Computer-Aided Size Optimization of Steel Trusses Subjected to Stress and Displacement Constraints

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Abstract
The analysis and optimum design of steel elastic trusses are carried out using a computer aided programme written in MATLAB. Karush-Kuhn-Tucker (KKT) conditions were employed and a mathematical model was developed. The adopted numerical solution technique used was finite element method. Since the aim of this computer aided analysis and optimum design is to provide, with due regard to economy, a structure capable of fulfilling its intended use and sustaining the specified loads for it intended life, the axial forces and nodal displacements in the members and nodes were calculated respectively. Then, the compressive members were subjected to compressive stress and buckling load constraints while the tensile members were subjected to tensile stress resistance and the nodal displacement were subjected to maximum allowable displacement based on BS5950. The comparison of the algorithm with existing methods in the literature using a benchmark 10-bar truss shows the advantage of this approach.

Keywords: Structural optimization, finite element analysis, augmented Lagrangian (modified objective function), Karush-Kuhn-Tucker Conditions.

Introduction

An increase in urbanization is putting more pressure on the use of construction materials for the provision of structural facilities including buildings (residential, industrial, and communal), bridges and dams. Steel, unlike concrete, timber and ceramics is essentially a manufactured structural material. By implication, therefore, it is a lot more expensive to produce than most other structural materials. The cost of production in terms of capital, human efforts and time investment logically calls for optimal use. Optimization in general, entails the process of achieving the 'best' out of a number of candidate's options available as solution to a given problem. In the context of this research, optimization procedure is applied in achieving trusses of minimum weight/volume that will satisfy strength conditions and other prevailing constraints.

Over the past decades, considerable progress has been achieved in the optimum design of structural members via mathematical programming methods such as the Lagrangian multipliers method, convex programming, linear programming, and sequential unconstrained minimization techniques and evolutionary algorithms. While for the structural optimization methodology in general and the weight/volume optimization approach in particular, to be embraced by the structural engineering community, for instance (Fan *et al.,* 2005; Ercan and Gregory, 2007), the focus of future research should be on complex structures such as trusses subjected to the actual constraints of commonly used design codes rather than structures subjected to non-realistic constraints.

Though design of steel structure has undergone considerable change in method over time, its philosophy still focuses on safety and economy as the driving force. Economy in design can be achieved through an optimization procedure by aiming at providing, structures capable of fulfilling intended functions and sustaining the specified loads for their intended lives (BS 5950-1; 2000) to achieve the most efficient and effective structure that will satisfy the chosen criteria. The increasing demand on engineers to lower production cost to withstand global competition has prompted them to look for various methods of decision making, such as optimization methods, to design and produce products and systems both economically and efficiently.

Structural optimization techniques are effective tools that can be used to obtain lightweight, low-cost and high performance structures. Optimum design of truss structures has been widely studied by many researchers as they represent a common and complex category of engineering structures. According to (Silih *et al.,* 2010; Thong and Liu, 2001), the size and topology optimization of truss structures is a mixed variable optimization problem, which deals simultaneously with discrete and continuous design variables. Such problems are usually non-convex by nature and, therefore, must be solved by appropriate optimization methods. Topology optimization studies are usually based on the assumption of an initial ground structure that contains all possible joints and members. While most of conventional mathematical optimization methods are suited and developed for continuous design variables (e.g. Hajirasouliha *et al*. 2011; Rajeev and Krishnamoorthy, 1992), in practice many structural design variables are chosen based on discrete values due to manufacturing constraints. Zhang *et al.* (2013) presented a comprehensive study on discrete optimization using generalized shape function-based parameterization.

Hasançebi (2007) used a different method for optimization of truss bridges by combining various variable-wise versions of adaptive evolution strategies under a common optimization routine. They carried out size and shape optimizations by using discrete and continuous evolution strategies, respectively. Ant System algorithm is another method that is used by Luh and Lin (2008) to find optimal truss structures for achieving minimum weight under stress, deflection, and kinematic stability constraints. The results of their study indicated that multiple truss topologies with almost equal overall weight can be found concurrently as the number of members in the ground structure increases. Dede *et al.* (2011) combined GA with value and binary encoding for continuous and discrete optimization of trusses to minimize structural weight based on stress and displacement constraints. They showed that the value encoding method requires less computer memory and computational time to achieve optimum solutions.

