



MATHEMATICAL MODELLING OF HYBRID RENEWABLE ENERGY SYSTEM

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Abstract

Hybrid renewable energy systems (HRES) are becoming popular for remote area power generation applications due to advances in renewable energy technologies and subsequent rise in prices of petroleum products. Economic aspects of these technologies are sufficiently promising to include them in developing power generation capacity for developing countries." Research and development efforts in solar, wind, and other renewable energy technologies are required to improve their performance, establish techniques for accurately predicting their output and reliably integrating them with other conventional generating sources. "This study summarizes the mathematical modelling of Hybrid renewable energy systems such as solar, wind, and storage devices". Wind and solar system required special technique to obtain maximum power due to their nonlinear power characteristics. "Due to the integration of two or more different source of power, an hybrid system has

a complex control system. Of which its complexity increases with the maximum power point tracking (MPPT) techniques used within their subsystems."We describe

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System,

methodologies to model HRES components, HRES designs and their evaluation". "The trends in HRES design show that the hybrid PV/wind energy systems are gaining popularity.

Introduction

Nowadays the price of fuels is increasing drastically mainly because of scarcity and high demand. Moreover these fuels eject out poisonous gases which is slowly destroying the environment. More attention are now been drawn to the fact that the usage of renewable energy is the way forward than the traditional fossil fuel system. Though people still depends on natural resources for energy to many applications like cooling, heating, electricity, etc. Presently, energy generate from renewable energy sources increases every year". The availability of resources plays a"major factor in producing energy from this sources". "Among all renewable energy sources solar and wind energy are more attractive because of their free cost availability".

However, the common drawback in exploring solar and wind energy can be traced back to the unpredictable weather and climatic changes along with the required load rate". "Decrease in the performance of the energy system, may sharply reduce the lifespan of the batteries. It should be of notes that only an integrated system of these energy sources will keep the cost of the system design low. "Stand-alone solar or wind energy system is insufficient to supply continuous power citing the frequent "seasonal and periodical changes (Yang et al, 2008)". " The drawbacks from single source of power leads people to the thinking of hybrid system. This hybrid system can get enough energy from both sources and even if the energy from one source is low at the point it will be compensated by the other. Hybrid systems has gained popularity in the past, for its application in remote systems such as radio telecommunication and satellite earth stations, or at localities inaccessible to the conventional power grids (Yang et al, 2009; Diafa et al 2008).

Upgrading the current single source systems (Solar, Hydro or Wind) into hybrid systems for grid-connection applications has been a major area of focus (Bakos and Tsagas, 2003)."The feasibility of integrating wind-powered generator and photo-voltaic (PV) array was discussed by Castle et al in 1981."

Due to scarcity and increase in price of fuel because of high demand, and moreover the gases emitted are dangerous to human health and also the environment, there is need for adequate precaution to be taken to make a pollution free surrounding and environmental friendly, it is necessary to find solution to all this problem by using renewable energy as the best alternate solution to the problem. In 2012 the United State National Renewable Energy Laboratory performed a study of a range of RE scenarios leading to 80% of

electricity being provided by renewable by 2050". "This would be a major step towards cutting global CO₂ and other greenhouse gas emissions by 80%"

Hybridization could result in increased reliability, however proper technology selection and generation unit sizing are essential in the design of such system for improved operational performance and dispatch and operational control. Different generation source may also help each other to achieve higher total energy efficiency or improved performance. (Ekren and Ekren, 2009) mentioned that wind and solar energies are mostly available for remote regions as renewable energy sources. However, neither standalone wind energy nor a solar energy system can produce continuous energy supply due to periodical and seasonal changes."

The table below gives a summary of different Renewable Energy power technologies and different energy storage scheme's which may be used in hybrid system.

Main Renewable Energy	Energy storage type
Biomass	Batteries
Hydro	Flywheel
Ocean (tidal/wave)	Hydrogen
Geothermal	Compressed Air
Solar PV/ thermal	Pumped Hydro
Wind	SMES

Any combination of the renewable energy power generation along with proper storage and possibly combined with a conventional generation technology example a diesel generator could form a hybrid energy system. A hybrid system could have any combination of wind, PV, MH, MT, conventional diesel generator, storage battery and Fc electrolyze hydrogen storage in grid-connected or standalone configuration often refer to as micro-grid.

"The outputs from various generation sources of a hybrid energy system need to be coordinated and controlled to realize their full potential/benefit. Proper optimization techniques and control strategies are needed for sizing and for power dispatch from the energy sources to make the entire system sustainable to the maximum extent, while facilitating maximum reduction in environmental emissions, and at the same time minimizing cost of energy production. An analysis of hybrid energy system usability was done by using simulation to optimize the electricity generated by the system (Agustin and Lopez, 2009). It showed that standalone hybrid energy systems are more suitable than only wind or

photovoltaic in terms of higher reliability and lower cost. However optimization and control of the hybrids systems are usually complex due to the higher variable number and non-linearity."

CONFIGURATION OF HYBRID RENEWABLE ENERGY SYSTEM.

Integration schemes

"Renewable energy source have different operating characteristics, it is therefore essential to have a well-defined and standardized frame work for connecting them to form a hybrid system, or more widely micro- grid, where a local cluster of DG source energy storage and loads are integrated together and capable of operating autonomously."

"There are two ways of integrating different Renewable Energy power generation sources to form a hybrid system. The method can be generally classified into three categories

- I. DC- coupled
- II. AC- coupled
- III. Hybrid coupled

- I. DC- coupled: in a DC coupled configuration, different Alternate Energy source are connected to a DC bus through appropriate power electronic (PE) interfacing circuits. The DC source may be connected to the DC bus through appropriate power electrons interfacing circuits. The DC sources may be connected to the DC bus directly if possible. If there are many loads, they can also be connected to the dc bus or directly, through dc converters, to achieve appropriate DC voltage for the DC loads. The system can supply power to the account loads (50 / 60Hz), or be interfaced to a utility grid through an inverter, which can be designed and controlled to allow bidirectional power flows. The DC coupling scheme is simple and no synchronization is needed to Integrate the different energy sources but it also has its own drawback, for instance if the system inverter is out of service then the whole system will not be able to supply AC power, to avoid this situation, it is possible to connect several inverters with lower power rating in parallel, in which case synchronization of the output voltage of the different inverters, or synchronization with the grid, if the system is grid-connected a proper power sharing control scheme is also required to achieve a desired load distribution among the different inverters.

