

## A NUMERICAL SIMULATION FOR A DETERMINISTIC COMPARTMENTAL MATHEMATICAL MODEL OF HIV/AIDS DISEASE PANDEMIC WITH EFFECT OF PUBLIC AWARENESS/CAMPAIGN

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### Abstract

*In this work, an earlier deterministic mathematical model of HIV/AIDS is revisited and numerical solutions obtained using Euler's numerical method. Using hypothetical values for the parameters, a program was written in VISUAL BASIC programming language to generate series for the system of difference equations from the output of the program. Simulation results were presented in form of graphs. Finally, results were compared with the earlier analysis which was carried out using a modified version of Bellman and Cooke's theory. It was found that both results agree.*

**Keywords:** Deterministic, HIV/AIDS, model, simulation

### Introduction

AIDS is an acronym for Acquired Immunodeficiency Syndrome. The disease is not hereditary but develops after having contact with a disease causing agent called Human Immunodeficiency Virus (HIV).

It is characterized by weakening of the immune system. HIV destroys a certain kind of blood cell (CD4+ T cells) which is crucial to the normal function of the human immune system. In fact, loss of these cells in people with HIV is an extremely powerful predictor of the development of AIDS. Studies of thousands of people have revealed that most people infected with HIV carry the virus for years before enough damage is done to the immune system for AIDS to develop. However, sensitive tests have shown a strong connection between the amount of HIV in the blood and the decline in CD4+ T cells and the development of AIDS. Reducing the amount of virus in the body with anti-retroviral therapies can dramatically slow the destruction of a person's immune system.

It must be emphasized that there are no specific symptoms of HIV infection and infected persons may remain asymptomatic for years due to the long latency period associated with the infection. Clinical symptoms usually subside and infected patients may enter long latency periods of up to 10 years, during which the virus continue to replicate and viral load increases while the patient's CD<sub>4</sub> count decreases. Eventually the body's immune system is overwhelmed and the patient manifests with symptoms and signs of infections called 'opportunistic infections'. For example *pneumocytis carinii pneumonia*, disseminated *tuberculosis*, Oesophageal, and central nervous system *mycosis* and gastrointestinal infections causing chronic diarrhea. The presence of a sexually transmitted disease in a patient prompts the suspicion of HIV infection. This is because HIV is mainly transmitted sexually and patients with STDs like syphilis gonorrhea, lymphogranuloma venereum and genital herpes have an increased risk of developing HIV infection because they share the same



behaviour and the presence of genital facilitate viral entry.(Kaidal,2007).

ntly observed symptoms associated HIV/AIDS include unexplained fevers for more than one month, loss, skin lesion e.g. *seborrheic* itis, *prourgoi* fungal nail infections, nt mouth ulcers, oral thrash, oral hairy 'akia, persistent cough generalized *denopathy* purulent abscesses in the . A diagnosis of pulmonary or inated tuberculosis, Herpes Zoster, sarcoma and CNS Cryptococcus also suspicion of HIV infection.

s spread by sexual contact with an d person, by sharing needles and/or s (primarily for drug injection) with e who is infected, or, less commonly ow very rarely in countries where blood ened for HIV antibodies), through sions of infected blood or blood g factors. Babies born to HIV-infected i may become infected before or during r through breast-feeding after birth.

#### uture review

ost models , some kinds of computer n were used to estimate the parameters model and obtain the results. These of programs are more of tailor-made ns designed to solve only the problem l and might not be easily applicable for other contexts. In this subsection, we some of those programs that have been ped for generic use. The ms/software falling in this category s the following.

The EpiModel developed by the WHO Global Programme on AIDS (GPA) for short-term forecasting of AIDS. The model has been described in Chin *et al.* (1989), While EpiModel does not produce

an automatic projection of future prevalence, it does make it easier for users to prepare such an estimate." (Stover, 1997:12-13)

ii. AIDSTECH and AIDSproj (an Excel spreadsheet software) developed by Stover and incorporates AIDSTECH and the methodology in EpiModel as described in Chin and Lwanga (1991) used for estimating HIV prevalence and for subsequently calculating AIDS cases and AIDS deaths. AIDSProj has similar features as EpiModel. One key difference between the two is that AIDSproj uses as input the prevalence of HIV as a percentage of all adults rather than the number of infections (as done in EpiModel). In this way, the projection of future prevalence does not rely on a population projection (since it is expressed as a percentage) (Stover, 1997).

