A MODIFIED REVENUE MODEL TO HANDLE COSTING OF ETHANOL PRODUCTION FROM BIOMASS

BEING A PRESENTATION DURING AUST CONFERENCE, ABUJA, 25TH – 28TH JUNE 2018

By

[1]P.O. Evans and [2]N. Nyor

[1]Department of Mathematics, Federal Polytechnic, Bida

[2]Department of Mathematics, Federal University of Technology Minna

INTRODUCTION

Historically, fermentation products were mainly food products, but in recent years an increased interest has been observed in the production of bulk chemicals (ethanol and other solvents), specialty chemicals (Pharmaceuticals, industrial enzymes), biofuels and food additives (flavor modifiers), fermentation processes are also used in agriculture.

Mathematical optimization can be used as a computational engine to arrive at the best solution for a given problem in a systematic and efficient way. In the context of biochemical systems, coupling optimization with suitable simulation modules opens a whole new avenue of possibilities. Mendes and Kell (1998) highlight two types of important applications:

- Design problems: How to rationally design improved metabolic pathways to maximize the flux of interesting products and minimize the production of undesired by-products (metabolic engineering and biochemical evolution studies);
- Parameter estimation: Given a set of experimental data, calibrate the model so as to reproduce the experimental results in the best possible way.

Banana trunk biomass is a renewable polymer abundant in nature particularly in Nigeria, as Nigeria is ranked among the highest producers of banana in West Africa. The biomass is often wasted after harvesting. Currently there are trends in hydrolyzing banana trunk polymers, using enzyme processes to produce fermentable sugars and the fermentable sugar is further converted into ethanol which makes it cheap and can be used as renewable fuel.

However, the mathematical model for the optimization of the various parameters leading to the production of ethanol from this abundant and renewable polymer has not been achieved.