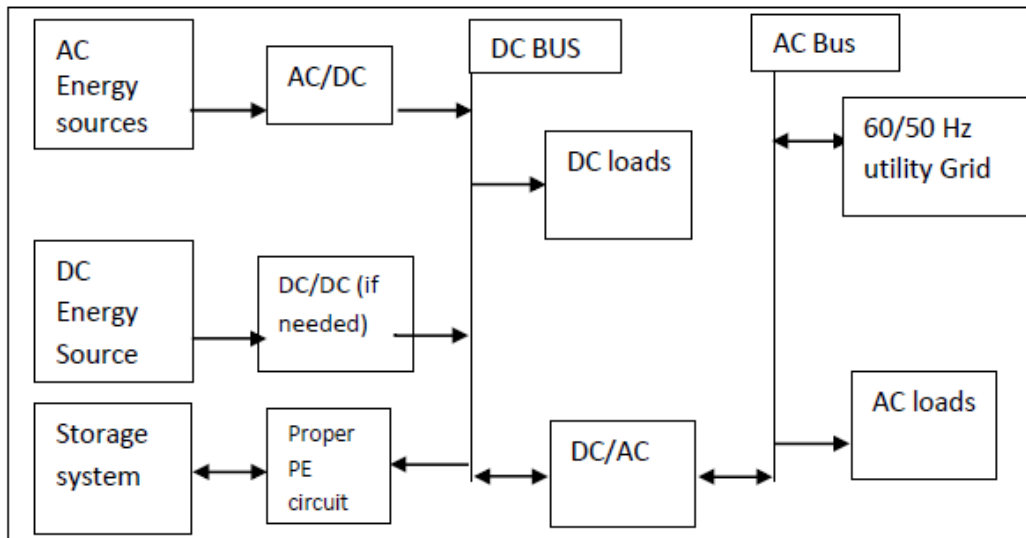


Figure 2.1: Schematic diagram of a dc-coupled hybrid energy system

II. AC- couple systems: Ac coupling can be divided into two subcategories PFAC-coupled and HFAC- coupled systems". The schematic of a PFAC- coupled system where the different energy sources are integrated through their own power electronic interfacing circuits to a power frequency ac bus coupling conductor may be needed between the power electronic circuits and the bus to achieve desired power flow management.

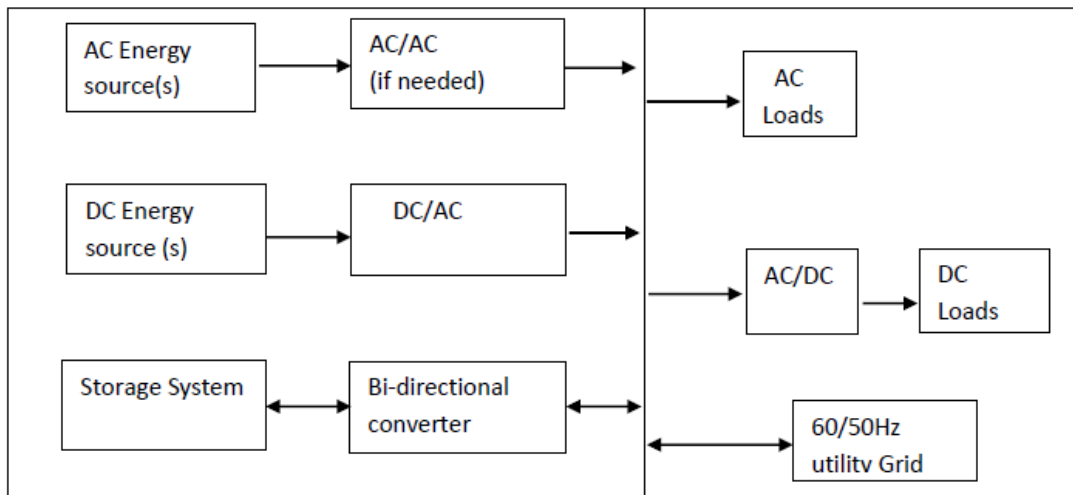


Figure 2.2: Showing schematic of ac-coupled hybrid energy system PFAC

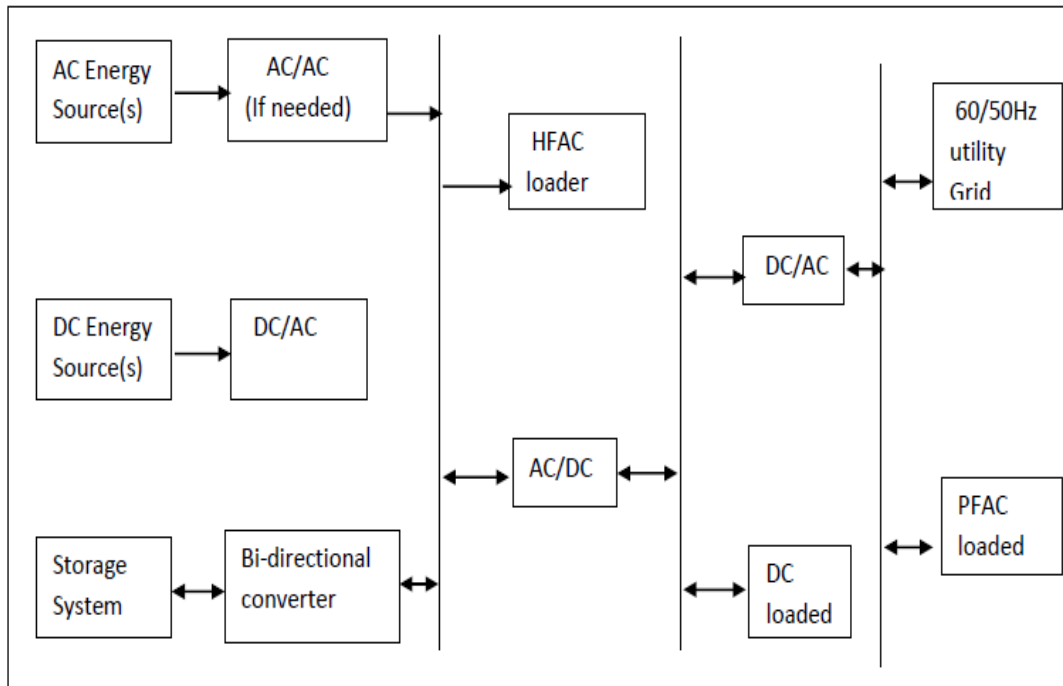


Figure 2.3: showing the Schematic of ac-coupled hybrid energy system of HFAC

II. III. Hybrid-Coupled Systems: "Instead of connecting all the DG sources to just a single dc or ac bus, as discussed previously, the different DG sources can be connected to the dc or ac bus of the hybrid system. Fig.3 shows a hybrid-coupled system, where DG resources are connected to the dc bus and/or ac bus. In this configuration, some energy sources can be integrated directly without extra interfacing circuits. As a result, the system can have higher energy efficiency and reduced cost. On the other hand, control and energy management might be more complicated than for the dc- and ac-coupled schemes. Different coupling schemes find their own appropriate applications. If major generation sources of a hybrid system generate dc power, and there are also substantial amounts of dc loads, then a dc-coupled system may be a good choice. On the other hand, if the main power sources generate ac (with reasonable power quality for the grid and the connected loads), then an ac-coupled system is a good option."