This paper aims to develop an efficient computer aided approach using KKT condition to optimize truss structures using both continuous and discrete design variables. To achieve a good convergence, Augmented Lagrangian function was introduced and the inequality constraints were effectively converted to equality constraint using slack variable. The method proposed work more effectively for continuous optimization of truss structure. The cross-sectional areas of the members are selected as design variables. The efficiency of the proposed methods to obtain reliable optimum solutions is investigated through simple investigation of the KKT conditions.

1. Optimization Methodology

1.1. Objective Function

There are several criteria for optimum design of truss structures including weight, volume, cost, displacements, maximum stresses, bucking strength, vibration frequencies, or any combination of these parameters. In this study, the objective function is to minimize the volume of the structure, as shown in Equation (1):

$$\text{Objective}: Minimize, f(A) = V = \sum_{i=1}^{n} A_i L_i \tag{1}$$

Subject to the following constraints:

$$\text{For } \sigma_i \geq 0; \frac{\sigma_i}{\sigma_a} - 1 \leq 0 \tag{2}$$

$$\text{For } \sigma_i < 0; \frac{\sigma_i}{\sigma_{cri}} - 1 \leq 0 \tag{3}$$

$$\frac{\delta_j}{\Delta_o} - 1 \leq 0 \tag{4}$$

After introducing *m* positive, slack variables, set

$$g_j(\mathbf{A}) + s_j^2 = 0 \qquad j = 1,2, \dots, m. \tag{5}$$

Where $g_j(A)$ are the inequality constraints
Lagrangian function (modified objective function):

$$L(A, \lambda, s) = f(A) + \sum_{j=1}^{m} \lambda_j [g_j(A) + s_j^2] \tag{6}$$

where *m* is the number of truss structural members, *L* is the length of each truss member, and *A* is the cross-sectional area of the members. During optimization process, A_i can either be continuous, chosen to be random number within a set region or can be discrete values extracting from cross-section types available in the market. The Augmented Lagrangian method is applied for solving the constrained optimization problem. To allow non-linear constraints, Karush-Kuhn-Tucker (KKT) conditions are utilized. Therefore, to minimize the objective function, the following KKT conditions equation should be satisfied:

$$\frac{\partial L}{\partial x_i} = 0 \Rightarrow \frac{\partial f}{\partial A_i} + \sum_{i=1}^{n} \lambda_j \frac{\partial g_j(A)}{\partial A_i} = 0, \quad i = 1,2, \dots, n \tag{7}$$

$$\frac{\partial L}{\partial \lambda_j} = 0 \Rightarrow g_j(A) + s_j^2 = 0, \quad j = 1,2, \dots, m \tag{8}$$

$$\frac{\partial L}{\partial s_j} = 0 \Rightarrow \lambda_j s_j = 0, \quad j = 1,2, \dots, m \tag{9}$$

Where $\lambda_j \geq 0 \; for \; j = 1,2, \dots, m; \; s_j \geq 0 \; for \; j = 1,2, \dots, m$ and

λ_j is Lagrange multipliers, $f(A)$ is the objective function, $g_j(A)$ are inequality constraint functions. Also, n and m denote the numbers of inequality and truss members, respectively.

1.2. Constraint handling

In this study, the constraints were member stress, nodal displacement, and buckling strength. In connection with member stress, the stress resulting from design load combinations should be within allowable limits, according to the materials used. In this study, a number of penalty functions were determined with regard to allowable tension and compressive stress of the truss members. For instance, the inequality constraints were normalized and a slack variable introduced to each of the constraints. This automatically scales the constraint making it easier to be handled. Equations (1) to (5) illustrate the application of the scale factor and the slackness property of the inequality constraints:

$$P_{1i}(A; X = 23, Com) = \max\left(\left| \frac{\sigma_i(X)}{\sigma_i{}^a(X)} - 1 \right|, 0 \right), \tag{10}$$

where the i^{th} member can be under tension or compression, *P1i(A, X)* is the penalty function value for the stress, σ_i and $\sigma_i{}^a$ are the member stress and allowable stress, respectively. In this study, FE analysis was used to calculate the member stress and nodal deflection of the truss structure in the optimization process. Similar to the stress constraints, if any one of the displacement constraints is not satisfied, a penalty function for the vertical displacement is assigned to the related chromosome by using Equation (3):

