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A FOUR DIMENSIONAL, DETERMINISTIC, COMPATMENTAL MATHEMATICAL MODEL OF HIV/AIDS DISEASE PANDEMIC

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Abstract
In this work we proposed a deterministic mathematical model of HIV/AIDS dynamics incorporating the effects of public awareness campaign in checking the spread of the infection. The population is partitioned into four compartments, namely, Susceptible, Removed, Latent and Infected. The analysis of the equilibrium state is carried out using the modified version of the Bellman and Cook Theorem.

Keywords: Susceptible; Removed; latent; infected

Introduction

AIDS is an acronym for Acquired Immunodeficiency Syndrome. The disease is not hereditary but develops after having contact with a disease causing agent called Human Immuno deficiency Virus (HIV)(Pictet et al, 1998). It is characterized by a weakening of the immune system and spread by sexual contact with an infected person, by sharing needles and/or syringes (primarily for drug injection) with someone who is infected, or, less commonly (and now very rarely in countries where blood is screened for HIV antibodies), through transfusions of infected blood or blood clotting factors. Babies born to HIV-infected women may become infected before or during birth or through breast-feeding after birth.

In this work, we proposed a deterministic mathematical model which is a system of ordinary differential equations. The population is partitioned into four compartments of the susceptible class S(t), this is the class in which members are virus free but are prone to infection by interaction with the latent and the infected classes; The second class is the removed class R(t), this is the class of those not susceptible to infection, possibly due to their yielding to warnings or changed behavior as a result of public awareness campaign or enlightenment; The third class is the Latent L(t), this is the class of those that have contracted the virus, but have no symptom of the AIDS disease, the members of this class are still active in the population both sexually and economically. The fourth class is the Infected I(t); this is the class of those that have the manifestation of the symptoms of infection; this class is assumed to be generally weak and inactive.

It is assumed that while the new birth of S(t) and R(t) are born into S(t), the new birth of I(t) are born into I(t), the off-spring of L(t) are divided between S(t) and L(t) in the proportion θ and $(1-\theta)$ respectively, i.e. a proportion $(1-\theta)$ of the offspring of $L(t)$ are born with the virus while the remaining proportion θ are free from the virus. The three classes have a natural death modulus, while the infected class $I(t)$ has additional death modulus δ arising from the weight of infection.

Members of the class $S(t)$ move into the class $R(t)$ due to change in the behavior or/and as a result of effective public campaign at a rate γ. Members of the class $L(t)$ move into the class $I(t)$ at the rate τ. Members of the class $S(t)$ moved into the class $L(t)$ at the rate α by interacting with $L(t)$ or $I(t)$

The model equations are presented in section one with the definition of parameters. We obtained the equilibrium states and the corresponding characteristic equations of the model in section two. We analyse the zero and the non zero equilibrium states for stability in section three. The result of the work is presented in section four in the form of concluding remarks.

The Model Equation

The model equations are given by equations (1.1) – (1.4)

$$S' = (\beta - \mu - \gamma)S(t) + \beta R(t) + \theta\beta L(t) - \alpha\, S(t)[L(t) + I(t)] \quad (1.1)$$

$$R' = [\gamma\, S(t) - \mu\, R(t)] \quad (1.2)$$

$$L' = [(1-\theta)\beta - \mu - \tau]L(t) + \alpha S(t)[L(t) + I(t)] \quad (1.3)$$

$$I' = (\beta - \mu - \delta)I(t) + \tau L(t) \quad (1.4)$$

with the parameters given by

β = natural birth rate for the population

μ = natural death rate for the population

δ = death modulus due to infection

α = rate of contracting the HIV virus

γ = rate of removal of the susceptible into the removed class due to public campaign