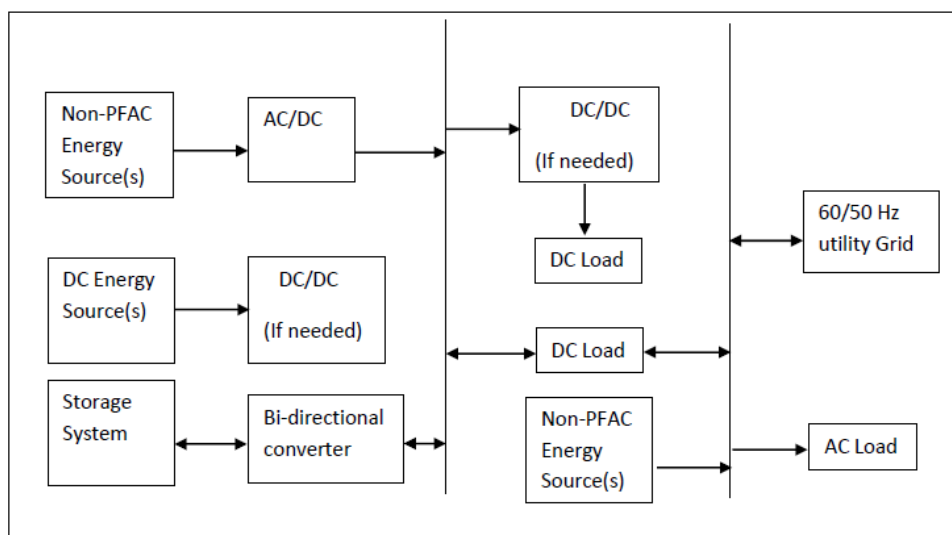


Figure 2.4: Showing the schematic of hybrid coupled energy system

"If the major power sources of a hybrid system generate a mixture of ac and dc power, then a hybrid-coupled integration scheme may be considered. It is worth mentioning that the power electronic interfacing circuits in Figs. 2 and 3 can be made as modular building blocks, which will give the systems more flexibility and scalability."

Unit Sizing and Technology Selection

"Component sizing of hybrid RE/AE systems is important and has been studied extensively. Selection of the most suitable generation technologies (i.e suitable mix of RE/AE/conventional sources) for a particular application is also equally important. Available application software can be used to properly select generation technologies and their sizes for specific applications. For example, with the aid of HOMER software, developed at the National Renewable Energy Laboratory (NREL) , a hybrid RE/AE system can be designed; and with the aid of the Distributed Energy Resource-Customer Adaption Model (DER-CAM) software, developed at Lawrence Berkeley National Laboratory(LBL), optimal technology selection for hybrid systems (to operate as independent micro-grids) can be achieved". "Unit sizing and technology selection can sometimes be as straightforward as meeting certain simple requirements such as using the available generation technology and not exceeding the equipment power rating, or it can be as complex as satisfying several constraints and achieving several

objectives to maximum extent at the same time. Normally, based on available statistical information about generation, load, financial parameters (e.g., interest rate), geographic factors, desired system reliability, cost requirements, and other case-specific information, generation technologies and their sizes can be optimized to satisfy specific objective functions, such as minimizing environmental impact, installation and operating costs, payback periods on investment, and/or maximizing reliability". "Power system optimization methods such as linear programming (LP), interior-point-method (IPM), and heuristic methods such as genetic algorithms and particle swarm optimization (PSO) can be used for component sizing and energy management of hybrid RE/AE systems". These techniques are especially attractive when multiple objectives are to be met, some of which may be conflicting, e.g., minimizing cost, maximizing system availability and efficiency, and minimizing carbon emission.

Storage

1) Storage Diversity: "Storage technology is critical for ensuring high levels of power quality and energy management of stationary hybrid RE systems. The ideal storage technology would offer fast access to power whenever needed, provide high capacity of energy, have a long life expectancy, and is available at a competitive cost. However, there is no energy storage technology currently available that can meet all these desirable characteristics simultaneously. In this section, the different types of energy storage devices and systems are covered without going into the details of operation of any specific device. The operational performance and applications of energy storage devices for advanced power applications (also, equally suited for hybrid RE power generation system applications)".

2) Storage Types: "In analogy to data storage in computer engineering, a classification in terms of access and capacity orientation may also be considered for energy storage. Among the different types of storage given in Table I, super capacitors, flywheels, and SMES offer fast access to the stored energy, have a very high cycle life of charge and discharge operations, and very high round-trip efficiency on the order of 95%. However, the cost per unit of stored energy is also very high. Therefore, all three technologies can be classify as access oriented and support power quality. The usage of SMES can here only be economically justified for applications involving comparatively high levels of power. Batteries could also be classified as high-power and/or high-energy types depending on their design".

"However, in general, their cycle life of charge/discharge is shorter than the high-access energy storage devices explained above. A promising capacity-oriented energy storage technology is the flow battery. In conventional batteries, chemical energy is stored in reactants, placed near the electrodes inside the battery cell, but in flow batteries, chemical energy is stored in the electrolyte solutions stored in two tanks outside the battery cell stacks. As the solution is pumped to circulate from one storage tank, through a cell stack, to the second tank, ion exchange takes place through the cell porous membrane, and electrons flow through the load to generate electrical power. Several different flow battery chemistries have been developed for MW/MWh-level utility applications. The available electrolyte chemistries include zinc-bromine flow batteries (ZBFB) and vanadium Redox Battery (VRB). Other chemistries are under development. An advantage of flow batteries is that their power and energy capacity can be designed independently. A battery power rating can be increased by increasing the cell area where energy conversion takes place, i.e., by increasing the number of cell stacks, while its energy capacity can be increased by using larger volume of electrolyte solutions in larger tanks. Furthermore, flow batteries can be stored and shipped completely discharged as the reaction only takes place when the electrolyte circulation pumps are turned ON. Conventional lead-acid batteries are the least expensive for hybrid energy system applications, but they suffer from a low cycle life. Nickel metal hydride (Ni-MH) batteries and those with sodium sulfur (NaS) chemistry offer significant improvements over lead acid batteries. Popular commercial applications for Ni-MH batteries have included usage in hybrid electric vehicles (HEVs) and distributed RE systems. NaS batteries have been used in Japan in distributed energy systems and to firm up wind energy in the grid on a large scale, up to 34 MW of power and 245 MWh of energy. The operating temperature of NaS batteries is of the order of 300 C to 350 C, which does not make them attractive for mobile applications". "This is in contrast with zinc–bromine batteries that operate near ambient temperature".

Hybrid Centralized and Distributed Control Paradigm

"A more practical scheme, hybrid control paradigm, combines centralized and distributed control schemes. The distributed energy sources are grouped within a micro-grid; centralized control is used within each group, while distributed control is applied to a set of groups. With such a hybrid energy management scheme, local optimization is achieved via centralized control within each group, while global

coordination among the different groups is achieved through distributed control. This way, the computational burden of each controller is reduced, and single-point failure problems are mitigated".

