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A Mathematical Modelling of Tuberculosis infection Dynamics with Effects of Case Detection and Drug Resistance

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Abstract

A deterministic mathematical model of tuberculosis incorporating case detection and drug resistance with constant recruitment rate was developed. The population was subdivided into six compartments according to their disease status. The basic reproduction number of the model was obtained using the next generation matrix. The existence of disease free and endemics equilibrium points were shown and the conditions for their stability was also esterblished. The results show that the disease free equilibrium points are locally asymptotically stable if $R_0 < 1$ and globally stable if $R_0 \leq 1$. Also the results further show that the endemic equilibrium points are locally asymptotically stable if $R_0 > 1$ and globally stable if $R_0 \geq 1$. We obtain the approximate solution of the model using Homotopy Perturbation Methods. The graphical summaries of the solution were carried out and the result show that increase in case detection and sustained treatment can help to reduce transmission of tuberculosis disease.

Key words: Tuberculosis, Reproduction Number, Homotopy Perturbation Method, Next Generation Matrix

1. Introduction

Tuberculosis is a bacterial disease which attacks some part of the human body such as lungs, bones, lymph nodes and brain. This disease is caused by a known mycobacterium tuberculosis that looks like rod-shape bacterium. Some of the symptoms are in the form of cough, chest pains, shortness of breath, loss of appetite, weight loss, fever, chills and fatigue.

Tuberculosis is the second leading cause of death from infectious disease worldwide after those caused by Human Immune Deficiency Virus (HIV) [4]. This disease affect over 2 billion of the world population. Approximately over nine million people develop active tuberculosis and up to 2 million death cases is recorded from tuberculosis every year. Also over 480 thousand people developed drug resistance to tuberculosis with 210 thousand of those who developed multi drug resistance tuberculosis result to death [6].

Tuberculosis (TB) infection is of two type, namely, latent infection and active infection. The latent infection in the body system is a condition in which a patient holds dormant (sleeping) Tuberculosis bacteria in the

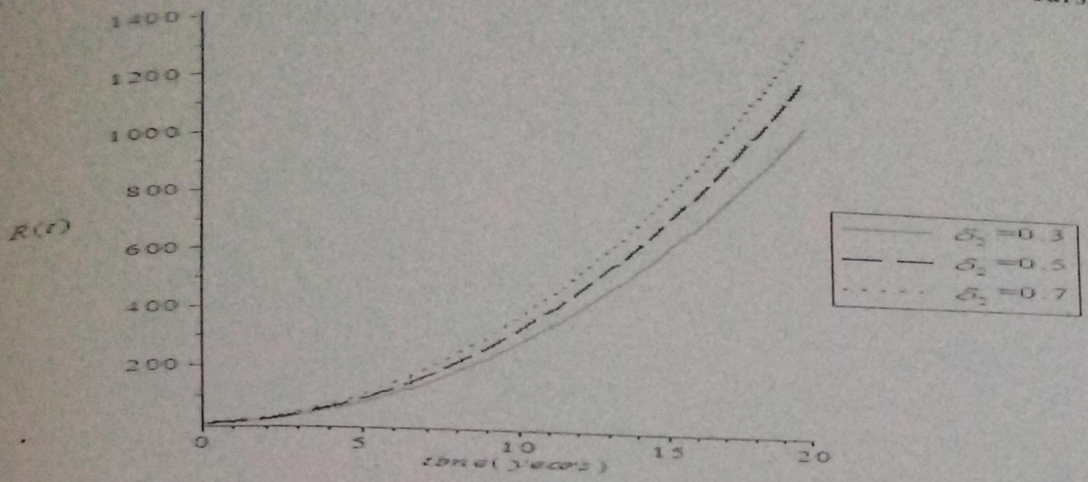

Figure 8 Graph of Recovered Individuals against time for different recovery rate due to second line treatment δ_2 .

It was observed that the population of recovered class increases as the rate of recovery due to second line of treatment increases. This shows that more people will move to recovered class from Resistance class due to second line of treatment.

4. Conclusion

This study presents a deterministic model for the effects of case detection and Resistance to tuberculosis diseases. It was shown that the model is mathematically and epidemiologically meaningful in the feasible region. The positivity of the solution was established, equilibrium points were obtained, and their stability analysis was performed. The conditions for local and global stability of both disease free equilibrium point and endemic equilibrium point were also established. The basic reproduction number using the next generation matrix was obtained was used to form the bases for stability of the equilibrium points. The analysis revealed that diseases free equilibrium is locally asymptotically stable if $R_0 < 1$ and globally stable if $R_0 \leq 1$. Also, the endemic equilibrium point is locally asymptotically stable if $R_0 > 1$ and globally asymptotically stable if $R_0 \geq 1$. Semi-Analytical solutions of the model using Homotopy Perturbation Method (HPM) were obtained graphical profiles of the solutions were presented.

From the results obtained, it was observed that when the case detection rate is high the infected population reduced drastically due to low possibility of contacts between the susceptible population and infectious individuals. Also the result revealed that resistant individuals to first and second line of treatment increase as resistance rate of both classes increases respectively mainly due to treatment failure. The result further revealed that recovered class increases as recovery rate of first and second line of treatment increases.

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