plate of capacitor $c_{2,}$ will meet, and as a result, the distance between T and $c_{2,}$ plate will be nill i.e. $x = y$ approximately, neglecting the thickness of $c_{2,}$).At this time the ratio of x to y (x/y) will be unity and this shows that the voltage, V_2 will be 0V just as calculated below.

$V_2 = 5 - \frac{5X}{4}$

$V_2 = 5 - \frac{5X}{4} X4$

$= 5 - 5$

$= 0$ V

Similarly, V_1 will be calculated thus

$V_1 = 5 + \frac{5x}{4}$

$= 5 + \frac{5}{4} x4$

$= 5 + 5$

$= 10$ V

At the point when the distance between C_2 and T plate is zero as x increases to y, V_2 will be 0V and V_1 will be 10V, thus assuming the entire value of the source voltage.

However, the adjustment of T plate to the midpoint of the two extreme capacitor plates again will decrease x to zero and makes the two end plates to have equal distance, y when measured from their positions to the zero line in the circuit diagram where the mid plate traces the centre of the two plates.

Also, at any time when x ceases to exist as a result of position of T at the centre, the capacitors C_1 and C_2 will develop equal output voltage each possessing half of the source voltage. See the illustration below.

Recall that

$V_1 = 5 - {}^{5}/_{y}\, x$

$V_2 = 5 - \frac{5x}{y}$

Then, when y= 4 and x = 0, we will have

$V_1 = 5 - \frac{5}{4}$ x 0

$= 5$V

$V_2 = 5 + \frac{5}{4}$x

$V_2 = 5$V

This indicates equality in the voltage value of capacitors, C_1 and C_2

Now, we can decide to divide the distance between the middle plate and each of the end plates into 40 units with a unit representing 1.0mm in order to establish 40mm length between T and C_1 and another 40mm distance between T and C_2 plates, thus, making a total of 80mm between C_1 and C_2 plates .

Assuming in every adjustment of plate T towards any of the end plates, that a small distance of 4mm is covered. It; therefore, means that we shall have exactly ten times adjustment to make, in which, x is always added a distance value of 4mm behind the T plate from the centre where x = 0.0mm.At the end ,we shall cover a distance of 40mm from any of the two capacitors to the centre.

Therefore, for the ten times adjustment, we will definitely have the following range of distance values to cover as we discretely adjust the T plate from the center. Δx : 0.0mm, 4mm, 8mm, 16mm, 20mm, 24mm, 28mm, 32mm,36mm, 40mm.

Note that initially before adjustment of T, the distance of any of the end plate from the centre is y and this is 40mm in length. With this we shall plot a graph of the capacitor C_2 and C_1 voltages (V_1 and V_1) against *x* displacement of the mid plate and illustrate the interdependence of the two variables in transduction operation of the 3-plate capacitive equipment.

Thus,

$V_1 = 5 - {}^{5}/_{y}\, x$

But, y = 40mm and x varies in this order: 0.0 mm, 4mm, 8mm,…..,40mm.

From the analysis, the table break down of the calculated values are presented as shown

X(mm)	0.0	4	8	12	16	20	24	28	32	36	40
V_1	5V	4.5V	4V	3.5V	3V	2.5V	2V	1.5V	1V	0.5V	0.0V
V_2	5V	5.5V	6V	6.5V	7V	7.5V	8V	8.5V	9V	9.5V	10V

Table 1:

From the table, when the diaghram, T is steady at the midpoint of capacitor C_1 and C_2 (i.e. at X = 0.0mm), the voltage of the two capacitors is 5V each. As it is adjusted to 4mm away from the centre, V_1 and V_2 become 4.5V and 5.5V respectively. The distance of the T plate from the centre to 8mm length gives the C_1 and C_2 capacitors the voltage values of 4V and 6V respectively. 12mm away from 0.0mm point (from the centre) yields 3.5V and 6.5V respectively for C_1 andC_2. At 16mm distance we have 3V for C_1 and 7V for C_2. Capacitor C_2 has a voltage value of 7.5V while C_1 produces 2.5V at 20mm along the line between the two end plates. Capacitor C_1 gives 2.0V while C_2 gives 8.0V at 24mm distance away from the centre. When x is 28mm, C_1 output voltage becomes 1.5V and C_2, 8.5V. At 32mm, C_1 gives 1.0V while C_2. gives 9.0V. As X increases to 36mm, C_1 yields 0.5V and C_2, 9.5V. Lastly when X is 40mm, and enables a facial contact between C_1 and the diagram, T, C_1, gives 0.0V while C_2.produces 10V and at this point X is assumed to be equal to y, neglecting the thickness of capacitor C_1.