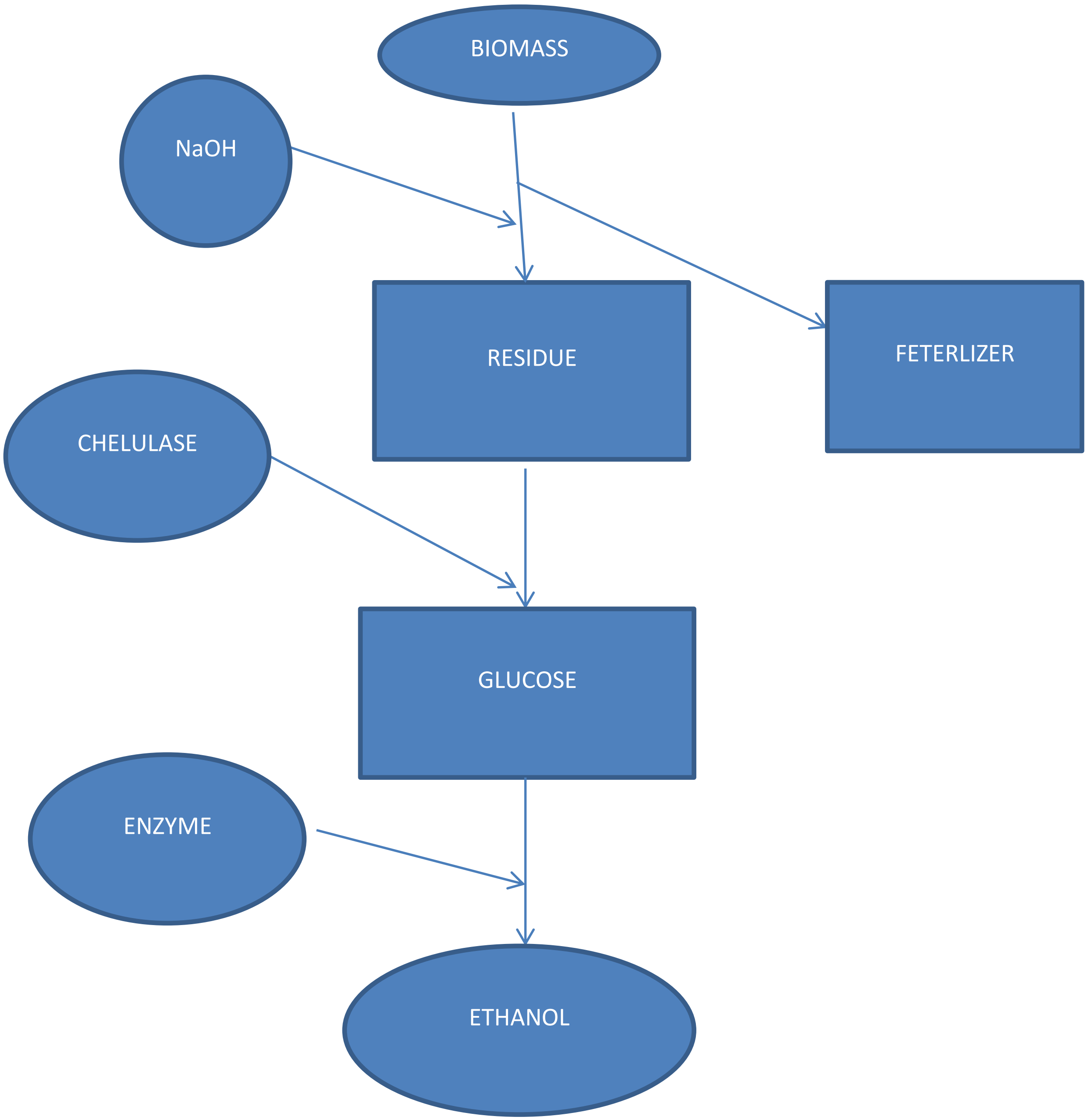

FIGURE 1: A FLOW CHART OF BATCH PRODUCTION OF ETHANOL

1.1 AIM AND OBJECTIVES OF STUDY

- The aim of this work is to study mathematically the optimum revenue of ethanol production from biomass.
- The OBJECTIVES are:

1. To formulate a mathematical optimization model of ethanol production with a cost function.

2. To solve the resulting equation using local search.

2.0 Literature review

Bioethanol is currently produced from sugars and starch materials. Ethanol made from sugar cane biomass (bagasse & straw) as well as other lignocelluloses materials provide unique environmentally sustainable energy sources, economic strategic benefits and can be considered as a safe and cleanest liquid fuel alternative to fossil fuels. So lignocellulose biomass can act as a cheap substrate with constant supply as a substrate for bioconversion to fuel ethanol.

Let X, Y, and Z be Banach spach, and let $\leq_k$ be the linear partial order in Y induced by a close, nonempty, convex cone K in Y in $y_1 \leq_k y_2$ iff $(y_2 - y_1) \in K$. We denote the polar cone of K by $K^+ : (y^*,y) \geq 0$, ¥ $y \in K$. Consider the following class of constrained optimization problems, for $(y; z) \in Y \times Z$,

$$P(y; z) : \min f(x) \text{ s:t: } g(x) \leq K \ y;\ h(x) = z;\ x \in C;$$

where C is a closed subset of X, $f : X \to IR$ is lower semicontinuous, $g : X \to Y$ is lower semicontinuous with respect to $\leq K$ and $h : X \to Z$ is continuous. Borwein and Zhu, 2005, Mordukhovich, 2006, Penot, 2014.

3.0 RESEARCH METHODOLOGY

To maximize or minimize a multivariable function f(x, y,…) subject to the constraint that another multivariable function equals a costant, g(x,y,…) = C. The following steps are applicable:

1. Introduce a new variable λ, and define a new function L as follows,

$$L(x,y,\dots) = f(x,y,\dots) - \lambda(g(x,y,\dots) - C)$$

The function L is called the "Lagrangian", and the new variable λ, is referred to as "Lagrangian multiplier".

2. Set the gradient of L equal to the zero vector

$$\nabla L(x,y,\ldots \lambda) = 0$$

In other words, find the critical points of L.

3. Consider each solution ,which will look like $(X_0, Y_0, \ldots, \lambda_0)$. Plug each one into f, whichever one gives the greatest or smallest value is the maximum or minimum point we are seeking. According to Cobb, and Douglas. 1928. **Production Function is given by**

$Q = AK^aL^b$ where a and b are positive fractions.

$Q = AK^aL^{(1-a)}$

We modified a revenue model in order to handle the costing with a heuristic approach for the production of ethanol from biomass. The methodology of heuristic combined Lagrange multiplier that involves budgetary constraints with local search was employed to formulate Lagrange cost model as multi-item, multi-level capacitated revenue generation. We updated the Lagrange multiplier are by using surrogate sub gradient method that ensures the convergence of the approximate solution. A feasible solution of the original problem is constructed from the solution of the Lagrange multiplier problem at each iteration which is later improved by local search that changes the values of one or more of the variables at each time.

BUDGETARY CONSTRAINTS

Suppose we are running a factory producing some ethanol that requires Biomass as a raw material.

Our cost is predominantly human labour which is =N= 120 per hour for workers, initial Biomass weight is 500kg, 40% of residue and 70% of the glucose is produced from the biomass per 2500ml of ethanol. We want to maximize the function R,

$R(h,b,r,g) = Kh^{m}b^{n}r^{p}g^{q}$, $_{m,n,p,q\,=}$ (0,1); K= 200

Subject to, $120h + 500b + 40r + 70g = 4000$.