$$P_{2i}(A) = \max\left(\left| \frac{\Delta_i}{\Delta_i{}^a} - 1 \right|, 0 \right); i = 1, \dots, N_{joint} \tag{11}$$

where $P_{2i}(A)$ is the penalty value of the active nodal displacement, Δ_i is the displacement in the direction of the degree of freedom, and $\Delta_i{}^a$ is the allowable displacement in the direction of the degree of freedom. In general, the failure of a truss structure could be due to failure of a structural component, material failure or structural instability. In this study, tubular hollow sections were used for all truss members with outer width (*D*), mean diameter (d_m), inner width (*c*) and sectional thickness (*t*). The buckling strength of each member was calculated based on the derivation according to the equation:

$$y = \frac{d_m}{2} \sin\theta \text{ and } \sigma_b = \frac{P}{A} \tag{12}$$

σ_b is the maximum allowable buckling stress in any compressive member.

To avoid buckling in the compressive members, the following criterion should be satisfied:

$$x = \frac{N_{ED}}{N_{b,Rd}} \leq 1.0 \tag{13}$$

Where N_{ED} is the design axial load and $N_{b,Rd}$ is the member buckling resistance determined based on the following equation:

$$N_{b,Rd} = \frac{xAf_y}{\gamma M_1}, \frac{xA_{eff}f_y}{\gamma M_1}, \quad x \leq 1.0 \tag{14}$$

, where the first term corresponds to Classes 1, 2 and 3 of the Eurocode 3 (2010) cross sections, while the second term is for Class 4 sections. Also, A is the reduction factor of the relevant buckling mode, *A* is the gross area, A_{eff} is the reduced effective area, and γM_1 is the partial safety factor for buckling resistance calculations. For members under compression, the value of *x* should be determined for the appropriate non-dimensional slenderness ratio λ from the relevant buckling curve, according to Eqn (6):

$$x = \frac{1}{\varphi + \sqrt{(\varphi^2 - \lambda^2)}};\ x \leq 1.0 \qquad (15)$$

$$\varphi = \frac{1}{2}[1 + \boldsymbol{\alpha}(\lambda - 0.2)\lambda^2]\ and\ \lambda = \sqrt{\frac{Af_y}{N_{cr}}},\ \sqrt{\frac{A_{eff}f_y}{N_{cr}}}.$$

Where the first term is applied for Class 1, 2 or 3 and the second term is applied for Class 4 cross sections. P_{cr} is the elastic critical bucking load based on the gross cross-sectional properties, $N_{cr} = \frac{\pi^2 EI}{L^2}$ .The imperfection factor, $\boldsymbol{\alpha}$, depends on the cross-section type. In this study, $\boldsymbol{\alpha}$ was considered to be 0.49. Based on Eurocode 3 (2010), for $\boldsymbol{\lambda}$ less than or equal to 0.2, buckling effects can be ignored.

1.3. Augmented Lagrangian KKT Conditions for continuous optimization approach

ALKKT was proposed for solving nonlinear optimization of truss structures with nonlinear constraints. This method helps to avoid conducting extensive numerical calculations to find the appropriate value for the penalty function coefficient. In this way, each constraint is separately allocated to its own adjusted penalty function coefficient. The advantage of using ALKKT is to include a set of Lagrange Multipliers, instead of a single coefficient penalty function. The fitness function and nonlinear constraint functions are combined by using the Lagrangian and the penalty parameters for a sequence of sub-problems. Subsequently, each sub-problem is solved by using genetic algorithm. The algorithm starts by setting an initial value for the penalty parameter (i.e. initial penalty). The sub-problem formulation in (6) can also be defined by Equation (16):

$$\varphi(A,\lambda,s) = f(A) - \sum_{i=1}^{m} \lambda_i s_i \log\big(s_i - g_i(A)\big) \qquad (16)$$

Where *m* is the number of nonlinear inequality constraints. The components $\boldsymbol{\lambda}_i$ of the vector $\boldsymbol{\lambda}$ are known as Lagrange multiplier estimates, the elements s_i of the vector **s** are nonnegative shifts.