iii. AIM also developed by Stover and used to illustrate the future consequences of the epidemic.included as a module in integrated software, Spectrum, developed by Futures Group International.The iwgAIDS (Interagency Working Group on AIDS) model is a complex simulation model of the spread of HIV through a population as a result of the behavior of various population subgroups (Stanley *et al.*, 1989). The US Census Bureau approach to projecting HIV prevalence is based on this.

iv. The Doyle model, ASSA500 and ASSA600 models are interrelated and were all developed in South Africa. The Doyle model was produced by Metropolitan Life and

was later adapted by the Actuarial Society of South Africa (ASSA) to produce ASSA500 and ASSA600 spreadsheet models. At the core of the ASSA models, The model has been widely used in South Africa

by various modellers and interest groups.

### Mathematical model

We revisit the Mathematical Model equations of HIV/AIDS disease pandemic presented by Enagi (2008).

$$P = S + R + L + I$$

$$S' = (\beta - \mu - \gamma)S(t) + \beta R(t) + \theta \beta L(t) - \alpha S(t)[L(t) + I(t)] \quad (2.1)$$

$$R' = [\gamma S(t) - \mu R(t)] \quad (2.2)$$

$$L' = [(1 - \theta)\beta - \mu - \tau]L(t) + \alpha S(t)[L(t) + I(t)] \quad (2.3)$$

$$I' = (\beta - \mu - \delta)I(t) + \tau L(t) \quad (2.4)$$

with the parameters given by

$\beta$  = natural birth rate for the population

$\mu$  = natural death rate for the population

$\delta$  = death modulus due to infection

$\alpha$  = rate of contracting the HIV virus

$\gamma$  = rate of removal of the susceptible into the removed class due to public campaign

$\tau$  = rate of flow from the latent class into the infected class

$\theta$  = the proportion of the off-spring of the latent which are virus free at birth  $0 \leq \theta \leq 1$

t = time

Using Euler's numerical method with step size h=1, we obtained the following Difference equations from (2.1) – (2.4)



$$S_{n+1} = S_n + S'_n$$

where

$$S'_n = (\beta - \mu - \gamma)S_n(t) + \beta R_n(t) + \theta I_n(t) - \alpha S_n(t)[L_n(t) - I_n(t)]$$

hence

$$S_{n+1} = S_n + (\beta - \mu - \gamma)S_n(t) + \beta R_n(t) + \theta I_n(t) - \alpha S_n(t)[L_n(t) - I_n(t)] \quad (2.5)$$

$$R_{n+1} = R_n + R'_n$$

where

$$R'_n = [\gamma S_n(t) - \mu R_n(t)]$$

hence

$$R_{n+1} = R_n + [\gamma S_n(t) - \mu R_n(t)] \quad (2.6)$$

$$L_{n+1} = L_n + L'_n$$

where

$$L'_n = [(1 - \theta)\beta - \mu - \tau]L_n(t) + \alpha S_n(t)[L_n(t) + I_n(t)]$$

hence

$$L_{n+1} = L_n + [(1 - \theta)\beta - \mu - \tau]L_n(t) + \alpha S_n(t)[L_n(t) + I_n(t)] \quad (2.7)$$

$$I_{n+1} = I_n + I'_n$$

where

$$I'_n = (\beta - \mu - \delta)I_n(t) + \tau L_n(t)$$

hence

$$I_{n+1} = I_n + (\beta - \mu - \delta)I_n(t) + \tau L_n(t) \quad (2.8)$$

### The programme

we write a program in VISUAL BASIC programming language which generates series for the above system of difference equations.

Project: "myProject"

Revision Number: 1.0.0

Revision Date: 05/07/2010 4:47:08 AM

File Name: "myProject.exe"

Location: C:\VB Projects\VB Projects 2010\myProject\myProject.vbp

Module1.bas

(Declarations)

Sub Main

frmMain.frm

(Declarations)