τ = rate of flow from the latent class into the infected class

θ = the proportion of the off-spring of the latent which are virus free at birth $0 \le \theta \le 1$

t = time

Equilibrium State of the Model

At equilibrium state, let

$$(S(t), R(t), L(t), I(t)) = (\omega, x, y, z) \quad (2.$$

at equilibrium we have that

$$(\beta - \mu - \gamma)\mathrm{w} + \beta\, \mathrm{x} + \theta\, \beta\, \mathrm{y} - \alpha\, \mathrm{w}[y + z] = 0 \quad (2.2)$$

$$\gamma\, \mathrm{w} - \mu\, \mathrm{x} = 0 \quad (2.3)$$

$$[(1-\theta)\beta - \mu - \tau]y + \alpha\, \mathrm{w}[y + z] = 0 \quad (2.4)$$

$$(\beta - \mu - \delta)z + \tau\, y = 0 \quad (2.5)$$

Solving these simultaneously gives (w,x,y,z) = (0,0,0,0) as the zero equilibrium state and the non-zero equilibrium state (w,x,y,z) given by

$$w = \frac{(\mu + \delta - \beta)[(1-\theta)\beta - \mu - \tau]}{\alpha(\beta - \mu - \delta - \tau)}, \quad (2.6)$$

$$\mathrm{x} = \frac{\gamma(\mu + \delta - \beta)[(1-\theta)\beta - \mu - \tau]}{\alpha\mu\,(\beta - \mu - \delta - \tau)}. \quad (2.7)$$

$$\mathrm{y} = \frac{[\mu(\beta - \mu - \gamma)(\beta - \mu - \delta - \gamma) + \beta\gamma(\beta - \mu - \delta - \tau)](\mu + \delta - \beta)[(1-\theta)\beta - \mu - \tau]}{\alpha\mu(\beta - \mu - \delta - \tau)(\beta - \mu - \delta - \gamma)(\mu + \tau - \beta)}, \quad (2.8)$$

$$Z = \frac{\tau[\mu(\beta - \mu - \gamma)(\beta - \mu - \delta - \gamma) + \beta\gamma(\beta - \mu - \delta - \tau)][(1-\theta)\beta - \mu - \tau]}{\alpha\mu\,(\beta - \mu - \delta - \tau)(\beta - \mu - \delta - \gamma)(\mu + \tau - \beta)}, \quad (2.9)$$

The Characteristic Equation

The Jacobian determinant for the system with the eigen value λ is given by

$$\begin{vmatrix} \beta-\mu-\gamma-\alpha y-\alpha z-\lambda & \beta & \theta\beta-\alpha w & -\alpha w \\ \gamma & -\mu-\lambda & 0 & 0 \\ \alpha y+\alpha z & 0 & [(1-\theta)\beta-\mu-\tau+\alpha w] & \alpha w \\ 0 & 0 & \tau & \beta-\mu-\delta-\lambda \end{vmatrix} = 0 \qquad (2.10)$$

Expanding the determinant gives

$$[\beta-\mu-\gamma-\alpha y-\alpha z-\lambda](-\mu-\lambda)\{[(1-\theta)\beta-\mu-\tau+\alpha w-\lambda][\beta-\mu-\delta-\lambda]-[\alpha\tau w]\}$$
$$-\beta\gamma\{[(1-\theta)\beta-\mu-\tau+\alpha w-\lambda][\beta-\mu-\delta-\lambda]-[\alpha\tau w]\}+(\theta\beta-\alpha w)(\mu+\lambda)(\alpha y+\alpha z)(\beta-\mu-\delta-$$
$$\lambda)+\alpha w(\mu+\lambda)(\alpha y+\alpha z)\tau = 0$$

Hence the characteristic equation is given by

$$\{[\beta-\mu-\gamma-\alpha y-\alpha z-\lambda](-\mu-\lambda)-\beta\gamma\}\{[(1-\theta)\beta-\mu-\tau+\alpha w-\lambda][\beta-\mu-\delta-\lambda]-[\alpha\tau w]\}+\alpha(y+z)(\mu+\lambda)\{(\theta\beta-$$
$$\alpha w)(\beta-\mu-\delta-\lambda)+\alpha\tau w\} = 0 \qquad (2.11)$$

Stability of the Equilibrium State

The zero equilibrium state

At the zero equilibrium state (w,x,y,z) = (0,0,0,0)

The characteristic equation takes the form

$$\{[\beta-\mu-\gamma-\lambda](-\mu-\lambda)-\beta\gamma\}\{[(1-\theta)\beta-\mu-\tau-\lambda][\beta-\mu-\delta-\lambda]\} = 0 \qquad (3.1)$$