1.5. Procedure for obtaining the optimum solution

Fig. 1 shows the flow chart of the proposed optimization methods. It is very important in any structural analysis and design to select a mathematical model that adequately simulates the response of the structure. The accuracy of assumptions made here determine the accuracy of the model as a true representation of the structure. It is assumed in this study, the loads from traffic and the bridge deck are applied at the nodes. The truss is then analysis as a plane truss instead of space truss. There is also tremendous advantage in the simplification since it is much easier to handle plane trusses compared to space trusses. A viable optimization model is formulated and the results obtained from the analysis are then inputted into the optimization model. The model is enacted with specific interest on the members' stresses and nodal displacements since the constraint is already built in the model. Various cases of the Karush-Kuhn-Tucker conditions (KKT) were considered. After each case was examined, its validity is checked based on the feasible direction and nonnegative criteria of the KKT conditions. The optimum design is reached when all the KKT conditions were satisfied and also, the prevailing criteria are met.

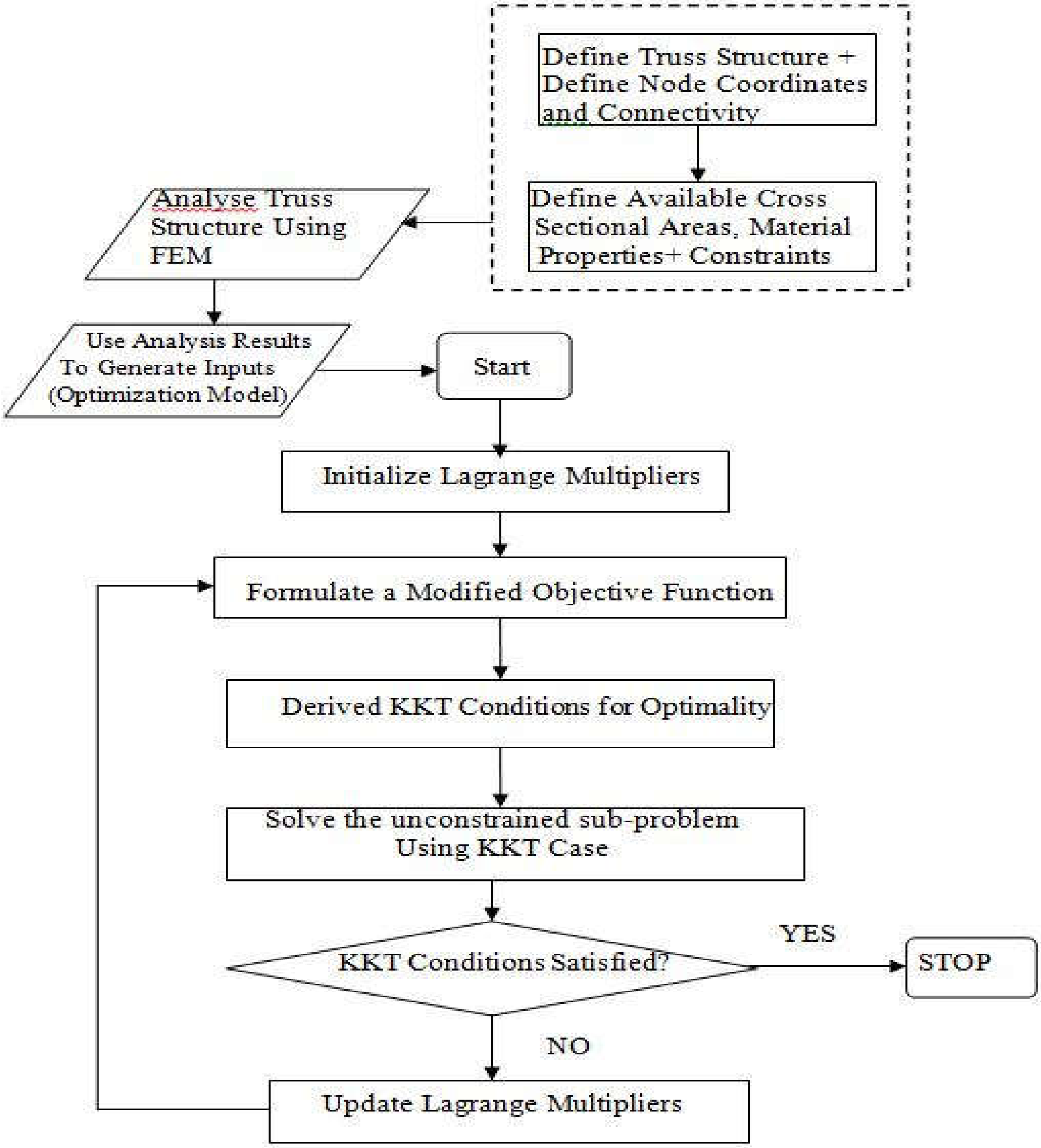

Fig. 1: Proposed Flowchart Optimization Algorithm Using KKT Conditions

2 Case study example

The performance of the proposed method is tested with a typical truss shown in **Fig. 2** and the node/element identification with the magnitude of loads at the nodes as follows:
Load and other fixed parameters:

$$P = 155kN, KEL = 120kN, E = 200kN/mm^2,$$

$$Density, \rho = 7860kg/m^3, Initial\ weight = 15720\ kg$$

It is worthy of note that, P is uniformly distributed load (UDL) converted to act at the nodes KEL is the knife edge load applied at the mid-span of the truss.

The results are as tabulated in Table1.

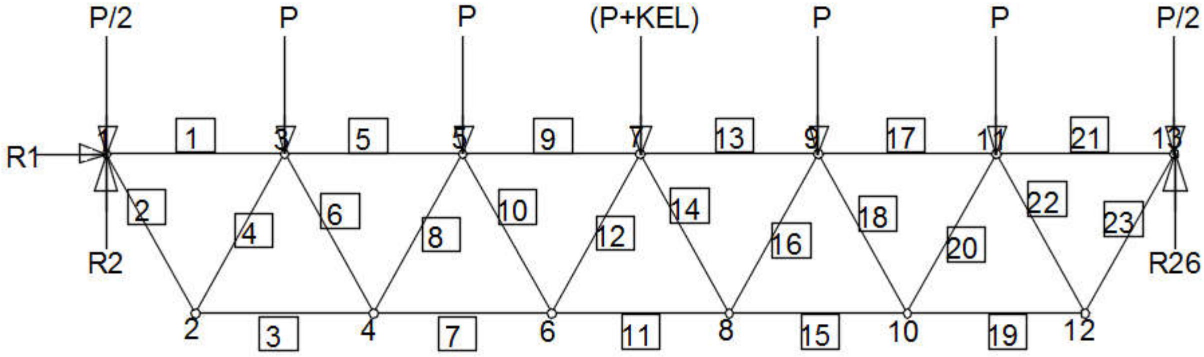

Fig. 2: Geometry of the truss structure with member and node identification

2.1. Size optimization using continuous cross-sectional areas.

From the optimization method proposed a computer programme written in MATLAB programming language as in section 3.16, the areas of truss members were obtained and the result was as tabulated as shown in Table 1. With the areas of individual truss elements known, the volume of the truss was calculated with the aid of equation (1.0) to be $1639119.175mm^3$ and $1739100mm^3$ that is, weight of truss is $12890kg$ and $13676.4kg$ for continuous and discrete variables of area respectively. This is the optimal solution of the bridge truss optimization problem, hence, the values of Lagrange multipliers, $\boldsymbol{\lambda}$ are all positive, that is, 209.339, 3879.245, 46.778 and 567.298 for $\lambda_1, \lambda_2, \lambda_3 and\ \lambda_4$ respectively. This has offered a designer an option to even decide whether the steel truss should be fabricated on not.

Table 1: Result of Optimized Area of Members for continuous and discrete

Member	Length(cm)	Area of continuous Section (cm^2)	Area of discrete section (cm^2)
1	425	225.2296	247
2	425	118.8457	125
3	425	215.8450	222
4	425	118.8457	125
5	425	225.2296	247
6	425	118.8457	125
7	425	215.8450	222
8	425	118.8457	125
9	425	225.2296	247
10	425	118.8457	125
11	425	215.8450	222
12	425	118.8457	125
13	425	225.2296	247
14	425	118.8457	125
15	425	215.8450	222
16	425	118.8457	125
17	425	225.2296	247
18	425	118.8457	125
19	425	215.8450	222
20	425	118.8457	125

21	425	225.2296	247
22	425	118.8457	125
23	425	118.8457	125
Volume (mm^3)		**1639119.175**	**1739100**

2.2 Comparison of the Proposed ALKKT Optimization Method with Previous Works in Literatures.

The performance of the proposed optimization method is tested for the benchmark 10-bar cantilever truss shown in Fig. 3. The results are compared with several other research studies using continuous design variables. The continuous variables were presented by the KKT cases as in-built in the programme coding. The objective function is to minimize total volume or equivalently, the cross-sectional areas of the truss members subject to design constraints. The lower and upper limits of the cross-section areas vary between 0.645 and 64.516cm^2 respectively.