Where,

K = a constant,

h = hour of human labour,

b = mass of Biomass,

r = quantity of residue used and

g = quantity of glucose used.

We need to maximize a function

Our objective function is give by

$R(h,b,r,g) = Kh^m b^n r^p g^q$

subject to a constraint,

$120h + 500b + 40r + 70g = 4000.$

We begin by writing the Lagrange function for the set up above.

Assuming,

$m = \frac{3}{4}$, $n = \frac{1}{2}$, $p = \frac{1}{6}$, $q = \frac{1}{12}$

the constrained optimization problem above becomes and unconstrianed problem,

$$L(h,b,r,g) = 200h^{3/4}b^{1/2}r^{1/6}g^{1/12} - \lambda(120h + 500b + 40r + 70g - 4000)$$

We set the gradient ∇L equal to 0 vector.

This is the same as setting each partial derivative equal to 0:

$$\frac{\partial L}{\partial h} = 0,$$

$$\frac{\partial L}{\partial b} = 0,$$

$$\frac{\partial L}{\partial r} = 0,$$

$$\frac{\partial L}{\partial g} = 0,$$

$$\frac{\partial L}{\partial \lambda} = 0,$$

Differentiating with respect to h, we have:

$$\frac{\partial}{\partial h}\left[200h^{3/4} b^{1/2} r^{1/6} g^{1/12} - \lambda(120h + 500b + 40r + 70g - 4000) \right\} = 0$$

$$600/4h^{-1/4} b^{1/2} r^{1/6} g^{1/12} - \lambda 120 = 0$$

Differentiating with respect to b, we have:

$$\frac{\partial}{\partial b}\left[200\, h^{3/4} b^{1/2} r^{1/6} g^{1/12} - \lambda(120h + 500b + 40r + 70g - 4000) \right] = 0$$

- $$200/2\, h^{3/4} b^{-1/2} r^{1/6} g^{1/12} - 500\,\lambda = 0$$

Differentiating with respect to g, we have:

$$\frac{\partial}{\partial r}\left[200\, h^{3/4} b^{1/2} r^{1/6} g^{1/12} - c(120h + 500b + 40r + 70g - 4000)\right] = 0$$

$$200/6\; h^{3/4} b^{1/2} r^{-5/6} g^{1/12} - 40_{\lambda} = 0$$

Differentiating with respect to r, we have:

$$\frac{\partial}{\partial r}\left[200\, h^{3/4} b^{1/2} r^{1/6} g^{1/12} - c(120h + 500b + 40r + 70g - 4000)\right] = 0$$

$$200/6\; h^{3/4} b^{1/2} r^{-5/6} g^{1/12} - 40_{\lambda} = 0$$

Differentiating with respect to g, we have:

$$\frac{\partial}{\partial r}\left[200\, h^{3/4} b^{1/2} r^{1/6} g^{1/12} - c(120h + 500b + 40r + 70g - 4000)\right] = 0$$

$$200/12\; h^{3/4} b^{1/2} r^{1/6} g^{-11/12} - 40_{\lambda} = 0$$

Differentiating with respect to λ , we have:

$$\frac{\partial}{\partial r}\left[200\, h^{3/4} b^{1/2} r^{1/6} g^{1/12} - \lambda(120h + 500b + 40r + 70g - 4000)\right] = 0$$

$$-120h - 500b - 40r - 70g + 4000 = 0$$

the maximum revenue can be obtained, using the relationship below,

$$R(h) = 200\, h^{3/4} b^{1/2} r^{1/6} g^{1/12}$$

Results and discussions

```
T := proc(x, d :: posint)
 Digits := max(Digits, d, length(x));
 trunc(x) + evalf[d](frac(x));
end proc:
```

> $$prob1 := \left(\frac{600}{4} \cdot h^{-\frac{1}{4}} \cdot b^{\frac{1}{2}} \cdot r^{\frac{1}{6}} \cdot g^{\frac{1}{12}} - 120 \cdot \lambda = 0, \frac{200}{2} \cdot h^{\frac{3}{4}} \cdot b^{-\frac{1}{2}} \cdot r^{\frac{1}{6}} \cdot g^{\frac{1}{12}} - 500 \cdot \lambda = 0, \frac{200}{6} \cdot h^{\frac{3}{4}} \cdot b^{\frac{1}{2}} \cdot r^{-\frac{5}{6}} \cdot g^{\frac{1}{12}} - 422 \cdot \lambda = 0, \frac{200}{12} \cdot h^{\frac{3}{4}} \cdot b^{\frac{1}{2}} \cdot r^{\frac{1}{6}} \cdot g^{-\frac{11}{12}} - 383 \cdot \lambda = 0, -120 \cdot h - 500 \cdot b - 422 \cdot r - 383 \cdot g + 4000 = 0 \right);$$