However, the differential output of the two capacitors, (V_1-V_2) can be computed using the expression as shown; or applying the already derived expression, $\frac{Vx}{y}$ (ΔV).

Where,

ΔV = $^{Vx}/_{y}$

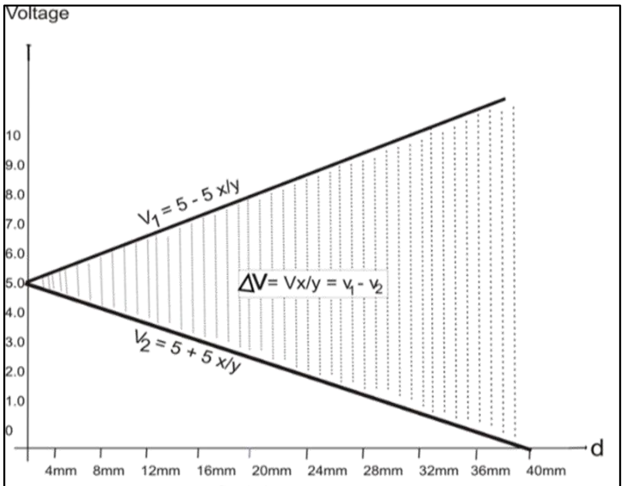


fig.1.4
Diagram showing a graphical output characteristics of the adjustable capacitive transducer

From the graph, it is very easy to note the relationship that exists among the variables$X$, V_1, $V_2$and ΔV. Of course, these variable depend on one another for issuance of the required operation of the transducer.

The adjustment of x variable from the centre manifests the two straight diagonal lines that intersect at the middle of the vertical axis which is taken as the voltage axis The intersection of these straight lines makes a seemingly “V” looking shape that is shaded in the graph. It is within the V shaded portion of the graph that all the outputs values fall.

X(mm)	0.0	4	8	12	16	20	24	28	32	36	40
Y(mm)	40	40	40	40	40	40	40	40	40	40	40
V_s	10V	10V	10V	10V	10V	10V	10V	10V	10V	10V	10V
ΔV	0.0V	1V	2V	3V	4V	5V	6V	7V	8V	9V	10V

Table 1.1:

The intersection of these straight lines makes a seemingly “V” looking shape that is shaded in the graph. It is within the V shaded portion of the graph that all the outputs, ΔV of the transducer are traceable as the mid plate, T is being adjusted through all the X discrete values from the centre to the end plate of C_2 and C_1 capacitors. Thus, with the transducer output, ΔV, sweeping across inbetween the two diagonally divergent lines that originate from the 5V of the voltage axis, every other voltage like $V_1$and V_2 which is made by the two end capacitors with the mid plate T are all obtained outside the transducer output. The C_1 expression forms the lower diagonal line with gradient of $^{5}/_{40}$ $(ie\ ^{1}/_{8})$ while C_2 from the equation forms the upper diagonal line whose gradient is --$^{1}/_{8}$ in value

III. OPERATIONAL MECHANISMS

From the mathematical analysis of the modeled system ,it is quite obvious that the movement of the third plate from the center of the two end plates of the equipment makes a good impart that establishes a linear relation between the source ,V_s and the system modeling variables .Such parameters as the adjustable distance , X of the diaphragm from the center of the two end plates A and B as well as distance , y of each of the plates to diaphragm, T when T is right at the middle of the plates are all related to the differential output of the device considering the voltages developed at the output of the two capacitor C1 and C2 with position of the diaphragm taken as a point of reference. As a result, the difference between the output voltages of the capacitors will, in effect, emerge the output voltage of the transducer and this, as a matter of facts, is referred as the differential output of the equipment which is the true signal of the device that represents the actual reflection of every motion that is effected between plates A and B by the third mid-plate otherwise known as the diaphragm in this work. The plate motion in this case and the distance, x covered within the plates can be duly regarded as the input of the capacitive transducer whose corresponding differential quantity at the output can be used in most Engineering field application.