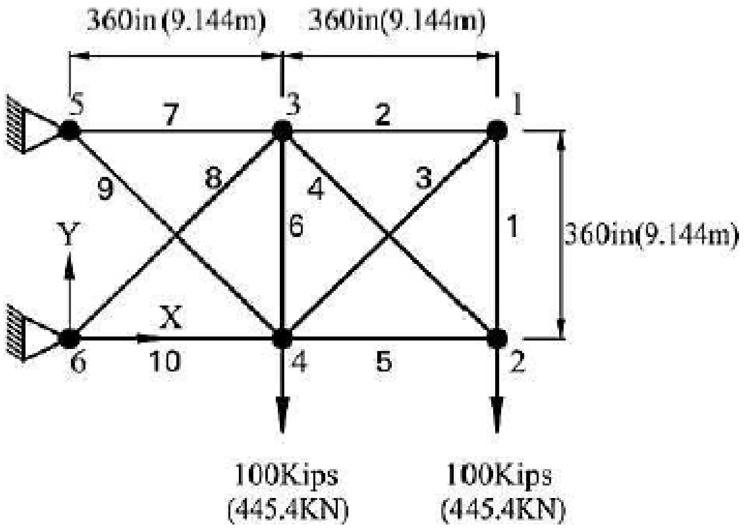

Fig. 3: Geometry of the Benchmark 10-bar Truss

2.2.1 Sizing Optimization Using Continuous Cross-sectional Areas

In this section, size optimization was conducted to determine the optimal cross-sectional area of each element with KKT conditions cases as presented in the programme code. In this study, mild steel was used in the truss members. To take into account the nonlinear constraints applied to the structure, Lagrange Multipliers with slack variables were utilized. Table 2 shows the comparison of size optimization results with those of other research studies. The result of this study for minimum displacement is shown in the present work column. It should be noted that some of the studies, for instance, Romero *et al*. 2004 and Burton (2004), did not consider without due regards to displacement constraints in the optimization process, therefore, obtained a relatively higher deflection.

Table 4.5 Comparison of the continuous size optimization results with other references (maximum deflection=16.688*cm*)

Area	**Auer (2006)**	**Romero *et al.* (2004)**	**Burton (2004)**	**Haftka Gurdal (1982)**	**de Souza and Fonseca (2008)**	**Noii *et al.* (2017)**	**Present work**
A1	0.645	0.645	0.645	0.645	0.645	0.645	0.645
A2	0.645	0.645	0.645	0.645	0.645	1.419	0.645
A3	0.751	0.645	0.903	0.903	0.839	1.710	1.551
A4	35.878	35.928	23.742	23.742	24.903	36.161	37.214
A5	25.370	25.405	52.258	25.161	25.161	31.632	29.572
A6	0.645	0.645	0.645	0.645	0.645	0.645	0.645
A7	51.177	51.212	50.968	50.968	50.968	53.503	37.290
A8	35.878	35.928	35.548	35.548	35.613	36.581	50.968
A9	37.114	37.063	37.419	37.419	37.290	31.110	35.613
A10	52.057	52.013	25.161	52.258	52.193	48.574	52.353
Weight (kg)	722.765	722.647	679.251	679.251	682.783	732.118	741.261
Max. Disp. (cm)	18.288	18.288	22.067	20.574	20.274	17.597	16.688
Max. Stress (MPa)	172.368	172.372	352.463	258.372	246.562	215.582	247.084

4.5.2 Sensitivity Analysis of 10-bar Truss Structure

The response of the optimal area distribution to an adjustment of constraints (perturbation) for the 10-bar truss benchmark is performed. This is achieved by simultaneously increasing the cross-sectional area of members 7 and 9 (lowest and highest cross-sectional area). Figure 4.2 presents the maximum nodal displacement of the truss structure with respect to the cross-sectional area of the member 7 and 9, respectively. It is shown that the gradient of maximum displacement reaches the minimum value for cross-sectional area equal to 56.7cm^2 and 130.00cm^2 in members 7 and 9, respectively. Maximum displacement at this point is 7 cm, which is around 33% above the optimum solutions shown in Table 4.5. The results also indicate that the maximum displacement is more sensitive to the variations in the cross-sectional area of member 7 compared to member 9. Subsequently, maximum displacement corresponding to member 7 reaches the constant value earlier than member 9.