Sub Form\_Load

Sub cmdNew\_Click

Sub InitialiseVariables

Sub mnuCopySn\_Click

Sub mnuCopyN\_Click

Sub mnuCopyL\_Click

Sub mnuCopyR\_Click

Sub mnuCopyIn\_Click

Sub mnuData\_Click

Sub cmdCompute\_Click

Sub mnuFileNew\_Click

Sub mnuHelpAbout\_Click

Sub mnuN\_Click

Sub mnuViewStatusBar\_Click

Sub ClearTextBoxes

Sub ClearListView

Sub mnuFileExit\_Click



Sub Form\_Unload

frmOptions.frm  
 (Declarations)  
 Sub cmdApply\_Click  
 Sub cmdCancel\_Click  
 Sub cmdOK\_Click  
 Sub Form\_Load

Objects:

Microsoft Common Dialog Control 6.0 (SP6)  
 Path: C:\WINDOWS\system32\comdlg32.ocx  
 Microsoft Windows Common Controls 6.0 (SP6)  
 Path:  
 C:\WINDOWS\system32\MSCOMCTL.OCX

References:

OLE Automation  
 Path: C:\WINDOWS\system32\stdole2.tlb

Summary:

Blank Lines 13  
 Lines of Code 114  
 Lines of Comments 6

Total Lines (Comments & Code) 120

```

/*****
// File Name: frmMain.frm
// File Size: 23.4 KB
// File Date: 7/5/10 4:45:12 AM
// Printed On: Mon. July 5, 2010 4:47:35 AM
*****/
    
```

```

Private Sub Form_Load()
N = GetSetting(App.Title, "Settings", "N", N)
A = GetSetting(App.Title, "Settings", "A", A)
ClearTextBoxes
End Sub
    
```

```

Text(6).Text = 0.025
Text(7).Text = 60
Text(8).Text = 20
Text(9).Text = 10
Text(10).Text = 10
    
```

```

Private Sub cmdNew_Click()
'clear textboxes
ClearTextBoxes
ClearListView
End Sub
    
```

```

AP = Text(0).Text
BT = Text(1).Text
TT = Text(2).Text
GM = Text(3).Text
MU = Text(4).Text
DT = Text(5).Text
TA = Text(6).Text
S(0) = Text(7).Text
R(0) = Text(8).Text
L(0) = Text(9).Text
i(0) = Text(10).Text
    
```

```

Sub InitialiseVariables()
Text(0).Text = 0.01
Text(1).Text = 0.02
Text(2).Text = 0.5
Text(3).Text = 0.1
Text(4).Text = 0.015
Text(5).Text = 0.01
    
```

End Sub

```
Private Sub mnuData_Click()
  InitialiseVariables
End Sub
```

```
Private Sub cmdCompute_Click()
  On Error Resume Next
```

```
Redim S(N), R(N), L(N), i(N)
```

```
ClearListView
```

```
For x = 0 To 10
  If Text(x).Text = "" Then
    MsgBox "Check your Input Parameters",
    vbExclamation, "Missing Value"
  Exit Sub
End If
Next
```

```
For K = 0 To N
```

```
Set lst = ListView.ListItems.Add(, , K, p, p)
lst.SubItems(1) = Format(S(K), "0")
lst.SubItems(2) = Format(R(K), "0")
lst.SubItems(3) = Format(L(K), "0")
```

```
lst.SubItems(4) = Format(i(K), "0")
```

```
S(K + 1) = S(K) + (BT - MU - GM) * S(K) +
BT * R(K) + TT * BT * L(K) - AP * (L(K) +
1)
```

```
R(K + 1) = R(K) + GM * S(K) - MU * R(K)
L(K + 1) = L(K) + ((1 - TT) * BT - MU - TA)
* L(K) + AP * S(K) * (L(K) + 1)
i(K + 1) = i(K) + (BT - MU - DT) * i(K) + TA
* L(K)
```

```
Next K
```

```
End Sub
```

```
Sub ClearTextBoxes()
```

```
'clears all textboxes
```

```
For i = 0 To 10
```

```
Text(i).Text = ""
```

```
Next
```

```
End Sub
```

```
Sub ClearListView()
```

```
On Error Resume Next
```

```
Dim i As Integer
```

```
For i = 1 To ListView.ListItems.Count
```

```
ListView.ListItems.Remove 1
```

```
Next i
```

```
End Sub
```

## Results of simulation



n	S <sub>n</sub>	R <sub>n</sub>	L <sub>n</sub>	I <sub>n</sub>
0	60	20	10	10
1	59	26	10	10
2	58	31	10	11
3	46	39	10	11
4	42	39	10	12
5	39	43	10	13
6	36	46	10	15
7	34	49	10	16
8	31	52	10	18
9	29	54	10	21
10	28	56	10	25
11	28	58	10	31
12	28	60	10	36
13	24	62	10	47
14	23	63	10	56
15	22	64	10	77
16	21	66	10	96
17	20	67	10	109
18	20	68	10	131
19	19	69	10	158
20	19	69	10	190
			1777	227

Figure 1 Showing the program inter phase  
 The graphs given below where obtained from the output of the program using Microsoft excel spread sheet package.