If $(\beta-\mu-\gamma-\lambda)(-\mu-\lambda)-\beta\gamma = 0$

we have

$$\lambda^2+(2\mu-\beta+\gamma)\lambda+[(\mu+\gamma)(\mu-\beta)] = 0$$

a quadratic equation in λ

Hence $\lambda_1 = \beta-\mu$ (3.2a)

$\lambda_2 = -\mu-\gamma$ (3.2b)

From (3.1)

$\lambda_3 = (1-\theta)\beta-\mu-\tau$ (3.2c)

$\lambda_4 = \beta-\mu-\delta$ (3.2d)

We note that $\lambda_2 < 0$, we have that the system will be stable at the origin if $\beta < \mu$; i.e when the death modulus is higher than the birth modulus.

The non zero equilibrium state

For the non zero equilibrium state,

$$(w,x,y,z) = \left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{\alpha(\beta-\mu-\delta-\tau)}, \frac{\gamma(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{\alpha\mu(\beta-\mu-\delta-\tau)},\right.$$

$$\frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)](\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{\alpha\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}$$

$$\frac{\tau[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau]}{\alpha\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}$$

We consider the characteristic equation (2.11) in the form
$H(\lambda)$ and apply Bellman and Cooke theorem [3] on it

$$H(\lambda)=\{[\beta-\mu-\gamma-ay-az-\lambda](-\mu-\lambda)-\beta\gamma\}\{[(1-0)\beta-\mu-\tau+aw-\lambda][\beta-\mu-\delta-\lambda]-[\alpha\tau w]\}+\alpha(y+z)(\mu+\lambda)\{(0\beta-aw)(\beta-\mu-\delta-\lambda)+\alpha\tau w\} \quad (3.3)$$

$$H(\lambda)=\lambda^4-\lambda^3\{[\mu-(\beta-\mu-\gamma-ay-az)]-[(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]\}$$
$$+\lambda^2\{[((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]-[\mu-(\beta-\mu-\gamma-ay-az)][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma]-\alpha(y+z)(0\beta-aw)\}+\lambda\{[\mu-(\beta-\mu-\gamma-ay-az)][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]+[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]$$
$$+[\alpha(y+z)(0\beta-aw)(\beta-\mu-\delta)-\mu\alpha(y+z)(0\beta-aw)+\alpha^2\tau w(y+z)]\}+\alpha\mu(y+z)(0\beta-aw)(\beta-\mu-\delta)+\alpha^2\mu\tau w(y+z)-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w] \quad (3.4)$$

We now set $\lambda=ip$, in $H(\lambda)$

$$H(ip)=(ip)^4-(ip)^3\{[\mu-(\beta-\mu-\gamma-ay-az)]-[(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]\}$$
$$+(ip)^2\{[((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]-[\mu-(\beta-\mu-\gamma-ay-az)][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma]-\alpha(y+z)(0\beta-aw)\}+(ip)\{[\mu-(\beta-\mu-\gamma-ay-az)][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]+[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]$$
$$+[\alpha(y+z)(0\beta-aw)(\beta-\mu-\delta)-\mu\alpha(y+z)(0\beta-aw)+\alpha^2\tau w(y+z)]\}+\alpha\mu(y+z)(0\beta-aw)(\beta-\mu-\delta)+\alpha^2\mu\tau w(y+z)-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]$$

$$H(ip)=p^4-ip^3\{[\mu-(\beta-\mu-\gamma-ay-az)]-[(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]\}$$
$$-p^2\{[((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]-[\mu-(\beta-\mu-\gamma-ay-az)][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma]-\alpha(y+z)(0\beta-aw)\}+(ip)\{[\mu-(\beta-\mu-\gamma-ay-az)][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]+[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]$$
$$+[\alpha(y+z)(0\beta-aw)(\beta-\mu-\delta)-\mu\alpha(y+z)(0\beta-aw)+\alpha^2\tau w(y+z)]\}+\alpha\mu(y+z)(0\beta-aw)(\beta-\mu-\delta)+\alpha^2\mu\tau w(y+z)-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w] \quad (3.5)$$