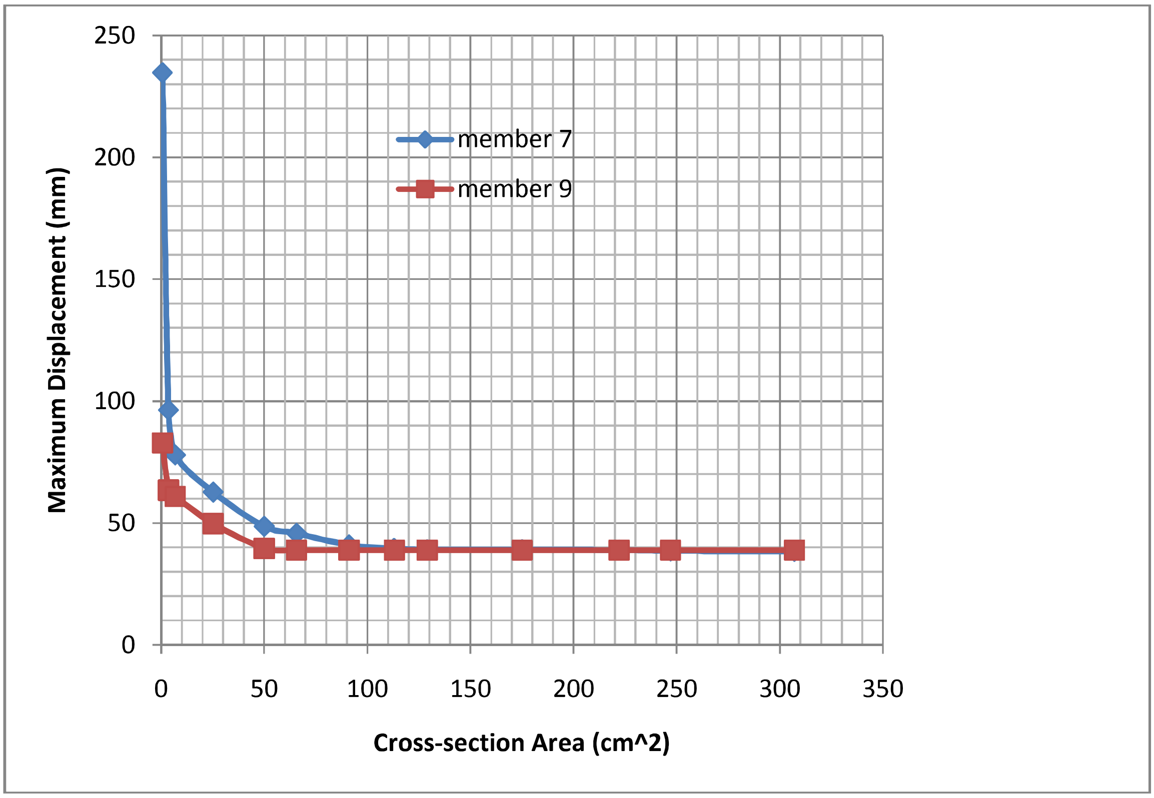

Figure 4.2 Maximum nodal displacement of the truss structure with respect to the cross-sectional area of the member 7 and 9.

Conclusion

This study has developed a computer-aided analysis and optimum design of steel elastic trusses based on the Karush-Kuhn-Tucker (KKT) optimality criteria. The performance of the approach has been demonstrated by optimizing a 10-bar steel truss structure. The numerical example verified the feasibility of the developed algorithm, and indicated that the adopted method can significantly reduce the structural weight/volume and maximum deflection over the conventional design route. It has been shown that the optimal design solution for the warren truss bridge using discrete optimization results in 13% weight reduction compared with the traditional design while it also exhibits 12% and 23% less maximum node displacement and maximum member stress under the design load respectively. Sensitivity analysis was conducted to show the reliability of the proposed optimization method, which should prove useful in optimum design of large-scale truss structure.

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