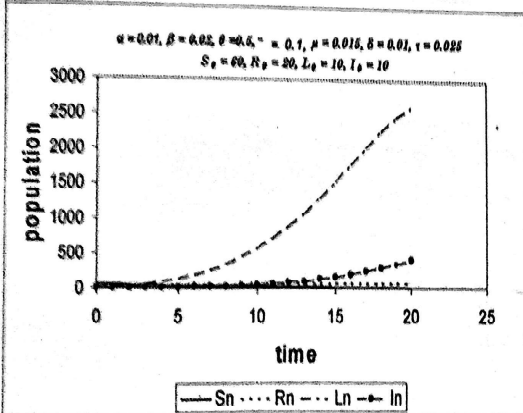


Figure 2 Showing graphical profile of each class for  $\alpha = 0.01, \beta = 0.02, \theta = 0.5, \gamma = 0.1, \mu, =0.015 \quad \delta=0.01, \tau = 0.025, S_0=60, R_0=20, L_0=10 \& I_0=10$

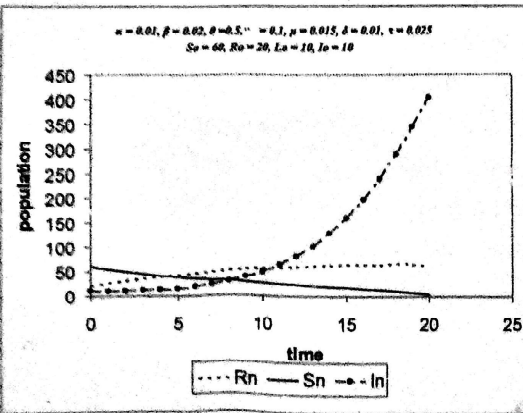


Figure 3 Showing a closer view of  $S_n, R_n$  and  $L_n$  for  $\alpha = 0.01, \beta = 0.02, \theta = 0.5, \gamma = 0.1, \mu = 0.015, \delta = 0.01, \tau = 0.025, S_0 = 60, R_0 = 20, L_0 = 10$  &  $I_0 = 10$

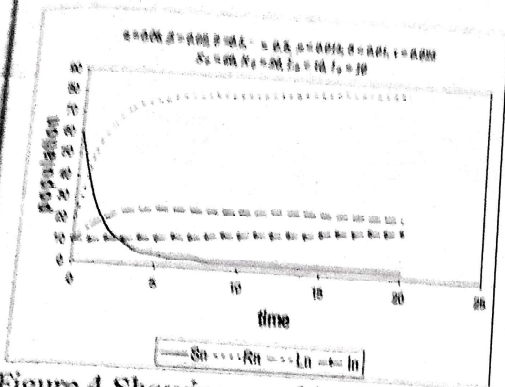


Figure 4 Showing graphical profile of each class for  $\alpha = 0.06, \beta = 0.02, \theta = 0.5, \gamma = 0.5, \mu = 0.015, \delta = 0.01, \tau = 0.025, S_0 = 60, R_0 = 20, L_0 = 10, I_0 = 10$

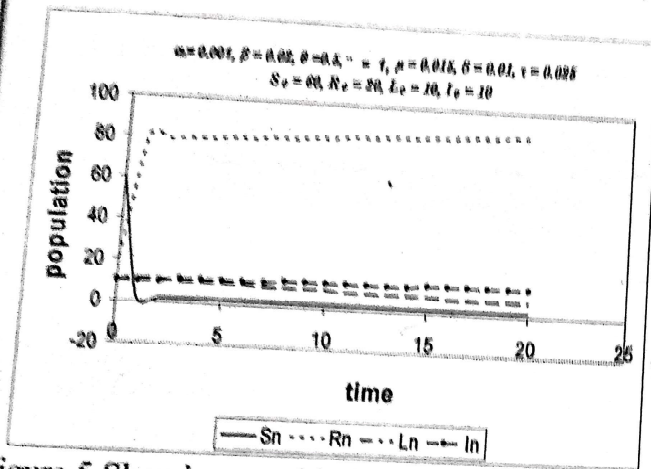


Figure 5 Showing graphical profile of each class for  $\alpha = 0.001, \beta = 0.02, \theta = 0.5, \gamma = 1, \mu = 0.015, \delta = 0.01, \tau = 0.025, S_0 = 60, R_0 = 20, L_0 = 10, I_0 = 10$