$$prob1 := \frac{150\sqrt{b}\, r^{1/6} g^{1/12}}{h^{1/4}} - 120\,\lambda = 0, \frac{100\, h^{3/4} r^{1/6} g^{1/12}}{\sqrt{b}} - 500\,\lambda = 0, \frac{100}{3} \frac{h^{3/4}\sqrt{b}\, g^{1/12}}{r^{5/6}} - 422\,\lambda = 0, \frac{50}{3} \frac{h^{3/4}\sqrt{b}\, r^{1/6}}{g^{11/12}} - 383\,\lambda = 0, -120\,h - 500\,b - 422\,r - 383\,g + 4000 = 0$$

> $$sol1 := solve(\{prob1\}, [h, b, r, g, \lambda])$$

$$sol1 := \left[\left[h = \frac{50}{3}, b = \frac{8}{3}, r = \frac{2000}{1899}, g = \frac{2000}{3447}, \lambda = \frac{1}{392751180}\, 2000^{1/4}\, 1899^{5/6}\, 3447^{11/12}\, 13500000^{1/4}\right]\right]$$

> $$h := \frac{50}{3}$$

> $$b := \frac{8}{3}$$

> $$r := \frac{2000}{1899}$$

> $$g := \frac{2000}{3447}$$

$$\lambda := \frac{1}{392751180}\, 2000^{1/4}\, 1899^{5/6}\, 3447^{11/12}\, 13500000^{1/4}$$

$$\lambda := \frac{1}{392751180}\, 2000^{1/4}\, 1899^{5/6}\, 3447^{11/12}\, 13500000^{1/4}$$

> $R[0] := 200 \cdot \left(\frac{50}{3}\right)^{\frac{3}{4}} \cdot \left(\frac{8}{3}\right)^{\frac{1}{2}} \cdot \left(\frac{2000}{1899}\right)^{\frac{1}{6}} \cdot \left(\frac{2000}{3447}\right)^{\frac{1}{12}}$

$$R_0 := \frac{200}{58912677}\, 50^{3/4}\, 3^{3/4} \sqrt{8}\, 2000^{1/4}\, 1899^{5/6}\, 3447^{11/12}$$

>

> $T(h, 2);$

$$16.67$$

> $T(b, 2);$

$$2.67$$

> $T(r, 2);$

$$1.053$$

> $T(g, 2);$

$$0.58$$

> $T(\lambda, 2);$

$$0.87$$

> $T(R[0], 2);$

$$2396.$$

>

- From the above result applying the revenue function formulae

$$R[16.67] = 200(16.67)^{\frac{3}{4}}(2.67)^{\frac{1}{2}}(1.057)^{\frac{1}{6}}(0.58)^{\frac{1}{12}}$$
$$= 2396$$

This means we should employ 16.67 hours of Labour, use 2.67 of the biomass, 1.057 of residue and 0.58 of glucose to obtain a maximum revenue of 2396 naira.

Table 1: Result of the variation of the parameter using Maple software.

k	m	n	p	q	h	b	r	g	λ	R
200	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	16.67	2.67	1.053	0.58	0.87	2396.
200	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	17.78	2.13	13.33	3.81	1.0	3119.
200	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	16.67	2.67	11.11	3.17	1.6	4530.
200	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	14.035	3.37	10.53	3.0075	1.6	4043.
200	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{8}$	13.68	3.28	10.26	4.40	1.8	4280.
200	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	30.90	0.27	2.58	0.74	4.0	1310.
200	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	30.90	0.27	2.58	0.74	4.0	1646.

Conclusion

The result obtained from the numerical experiment shows that the optimum λ = 1.6, which has a revenue function given as

$$\boldsymbol{R16.67 = 200(6.67)^{\frac{3}{4}}(11.11)^{\frac{1}{2}}(3.17)^{\frac{1}{6}}(1.6)^{\frac{1}{12}}}$$
$$\boldsymbol{= 4530}$$

REFERENCES

Cobb, C. W. and P. H. Douglas. 1928. A theory of production. American Economic Review 18(1):139–165. Supplement, Papers and Proceedings of the Fortieth Annual Meeting of the American Economic Association.

B. S. Mordukhovich, Variational Analysis and Generalized Di_erentiation I, II, Springer-Verlag, 2006.

J.M. Borwein and Q.J. Zhu, Techniques of Variational Analysis, Springer-Verlag, 2005.

J.-P. Penot, Calculus without Derivatives, Springer-Verlag, 2014.

Mendes, P. and Kell, D.B. 1998. Non-linear optimization of biochemical pathways: Applications to metabolic engineering and parameter estimation. Bioinformatics 14: 869-883.

Thank you