IV. MERITS, DEMERITS AND APPLICATIONS OF CAPACITIVE TRANSDUCERS

A. *Merits:*

1) These transducers have very high impedance, hence; loading effects are minimum in the measuring circuits.
2) These transducers have excellent frequency response (as high as 50 kHz), thus; it can be used for measurement of both static and dynamic phenomena.
3) These transducers are not affected by stray magnetic fields. That is why the capacitive transducers are used for applications where stray magnetic fields adversely affect inductive transducers.
4) These transducers are extremely sensitive.
5) A resolution of the order of 2.5 microns can be achieved with such transducers.
6) These can be operated with very small forces so they are very useful for small systems and require small power to operate.

B. *Demerits:*

1) Output impedance of capacitive transducer is very high; hence, its measuring circuit is very complicated.
2) Insulation resistance of the system cannot be neglected because of high output impedance of the transducer. This, reduces its sensitivity, moreover with change in physical conditions such as humidity, temperature etc. The resistance changes its value as such introduces error in measurement.
3) Stray capacitance in the system including that due to cables etc. in parallel with the output impedance of the transducer also causes error and introduces non-linearity. In order to reduce the effects of stray capacitances, the device should be subjected to proper earthing system
4) Electrostatic screening should be provided for capacitive transducers in order to avoid any pickup incident.
5) The screened cable connector to the transducer can be a source of error because its capacitance varies with the movement between the cable conductors and cable dielectric.
6) Capacitance of capacitive transducers changes with change in temperature or on account of presence of small external matter e.g. dusts particles and moisture etc. Hence error is introduced in measurement.
7) Since the displacement, in general, is small and a large sensitivity is usually needed, adequate design is required for accurate measurements.

V. APPLICATIONS OF CAPACITIVE TRANSDUCERS

1) Capacitive transducers can be employed for measuring both linear and angular displacements. The capacitive transducers are highly sensitive and can be employed for measuring extremely small displacements such as 0.01 pm. On the other hand they can be employed for measuring large distances up to about 30 m as in airplane altimeters.
2) Capacitive transducers can be employed for measuring force and pressure, which are first converted into displacement which makes the capacitance to change.
3) Capacitive transducers can also be employed for measuring pressure directly especially in the cases where permittivity of a medium changes with pressure. Example in case of Benzene permittivity varies by 0.5% in the pressure range of 1 to 1,000 times the atmosphere pressure.
4) Capacitive transducers can be employed for measuring humidity since the permittivity of gases varies with the variation in humidity. Though the variation in capacitance due to variation in humidity is quite small but is detectable.
5) Capacitive transducers can also be employed to measure density, volume, level of liquid, weight etc. but with mechanical modifiers
6) The most commonly used microphones are of three types namely:
    1) Capacitor microphones.
    2) Carbon microphones and.
    3) Dynamic microphones.

VI. CONCLUSION

The introduction of a mid-plate in the capacitive transducer makes a unique influence on the device .It diverts the component from natural behavior of ordinary two plate capacitive transducer equipment whose properties are technically restricted to generation of electrostatic field in-between its two end plates; thereby enabling storage of electronic charges over which electric voltage is built across the two end static plates. However; this transducer, due to the movable diaphragm between the two end plates is a specially designed electrostatic instrument whose ability to vary the resulted two separate and independently existing voltages on the external pates determines its operational dynamism in real-life application

REFERENCES

[1] Madou MJ (2002) Fundamentals of Microfabrication 2nd edn., CRC Press, New YorkGoogle Scholar
[2] Maluf N, Williams K (2004) An Introduction to Microelectromechanical Systems Engineering, 2nd edn., Artech House, Inc., Norwood, MAGoogle Scholar
[3] S, Ensell G, Kraft M, White N (2004) MEMS Mechanical Sensors, Artech House, Inc., Norwood, MAGoogle Scholar
[4] Busch-Vishniac I (1998) Electromechanical Sensors and Actuators, Springer, New York,MATHGoogle Scholar
[5] Gao D, Howe RT, Maboudian R (2003) High-selectivity Etching of Polycrystalline 3C-SiC Films using HBr-based Transformer Coupled Plasma. Applied Physics Letters.
[6] “Microsystems Science, Technology and Components Dimensions.” Sandi National Laboratories. 2005. http://www.mems.sandia.gov/
[7] Senturia S (2001) Microsystem Design, Kluwer Academic Publishers, Boston