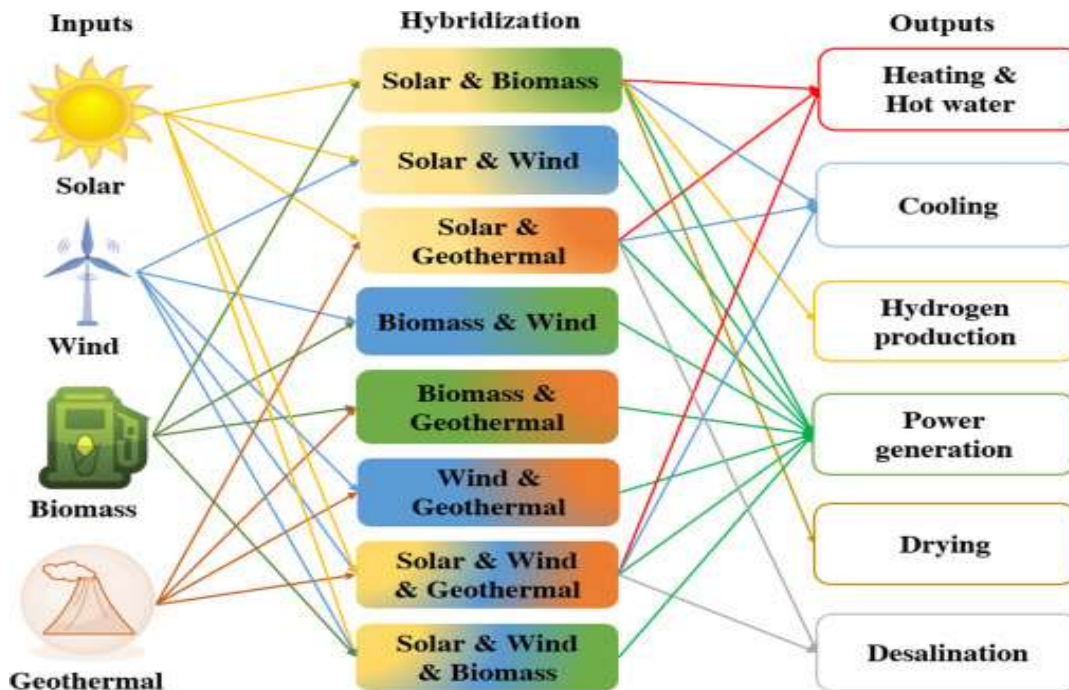


Fig. 2.5. Route toward HRE utilization

RESEARCH METHODOLOGY

A hybrid energy system may consist of various renewable energy conversion components such as wind turbines, PV arrays and hydro turbines, as well as conventional non renewable generators such as diesel generators, micro turbines and storage device like battery". A hybrid energy system could have all these components or only a part of them. In order to make the correct selection of components and subsystems for optimal sizing of the entire system, the first step is modelling the individual components. Very good mathematical models have been derived by many researchers to model hydro turbine/generator, solar cells, wind turbine/generator, diesel engine, battery storage banks, fuel cells. The performance of the individual component is either modelled by deterministic or probabilistic approaches". "The purpose of each model or approach is to look for an accurate mathematical model that more closely represents the final model. When an approximate mathematical model is available it is easier to test the

system for suitability without spending money prior to the fabrication. The modelling process allows identification and better understanding the characteristics of the components and it provides support in decision making. The detail modelling reflects by correct performance prediction, but the design of the perfect model is too complex or extremely time consuming. A sufficiently appropriate model should be between complexity and accuracy. From mathematical model, the next step is to develop the computer simulation model to test the system outputs under various input conditions, as well as to test the system for stability. These computer simulations can be used for further analysis or to change various components or to redesign the system for optimization before fabricating the final prototype model. A general methodology for mathematical modelling of a small hydro-solar-wind power system is described below".

A. PV System

Power output of a PV array is based on solar irradiance and ambient temperature. The power output is calculated as

$$P_{pv} = \eta_{pv} A_{pv} G_t \quad (3.1)$$

Where η_{pv} is PV generation efficiency, A_{pv} is PV generator area (m^2), and G_t is solar irradiation in tilted module plane (W/m^2). Further, η_{pv} is defined as

$$\eta_{pv} = \eta_r \eta_{pc} \left[1 - \beta (T_c - T_{cref}) \right] \quad (3.2)$$

Where η_r is the reference module efficiency, η_{pc} is power conditioning efficiency which is equals to one when MPPT is used, and β is temperature coefficient ((0.004-0.006) per $^{\circ}C$), and T_{cref} is reference cell temperature in $^{\circ}C$. The cell temperature (T_c) can be obtained by relation

$$T_c = T_a + \left(\frac{NOCT - 20}{800} \right) G_t \quad (3.3)$$

Where T_a is ambient temperature in °C, NOCT is nominal operating cell temperature in °C, and G_t is solar irradiation in tilted module plane (W/m^2). The total radiation in the solar cell considering normal and diffuse solar radiation can be estimated as

$$I_r = I_b R_b + I_d R_d + (I_b + I_d) R_r \quad (3.4)$$

B. System Modeling

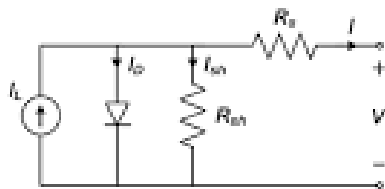


Fig. 3.1 Single diode PV cell model

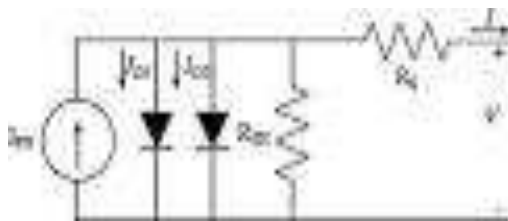


Fig. 3.2 Double diode PV model

Small solar cell is the basic building blocks of PV array system. PV cells are interconnected in series-parallel configuration to form a PV array. An ideal solar cell can be considered as a current source wherein the current produce is proportional to the solar irradiation intensity. Using ideal single diode (Fig. 3.1) for

an array with N_s series connected cells and N_p parallel connected cells, the array current may be related to the array voltage as

$$I = N_p \left[I_{ph} - I_{rs} \exp \left(\frac{q(V + IR)s}{AKTNs} - 1 \right) \right] \quad (3.5)$$

Where q is the electron charge ($1.6 \times 10^{-19} \text{C}$), K is Boltzmann's constant, A is the diode ideality factor, T is the cell temperature (K), I_{rs} is the cell reverse saturation current at T ,

$$I_{rs} = I_{rr} \left(\frac{T}{T_r} \right)^3 \exp \left[\frac{E_G}{A_K} \left(\frac{1}{T_r} - \frac{1}{T} \right) \right] \quad (3.6)$$

Where T_r is the cell referred temperature, I_{rr} is the reverse saturation current at T_r , E_G is the band gap energy of the semiconductor used in the cell. The photo current I_{ph} varies with the cell's temperature and radiation as follows

$$I_{ph} = I_{SGR} + K_i (T - T_r) \frac{S}{100} \quad (3.7)$$

Where I_{SGR} is cell short circuits current at reference temperature and radiation, K_i is the short circuit current temperature coefficient and S is the solar radiation in (mw/cm^2). Of which there are two models that are generally used to model solar cell: A **single diode circuit model**(Fig.3.1), and a **double diode circuit model** (Fig.3.2).