Since from our assumptions the population has constant natural Birth and Death rates  $\beta$  and  $\mu$  respectively, we set  $\beta = 0.02$  and  $\mu = 0.015$ . We also assumed that half of the offspring of the latent class are virus free at birth therefore  $\theta = 0.5$  we then vary the measure of effectiveness of public awareness and campaign ( $\gamma$ ) which resulted in changed of behaviour of members of susceptible class. Increase in ( $\gamma$ ) results in decrease in contraction rate ( $\alpha$ ) and vice versa. The movement rate from the Latent class to the

infected class ( $\square$ ) and the death rate with respect to infection ( $\delta$ ) were kept constant because they depend on  $\alpha$  they increase as  $\alpha$  increases and decrease as it decreases, hence we set  $\square = 0.025$  and  $\delta = 0.01$ . We take a sample population of 100, 60 of them are susceptible, 20 Removed class, 10 Latent and 10 infected as the initial populations.

### Discussion of results

Figure 2 gives graphical profiles of each class when  $\alpha = 0.01$  and  $\gamma = 0.1$ , with The initial



population of 100.  $S_0 = 60$ ,  $R_0 = 20$ ,  $L_0 = 10$  and  $I_0 = 10$ . We observed that the population of the Latent class increases rapidly, thus giving an unstable population.

Figure 3 gives a closer look at  $S_n$ ,  $R_n$  and  $I_n$ . We observed that the Susceptible class decreases gradually while the Infected class increases exponentially, this is because  $\gamma$  is low and  $\alpha$  is high. Figure 4 gives graphical profiles of each class when  $\alpha = 0.06$  and  $\gamma = 0.5$ . We observed that  $S_n$  decrease rapidly and became stable at  $n = 6$ , while  $R_n$  increases rapidly and also became stable at  $n = 6$ . This gives a stable population at  $n = 6$ .

Figure 5 gives gives graphical of each class when  $\alpha = 0.001$  and  $\gamma = 1.0$ .  $S_n$  decreased very rapidly and became stable at  $n = 2$ . There was a slight increase in  $L_n$  and a slight decrease in  $I_n$ . Thus a stable population at  $n = 2$ .

### Conclusion

The results from numerical simulations above agrees with that of Enagi (2008). It was observed that the population became stable at  $n = 6$  from figure 4.3 and at  $n = 2$  from figure 4.4, this shows that the higher the measure of effectiveness of public awareness/campaign ( $\gamma$ ) the lower the contraction rate ( $\alpha$ ) and the shorter the time before the stability of the population.

### References

Bellman, R. and Cooke K.L.(1963)  
*Differential Difference equations*;  
London: Academic Press.

Chin, J. and Lwanga, S. (1991). Estimation and projection of adult AIDS cases: a simple epidemiological model. *Bulletin of WHO*, 69:339-406.

Chin *et al.* (1989). The global epidemiology and projected short-term demographic impact of AIDS. *Population Bulletin of the United Nations*. No 27,1989:54-68.

Enagi, A. I. (2008). A Four Dimensional Deterministic Compartmental Model of HIV/AIDS Disease Pandemic. *Journal of Research in National Development*. Vol. 6 No. 2 : 123-129

Stanley, E. *et al.* (1991). The United States Interagency Working Group Approach: The iwgAIDS model for the heterosexual spread of HIV and the demographic impact of the AIDS epidemic. In: *The AIDS Epidemic and Its Demographic Consequences*. United Nations and WHO, New York.

Stover, J. (1997). The Future Demographic Impact of AIDS: What Do We Know? Revised version of paper prepared for the workshop on "AIDS in Development: The Role of Government" Château de Limelette, 17-19 June 1996 (<http://www.tfgi.com/wbpaper.doc>).

Stover, P and Way, P (1998). Projecting the impact of AIDS on mortality. *AIDS*, 12 (suppl 1): S29-S39.