Resolving into real and imaginary parts,
$H(ip) = F(p) + iG(p)$.
$F(p)$ and $G(p)$ are given respectively by

$$F(p)=p^4-p^2\{[((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]-[\mu-(\beta-\mu-\gamma-ay-az)][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma]-\alpha(y+z)(0\beta-aw)\}+\alpha\mu(y+z)(0\beta-aw)(\beta-\mu-\delta)+\alpha^2\mu\tau w(y+z)-[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w] \quad (3.6)$$

$$G(p)=-p^3\{[\mu-(\beta-\mu-\gamma-ay-az)]-[(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]\}+p\{[\mu-(\beta-\mu-\gamma-ay-az)][((1-0)\beta-\mu-\tau+aw)(\beta-\mu-\delta)-\alpha\tau w]+[\mu(\beta-\mu-\gamma-ay-az)+\beta\gamma][(\beta-\mu-\delta)+((1-0)\beta-\mu-\tau+aw)]+[\alpha(y+z)(0\beta-aw)(\beta-\mu-\delta)-\mu\alpha(y+z)(0\beta-aw)+\alpha^2\tau w(y+z)]\} \quad (3.7)$$

Differentiating with respect to p we have that

$F'(p) = 4p^3 - 2p\{[((1-\theta)\beta-\mu-\tau+\alpha w)(\beta-\mu-\delta)-\alpha\tau w]-[\mu-(\beta-\mu-\gamma-\alpha y-\alpha z)][(\beta-\mu-\delta)+((1-\theta)\beta-\mu-\tau+\alpha w)]-[\mu(\beta-\mu-\gamma-\alpha y-\alpha z)+\beta\gamma]-\alpha(y+z)(\theta\beta-\alpha w)\}$ (3.8)

$G'(p) = -3p^2\{[\mu-(\beta-\mu-\gamma-\alpha y-\alpha z)]-[(\beta-\mu-\delta)+((1-\theta)\beta-\mu-\tau+\alpha w)]\}+[\mu-(\beta-\mu-\gamma-\alpha y-\alpha z)][((1-\theta)\beta-\mu-\tau+\alpha w)(\beta-\mu-\delta)-\alpha\tau w]+[\mu(\beta-\mu-\gamma-\alpha y-\alpha z)+\beta\gamma][(\beta-\mu-\delta)+((1-\theta)\beta-\mu-\tau+\alpha w)]+[\alpha(y+z)(\theta\beta-\alpha w)(\beta-\mu-\delta)-\mu\alpha(y+z)(\theta\beta-\alpha w)+\alpha^2\tau w(y+z)]\}$ (3.9)

Setting p = 0 we have

$F(0) = \alpha\mu(y+z)(\theta\beta-\alpha w)(\beta-\mu-\delta)+\alpha^2\mu\tau w(y+z) - [\mu(\beta-\mu-\gamma-\alpha(y+z))+\beta\gamma][((1-\theta)\beta-\mu-\tau+\alpha w)(\beta-\mu-\delta)-\alpha\tau w]$ (3.10)

(3.11)

$G(0) = 0$ (3.12)

$F'(0) = 0$

$G'(0) = [\mu-(\beta-\mu-\gamma-\alpha(y+z))][((1-\theta)\beta-\mu-\tau+\alpha w)(\beta-\mu-\delta)-\alpha\tau w]+[\mu(\beta-\mu-\gamma-\alpha(y+z))+\beta\gamma][(\beta-\mu-\delta)+((1-\theta)\beta-\mu-\tau+\alpha w)]+[\alpha(y+z)[(\theta\beta-\alpha w)(\beta-2\mu-\delta)+\alpha\tau w]\}$ (3.13)

The condition for stability according to the Bellman and Cooke theorem [3] is given by

$F(0)\ G'(0) - F'(0)\ G(0) > 0$ (3.14)

Since G(0) = 0, (3.14) becomes

$F(0)\ G'(0) > 0$ (3.15)

The condition for (3.15) to hold is

Sign F(0) = Sgn $G'(0)$

From (2.6)

$$w = \frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{\alpha(\beta-\mu-\delta-\tau)} \quad (3.16)$$

and from (2.8) and (2.9)

$$(y+z) = \frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)](\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{\alpha\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)} + \frac{\tau[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau]}{\alpha\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}$$

$$= \frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau](\mu+\delta-\beta+\tau)}{\alpha\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)} \quad (3.17)$$