Single diode circuit model uses an additional shunt resistance in parallel to ideal shunt diode model. I-V characteristic of PV cell using one diode model can be derived as

$$I = I_{ph} - I_D \quad (3.8)$$

$$I = I_{ph} - I_0 \left[\exp \left(\frac{q(V + R_s I)}{AKT} - 1 \right) - \frac{V + R_s I}{R_{sh}} \right] \quad (3.9)$$

where I_{ph} is photo current (A), I_D is the diode current (A), I_0 is the inverse saturation current (A), A is the diode constant, q is the charge of the electron ($1.6 \times 10^{-19}C$), K is Boltzmann's constant, T is the cell temperature ($^{\circ}C$), R_s is the series resistance (Ohm), R_{sh} is the shunt resistance (Ohm), I is the cell current (A), and V is cell voltage (V). I-V characteristic of PV cell using two diode circuit models can be described as

$$I = I_{pv} - I_{D1} - I_{D2} - \left(\frac{V + IR_s}{R_{SH}} \right)$$

(3.10)

Where
$$I_{D1} = I_{01} \left[\exp \left(\frac{V + IR_s}{a_1 V_{r1}} \right) - 1 \right]$$
 (3.11)

$$I_{D2} = I_{02} \left[\exp \left(\frac{V + IR_s}{a_2 V_{r2}} \right) - 1 \right]$$
 (3.12)

I_{01} and I_{02} are reverse saturation current of diode 1 and diode 2, respectively, V_{r1} and V_{r2} are thermal voltage of respective diode, a_1 and a_2 represent the diode ideality constants. The maximum power output of a single PV module is given by

$$v_{oc} = \frac{V_{oc}}{cKT/q}$$
 (3.13)

$$p_{max} = \frac{v_{pc}}{ckt/q} - 1 \left(\frac{v_{oc}}{ckt/q} + 0.72 \right) \left(1 - \frac{R_s}{V_{oc}/I_{sc}} \right) \left(\frac{V_{oco}}{1 + \beta \ln \frac{G_o}{G}} \right) \left(\frac{T_o}{T} \right)^{\gamma} I_{sco} \left(\frac{G}{G_o} \right)^{\alpha}$$
 (3.14)

where v_{oc} is normalized value of the open-circuit voltage V_{oc} with respect to the thermal voltage $V_t = nkT/q$, n is the ideality factor ($1 < n < 2$), K is the Boltzmann constant, T is the PV module temperature (K), q is the electron charge, α is the factor responsible for all the non-linear effects that the photocurrent depends on, β is a PV module technology specific-related dimensionless coefficient, and γ is the factor considering all the non-linear temperature-voltage effects.

Equation (14) represents the maximum power output of a single PV module. A real system consists of the number of PV modules connected in series and parallel. The

total power output for an array with N_s series connected cells and N_p parallel-connected cells with P_M power of each module will be

$$P_{array} = N_s N_p P_M \quad (3.15)$$

3.2 Wind Power

The fundamental equation governing the mechanical power of the wind turbine is given by

(3.16)

$$P_w = \frac{1}{2} C_p(\lambda, \beta) \rho A V^3$$

Where C_p is power coefficient, ρ is air density (kg/m^3), A is intercepting area of the rotor blades (m^2), v is average wind speed (m/s), λ is tip speed ratio. The theoretical maximum value of the power coefficient C_p is 0.593, also known as Betz's coefficient. The Tip Speed Ratio (TSR) for wind turbine is defined as the ratio of rotational speed of the tip of a blade to the wind velocity, i.e.

$$\lambda = \frac{R\omega}{V} \quad (3.17)$$

Where R is the radius of the turbine (m), ω is the angular speed (rad/s), V is the average wind speed (m/s).

The energy generated by wind can be obtained by

$$Q_w = P \times (\text{Time}) [\text{kWh}] \quad (3.18)$$

Sometimes because of various factors the velocity of wind at any particular height cannot be obtained by direct measurement. In that case the data at any reference height can be interpolated or extrapolated to find the wind speed at any particular height. The wind velocity is measured at a lower height can be error prone due to vegetation, shading and obstacles in the vicinity.

$$v(z) \ln\left(\frac{z_r}{z_0}\right) = v(z_r) \ln\left(\frac{z}{z_0}\right) \quad (3.19)$$

Where z_r is the reference height (m), z is the height where the wind speed is to be determined, z_0 is the measure of surface roughness (0.1-0.25 for crop land), $v(z)$ is

the wind speed at height z (m/s), and $v(z_r)$ is the wind speed at reference height z (m/s).

$$P_w(v) = \begin{cases} \frac{v^k - v_C^k}{v_R^k - v_C^k} \cdot P_R & v_C \leq v \leq v_R \\ P_R & v_R \leq v \leq v_F \\ 0 & v \leq v_C; v \geq v_F \end{cases} \quad (3.20)$$

Where P_R is rated power, v_C is cut-in wind speed, v_R is rated wind speed, v_F is rated cut-out speed, and k is the Weibull shape factor. Kalantar and Mousavi used the value of k as 1, while (Daif et al 2008). and Yang et al (2009). used k as 2, Chedid et al (2000). took the value as 3. Cut-in speed is a very low wind speed at which the turbine first starts to rotate and generate power. Cut-out speed is the high wind speed at which the forces on the turbine structure are high as a result there is a risk of the damage to the rotor. To prevent damage, braking system is employed to bring the rotor to stand-still. Rated output speed is the wind speed between cut-in speed and cut-out speed where the power output reaches the maximum limit that the electrical generator is capable of and is called rated power output".

Drive Train Modeling

"Drive train transfers high aerodynamic torque at rotor to low speed shaft of generator through gearbox. Some generators are directly coupled with the rotor to reduce complexity so they do not need modeling of this part".

"Drive train can be modeled using one mass model and two mass model. Shah et al (2015). Developed mathematical model based on the torsional multi-body dynamic model".

$$\begin{bmatrix} \omega_r \\ \omega_g \\ T_{ls} \end{bmatrix} = \begin{pmatrix} \frac{-k_r}{j_r} & 0 & \frac{-1}{j_r} \\ 0 & \frac{-k_g}{j_g} & \frac{1}{n_g j_g} \\ \left(B_{ls} - \frac{k_{ls} k_r}{j_r} \right) & \frac{1}{ng} \left(\frac{k_{ls} k_r}{j_g} - B_{ls} \right) & -k_{ls} \frac{j_r + n_g^2 j_g}{n_g^2 j_g j_r} \end{pmatrix} \begin{bmatrix} \omega_r \\ \omega_g \\ T_{ls} \end{bmatrix} + \begin{bmatrix} \frac{1}{j_r} \\ 0 \\ \frac{k_{ls}}{j_r} \end{bmatrix} T_m + \begin{bmatrix} 0 \\ \frac{1}{j_g} \\ \frac{k_{ls}}{n_g j_g} \end{bmatrix} \quad (3.21)$$

One Mass Model

If a perfectly rigid low-speed shaft is assumed, a single mass model (as shown below) of the turbine can be considered

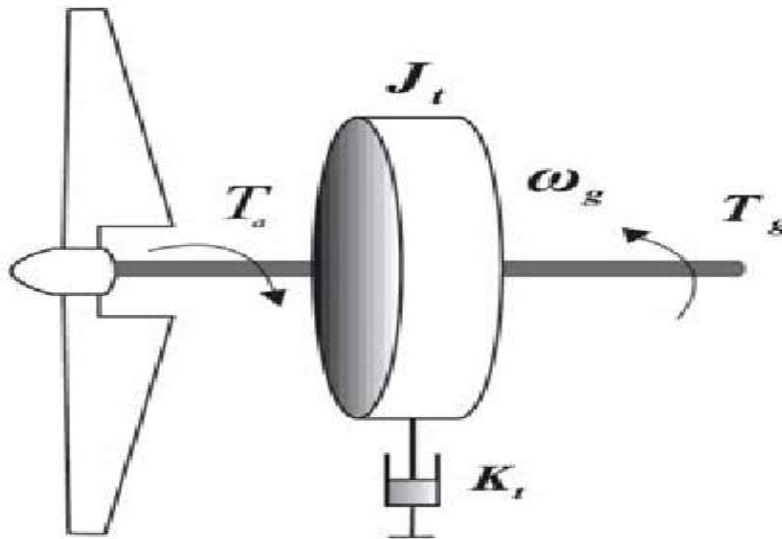


Fig. 3.5 One mass model of wind turbine system.

With equation given as

$$J_t \dot{\omega}_g = T_a - K_t \omega_g - T_g \quad (3.22)$$

And

$$\begin{aligned} J_t &= J_r + n_g^2 J_g \\ K_t &= K_r + n_g^2 K_g \\ T_g &= n_g T_{em} \end{aligned} \quad (3.23)$$

Where J_t is the turbine rotor moment of inertia in (kgm^2), ω_t is low shaft angular speed in (rad sec^2), K_t is the turbine damping coefficient in ($\text{Nm rad}^{-1}\text{sec}^{-1}$) representing aerodynamic resistance, and K_g is generator damping coefficient in ($\text{Nm rad}^{-1}\text{sec}^{-1}$) representing mechanical friction and windage.

Two Mass Model

The schematic of two mass wind turbine systems is shown in Fig. 6. Below

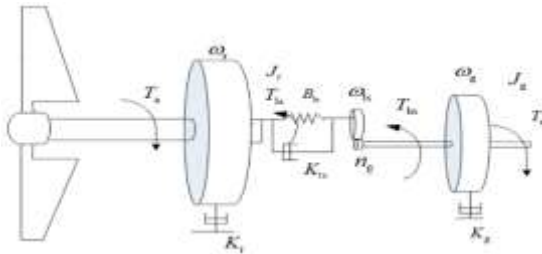


Fig. 3.6 Two mass model of wind turbine system.

Bati and Brennam (2002), presented modeling of wind turbine of the rotor side inertia J_r is given by

$$J_r \frac{d\omega_r}{dt} = T_m - T_{ls} - K_r \omega_r \quad (3.24)$$

The low speed shaft torque is given by

$$T = B_{ls}(\theta_r - \theta_{ls}) + K_{ls}(\omega_r - \omega_{ls}) \quad (3.25)$$

The generator inertia J_g is driven by the high speed shaft and braked by electromagnetic torque T_g of the generator

$$J_g \frac{d\omega_g}{dt} = T_{hs} - K_g \omega_g - T_g \quad (3.26)$$

If we assume the ideal gearbox with ratio n , then

$$n = \frac{T_{ls}}{T_{hs}} = \frac{\omega_g}{\omega_r} = \frac{\theta_g}{\theta_{ls}} \quad (3.27)$$

In which the notations are the same as those of one mass model. K_{ls} is the low-speed shaft damping coefficient in $(\text{Nm rad}^{-1}\text{sec}^{-1})$, ω_g is high-speed shaft angular

speed in (rad sec⁻²), T_m is turbine torque in (Nm), T_{ls} is low-speed shaft torque in (Nm), J_g is the generator rotor moment of inertia in (kgm²) and T_{hs} is high speed shaft torque in (Nm).

After eliminating T_{ls} time derivative from equation (3.25) and using equation (3.24) and (3.26), following dynamical system is derived.

$$\frac{dT_{ls}}{dt} = \left(B_{ls} - \frac{K_{ls}K_t}{J_t} \right) \omega_t + \frac{1}{n} \left(\frac{K_{ls}K_t}{J_g} - B_{ls} \right) \omega_g - K_{ls} \left(\frac{J_t + n^2 J_g}{n^2 J_r J_g} \right) T_{ls} + \frac{K_{ls}}{J_t} T_\alpha + \frac{K_{ls}}{n J_g} T_g$$

(3.28)

Where

$$K_{ls} = \frac{IG}{L_{ls}}$$

$$D_{ls} = \xi D_s$$

$$\xi = \sqrt{1 - \left(\frac{\omega}{\omega_n} \right)^2}$$

(3.29)

$$D_s = 2\sqrt{K_{ls}m}$$

(3.30)

And ω/ω_n is ratio of shaft frequency of oscillation to the un-damped natural frequency of shaft, m is mass of shaft, I is second momentum of area about the axis of rotation, L_{ls} is shaft length, G is modulus of rigidity, D_s is critical damping of shaft, and ξ is damping ratio of shaft

The output power and torque of turbine (T_t) in terms of rotational speed can be obtained by substituting (3.17) in (3.16).

$$\rho_w = \frac{1}{2} \rho A C_p (\lambda, \beta) \left(\frac{R \omega_{opt}}{\lambda_{opt}} \right)^3 \quad (3.31)$$

$$T_t = \frac{1}{2} \rho A C_p (\lambda, \beta) \left(\frac{R}{\lambda_{opt}} \right)^3 \omega_{opt} \quad (3.32)$$