Substituting (3.16) and (3.17) in to F(0) we obtain

$$F(0) = \mu\left\{\frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau](\mu+\delta-\beta+\tau)}{\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}\right\}\{\theta\beta -$$

$$\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\Bigg\}(\beta-\mu-\delta)+$$

$$\mu\tau\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\left\{\frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau](\mu+\delta-\beta+\tau)}{\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}\right\}$$

$$-\left\{\mu\left(\beta-\mu-\gamma-\left[\frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau](\mu+\delta-\beta+\tau)}{\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}\right]\right)+\beta\gamma\right\}\Bigg\{\Bigg((1-\theta)\beta-$$

$$\mu-\tau+\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\Bigg)(\beta-\mu-\delta)-\tau\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\Bigg\} \quad (3.27)$$

Substituting (3.16) and (3.17) in to G`(0) gives

$$G(0)=\left\{\mu-\left(\beta-\mu-\gamma-\left[\frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau](\mu+\delta-\beta+\tau)}{\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}\right]\right)\right\}\Bigg\{\Bigg((1-\theta)\beta-$$

$$\mu-\tau+\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\Bigg)(\beta-\mu-\delta)-\tau\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\Bigg\}$$

$$+\left\{\mu\left(\beta-\mu-\gamma-\left[\frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau](\mu+\delta-\beta+\tau)}{\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}\right]\right)+\beta\gamma\right\}\Bigg\{(\beta-\mu-\delta)$$

$$+\left((1-\theta)\beta-\mu-\tau+\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\right)\Bigg\}+$$

$$\left\{\left[\frac{[\mu(\beta-\mu-\gamma)(\beta-\mu-\delta-\gamma)+\beta\gamma(\beta-\mu-\delta-\tau)][(1-\theta)\beta-\mu-\tau](\mu+\delta-\beta+\tau)}{\mu(\beta-\mu-\delta-\tau)(\beta-\mu-\delta-\gamma)(\mu+\tau-\beta)}\right]\left(\left(\theta\beta-\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\right)(\beta-\right.\right.$$

$$\left.\left.2\mu-\delta)+\tau\left(\frac{(\mu+\delta-\beta)[(1-\theta)\beta-\mu-\tau]}{(\beta-\mu-\delta-\tau)}\right)\right)\right\} \quad (3.19)$$

Concluding Remarks

Using Qbasic programming language, we used hypothetical values for the parameters in (3.18) and (3.19) in order to gain insight into the sign of F(0) and $G'(0)$, we have the following table.

α	β	θ	γ	μ	δ	τ	F(0)	G'(0)	REMARK
.01	.02	.5	.1	.015	.01	.025	-1.073065E-07	3.182288E-05	UNSTABLE
.009	.02	.5	.2	.015	.01	.025	-3.441995E-07	1.19 487E-05	UNSTABLE
.008	.02	.5	.3	.015	.01	.025	-4.71596E-07	3.47[illegible]887E-06	UNSTABLE
.007	.02	.5	.4	.015	.01	.025	-5.724663E-07	-2.055574E-06	STABLE

								-6.384833E-06	STABLE
.006	.02	.5	.5	.015	.01	.025	-6.628787E-07	-1.010945E-05	STABLE
.005	.02	.5	.6	.015	.01	.025	-7.481069E-07	-1.348669E-05	STABLE
.004	.02	.5	.7	.015	.01	.025	-8.303894E-07	-1.664601E-05	STABLE
.003	.02	.5	.8	.015	.01	.025	-9.108385E-07	-1.965962E-05	STABLE
.002	.02	.5	.9	.015	.01	.025	-9.900689E-07	-2.257105E-05	STABLE
.001	.02	.5	1	.015	.01	.025	-1.068448E-06		

From the analysis in the previous section, we observe that the origin is stable when the birth rate (β) is less than the death rate (μ), this consequently leads to the extinction of the population. The analysis of the non-zero state resulting from Bellman and Cooke's theorem as shown in the table above shows that the population will be unstable when the measure of effectiveness of public awareness/campaign (γ) is low consequently leading to high contraction rate (α), and stable when (γ) is high consequently leading to low contraction rate (α).

The work hence gives credence to high level awareness campaign as an effective measure of steaming the scourge of the pandemic.

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