Efficiency of the wind turbine is

$$\eta = \frac{P_{output}}{P_{input}} \times 100 \quad (3.33)$$

Where η is efficiency (%)

P_{output} is the Power output in Watt

P_{input} is the Power input in watt

$$\text{The total power Hybrid} = (N_w \times P_{Generator}) + (N_s \times P_s) \quad (3.34)$$

N_w is the number of wind turbine

$P_{Generator}$ is the power generated by the wind turbine

N_s is the number of solar panel

P_s is the power generated by solar panels

Then the efficiency of the hybrid renewable energy is given by

$$\eta_{Hybrid} = \frac{P_{Hybrid}}{(E_{ST}^{Sw} \times A_c) + P_w} \times 100\% \quad (3.35)$$

P_{Hybrid} is the power generated by the hybrid system

A_c is the area of collector (m^2)

P_w is the power of the wind

Both solar panel and the wind turbine are arranged in two different methods in (Fig.3.7), the wind turbine is connected to the controller with only one solar panel. Therefore the wire with positive polarity and the wire with negative polarity that

came from both systems should be connected to the specific location on the controller, where they should finally be connected to the battery.

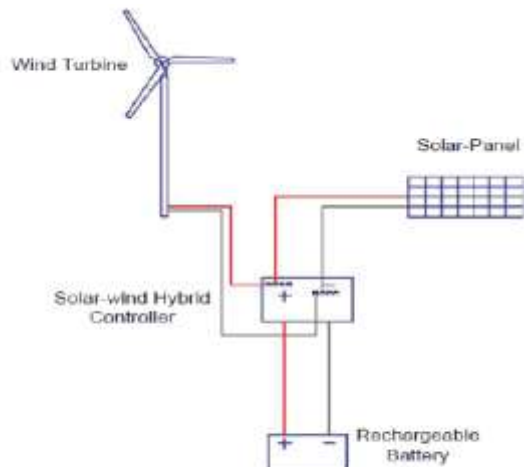


Figure 3.7: Hybrid system- big solar panel with turbine

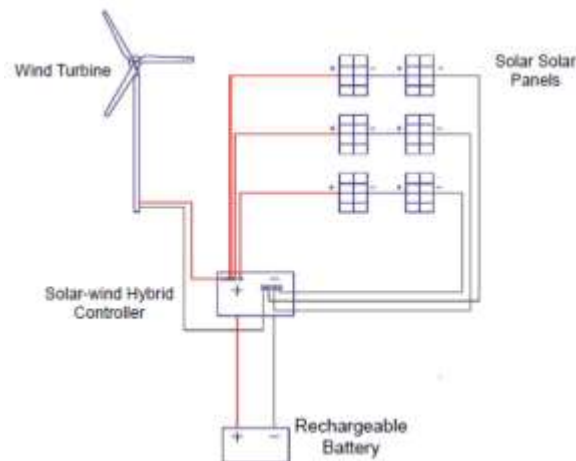


Figure 3.8: Hybrid system- small size solar panels with wind turbine

RESULTS AND DISCUSSIONS

Analysis of Result

"The power output from the big solar panel and the three blade horizontal axis of the wind turbine are 30W and 21.9W respectively, all components in both systems are being placed in one device which is the hybrid controller. In this experiment work, it took a day with this procedure repeated four times to get a new five sets of data where the average is being calculated. This is to make sure that no big

changes in the data as a result of changing of the weather condition and the system themselves".

	Date	Weather
Individual system Testing	Day one	Partial cloudy
	Day two	Sunny
	Day three	Rainy
	Day four	Rainy
	Day five	Cloudy
Hybrid System Testing	Day six	Sunny
	Day seven	Sunny
	Day Eight	Sunny

Table 4.1: Shows the readings of weather conditions during testing of models of hybrid system.

Time	Large solar Panel Volts (V)	Current (A)	Big Solar Panel
10:00am	14.01	1.55	21.71
10:30am	13.78	1.62	22.19
11:00am	13.49	1.81	24.35
11:30am	12.97	1.49	19.42
12:00pm	13.66	1.58	21.72
12:30pm	13.91	1.59	22.19
1:00pm	13.25	1.55	20.45
1:30pm	12.65	1.76	22.34
2:00pm	12.33	1.37	16.78
2:30pm	12.45	1.48	18.45
3:00pm	12.89	1.56	20.12
3:30pm	13.35	1.61	21.12
	13.22833	1.580833	20.90333

Table 4.2: shows the data for each individual renewable energy system. The duration of the testing was 5 days.

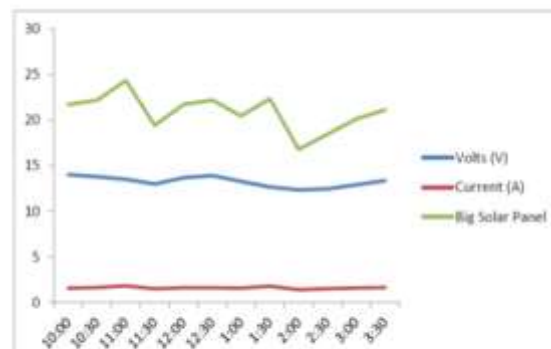


Figure 4.1: Showing the power output and efficiency of the three renewable energy systems tested individually. Where the power output is measure as a function of time for the three individual.

"In the testing day a suitable month was used, the weather was very sunny with passing clouds. The average temperature was 30°C and the average wind speed was 2.53m/s. As shown in table 2. The maximum wind turbine power output was 3.9W, furthermore there are times where the wind turbine, has no power output due to the absence of wind. However the solar panel had a continuous output through the whole testing times in a day. The large solar panels minimize power recorded as 16.9W and the maximum was 24.3W. For the small connected solar panel, the minimum power output was 9.1W and its maximum was 11.99W".

"From the experiment it can be seen that the wind turbine did not produce a continuous supply of energy on every testing day. The power output of the wind is zero in the absence of wind turbine, however the big solar panel resulted to have more power than the small solar panels. This is due to the type of connection of the small panel together which affect the voltage and current produce from the panel(s).

After all consideration using the individual systems as the baseline, the hybrid system is tested. The first hybrid system was the wind turbine connected to six small size solar panels"

Time	Wind Speed (m/s)	Volts (v)
10:00Am	3.61	12.90
10:30Am	3.66	13.02
11:00Am	0.50	13.55
11.30Am	4.00	13.42
12:00Pm	4.14	14.00
12:30Pm	7.78	13.72
1:00Pm	7.91	13.40
1:30Pm	6.09	14.62
2:00Pm	0.00	14.63
2:30Pn	5.50	14.00
3:00Pm	7.70	13.91
3:30Pm	7.80	12.62
Average	4.89	13.60
Time	Wind Speed (m/s)	Volts (v)
10:00Am	3.61	12.90
10:30Am	3.66	13.02
11:00Am	0.50	13.55
11.3Am	4.00	13.42
12:00Pm	4.14	14.00
12:30Pm	7.78	13.72
1:00Pm	7.91	13.40
1:30Pm	6.09	14.62
2:00Pm	0.00	14.63

Table 4.3 Show the results of second hybrid system that consist of the wind turbine and six small size solar panels under the same weather

Time	Wind Speed (m/s)	Volts (v)
10:00Am	2.62	14.00
10:30Am	3.15	14.78
11:00Am	2.10	13.90
11.30Am	0.50	13.72
12:00Pm	6.11	13.00
12:30Pm	7.02	13.20
01:00Pm	6.00	12.53
1:30Pm	5.20	12.53
2:00Pm	4.02	13.00
2:30Pm	6.11	14.30
3:00Pm	6.10	13.99
3:30Pm	6.20	14.70
Average	4.51	13.76
Time	Wind Speed (m/s)	Volts (v)
10:00Am	2.62	14.00
10:30Am	3.15	14.78
11:00Am	2.10	13.90
11.30Am	0.50	13.72
12:00Pm	6.11	13.00
12:30Pm	7.02	13.20
1:00Pm	6.00	12.53
1:30Pm	5.20	12.53
2:00	4.02	13.00
2:30	6.11	14.30
3:00	0.00	13.99
3:30	6.20	14.70

Table 4.4: The power output and efficiency of hybrid renewable energy system result, from connecting the wind turbines to the one large

Table 4: The power output and efficiency of hybrid renewable energy system results from connecting the wind turbine to the six small size solar panel, It is

compare with the three individual systems. The results of comparison are shown below.

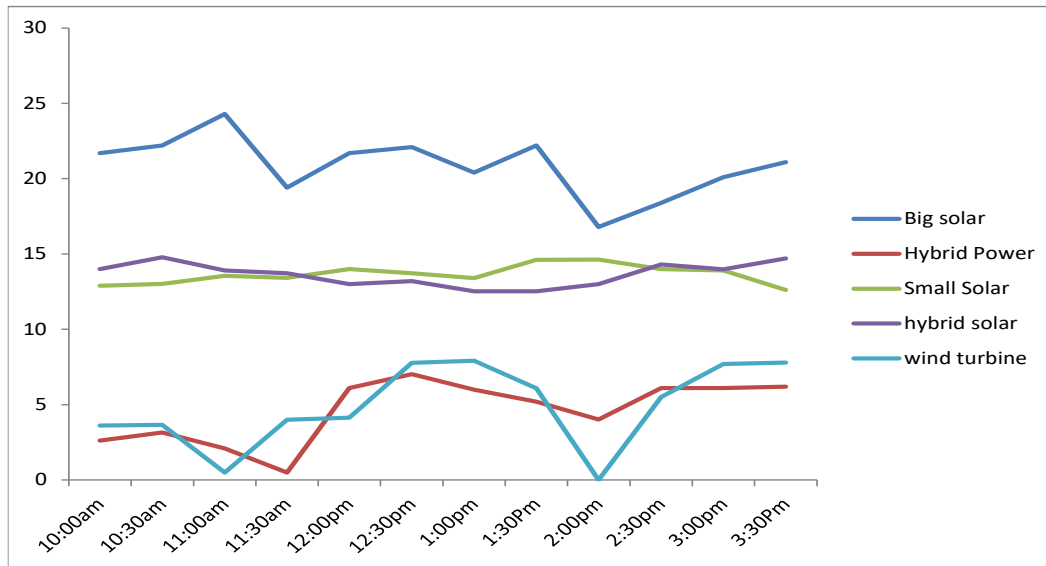


Figure 4.2: The power output and efficiency of hybrid renewable energy system results from connecting the wind turbine to the six small size solar panel, It is compare with the three individual systems.

"From the above we depict that the main Aim of a renewable hybrid system is that, the system should be able to provide continuous supplies of power in any weather condition". Firstly, if the individual testing of energy system of the big solar panel, the small solar panels and the wind turbine and the individual systems are tested in 6 days with a duration of five hours and a half from 10:00 AM until 3:30 PM. The readings are recorded after 30mins of testing the system individually, of which the systems have been tested again by combining the solar panel(s) to the wind turbine to create the hybrid system. Two types of hybrid systems are made, one is the wind turbine with the big solar panel (Hybrid System 1), and the other is the wind turbine with the 6 small solar panels connected together (Hybrid System 2). Moreover, the testing duration is the same as the testing duration of the individual systems which is from 10:00 AM until 3:30 PM (five and a half hours). This will show the comparison between having an individual system and a hybrid system. The testing started with the big solar panel with the wind turbine. The weather was sunny and clear at 31°C with an average wind speed of 4.89 m/s. The power

output of the hybrid system ranged from 19 W to 46 W, and the average is 39.11 W. The power output increased compared to the individual systems.

"Overall, the power output comparison can be seen in graphs shown in (Fig. 4.2). The hybrid system is compared to the individual systems with the most similar weather conditions. It shows that the power output increased drastically when having a hybrid system. However, the big solar panel gives higher power output than the small solar panels. Moreover, the power output is continuous in the hybrid system unlike the individual system, where the power is zero for the wind turbine when there is no wind. In the hybrid system, when there is no wind the power decreases only. It does not reach zero like a single system due on having another source of input power.

"Decision matrix is one of the most important management tools which is used to defend a decision based on the alternatives factors that can affect the decision such as cost, availability, quality, installation, durability and efficiency of the selected idea. This decision matrix shows that the comparison is between three listed solar panels that are consisting of Polycrystalline, Monocrystalline and Thin film"

Conclusion and Recommendations

"The aim of this research was to present suitable mathematical modelling on hybrid solar and wind energy systems using the literary works focusing on improving the HRES design and experimentally investigate the performance of a hybrid system that utilized both solar and wind energy as renewable sources. The advantage of maximizing the power output and keep a continuous source of power supply were proved. The experimental study started by studying different scenarios in connecting the two sources together. In all the scenarios, the power output was measured by the output voltage and ammeter during six days. Before testing the hybrid system, individual investigation for big solar panel, small solar panels and the wind turbine were done. The results during the six days in different weather conditions showed that a continuous and significant amount of power is hardly achieved using wind turbine only. However, the comparison was between the big solar panel and the small ones. Where the small solar panels had an equivalent area to the large solar panel, results showed better performance of the big solar panel among the smaller size solar panels".

"Finally, the data and results were recorded for hybrid system in two different cases, case one with its by using six solar panels and the other one is the big solar panel separately with the wind turbine. It showed that the power output increased with the both hybrid systems in general. However, the big solar panel gives higher power output than the small solar panels when connected to the wind turbine. In addition, the power output is continuous in the hybrid system in comparison with the wind turbine individually, and in hybrid system the power output was higher in comparison with the individual solar systems".

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