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MATHEMATICAL MODELLING OF PROCESS PARAMETERS OF A TYPICAL TANNERY EFFLUENT

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AUTHORS' CONTRIBUTIONS
This work was carried out in collaboration between both authors. Author OSA designed the study, wrote the protocol and interpreted the data. Author IMO anchored the field study, gathered the initial data and performed preliminary data analysis. While authors OSA and IMO managed the literature searches and produced the initial draft. Both authors read and approved the final manuscript.

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ABSTRACT

This study was aimed at developing mathematical models for the process parameters of a typical tannery effluent. Three model equations were developed using the empirical method of the least squares. A close agreement was observed between the simulated values and experimental values. The mean deviation of 2.17% was obtained for rainy season, 1.81% for harmattan and 2.26% for dry season. A correlation coefficient of 0.999, 1.000 and 1.000 were obtained for rainy season, harmattan and dry season respectively. A t-value of 1.581 was calculated for the rainy season, 0.011 for harmattan and 0.202 for dry season with corresponding p-values of 0.175, 0.992 and 0.848 respectively. The empirical models developed using the method of least squares can represent the process parameters of a typical tannery effluent and can be used to investigate the effect on the environment. And the pH of the effluent is within the limit for surface discharge.

Keywords: pH; mathematical modelling; process parameters; t-test.

1. INTRODUCTION

Industrial pollution is one of the major problems faced by many developed and developing countries in the world and there have been several attempts to control it. The effluents that industries generate are sources of pollution. Contaminated air, soil, and water by effluents from industries are associated with disease burden and this could be reasons for the current shorter life expectancy [1,2]. Heavy metals in industrial effluent have been found to be carcinogenic, while others are poisonous depending on the dose and exposure period [3,1]. These chemicals are poisonous to man and aquatic life resulting in food contamination [4,2].

Tanning is the process in which raw animal hides are converted into leather. During this process, the leather is made resistant to biological decay by stabilizing the collagen structure of the hide, using natural or synthetic chemicals [5].

The concentration of Sulphate in effluents is of environmental concern [1]. This is because it could lead to a poor air quality of an environment. The same is applicable to pH if water available for human use is not of the required quality [6].

A tannery effluent refers to the wastewater gotten from the process of transforming hides and skins into leather. The process of tanning involves a large

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volume of water, which is used in cleaning the hides and skins, or to serve as a medium of interaction between the hides and skin [7].

During the process tanning, large volume of waste water is discharged into the surrounding soil as well as water sources. These effluents may consist of several chemicals used during the tanning process such as sodium Sulphate, chromium Sulphate, non-ionic wetting agents and may accumulate in the immediate environment.

Furthermore, when these effluents are not properly managed and disposed, many pathogenic microorganisms in the effluents may prompt the inhabitants to severe health hazards [8].

Water plays a very important role in human activities, aquatic life and even, other living creatures. Thus, the development of mathematical model to evaluate the effect of process parameters of tannery effluent is necessary. This will help to know their effects on the environment. A mathematical model of a process is just the representation of the mathematical aspects of that process. Since constant monitoring of the effluents' discharge from the tannery is very necessary, modelling, which serves as the tool of control, becomes important in this study. Modelling could reduce the time and material wastage involved in carrying out experimental work. Modelling and simulation can be carried out using some software packages, like Microsoft Excel, Math LAB, MathCAD, SPSS, and so on. In this study, a mathematical simulation was performed using Excel and MathCAD. The aim of this research work is to develop a mathematical model of the process parameters of a typical tannery effluent to know their possible effects on the environment

Previously, [9] presented a model for the prediction of carbonate in a brewery effluent using the method of least square. The percentage average deviation of carbonate between the experimental and the simulated values was 9%. The research shows that the effect of Sodium was significant while the effect of temperature is not significant in the brewery effluent [9].

Similarly, [10] investigated the effect of effluent emanating from Kaduna refinery on river Kaduna. The effect of pH in the refinery effluent was studied using the model equations developed from the empirical method of the least squares. In their work, a model of pH as dependent variable against other parameters was presented [10].

The present research investigates the effect of pH as a function of other process parameters in the effluent of a tannery industry. In a similar manner to [9,10], the method of least square was adopted to study the effect of pH on the process parameters in the tannery effluent.

A comparison between simulated results and the experimental data was also made to know the deviation of the simulated data from the real data.

2. METHODOLOGY

Modelling of typical tannery effluent using pH as the optimization criterion is presented in this study. A model developed using the acidity or alkalinity (pH) of a solution was reported in the work of [11].

In this study, pH was modelled using the empirical method of least square of the form [10,12,11].

Hence PH is a function of temperature, TDS, BOD, COD, SO_4^{2-}, Cl^- and DO

Modelling Using Empirical Method of Least Square

pH of water is a reflection of temperature (T), Total Dissolved Solid TDS, Biological Oxygen demand (BOD), Chemical Oxygen demand (COD), Sulphate (SO_4^{2-}), $Chloride\ (Cl^-)\ and\ Dissolved\ Oxygen\ (DO)$

Hence

$$PH = f(T, TDS, BOD, COD, SO_4^{2-}, Cl^-, DO) \text{-- (1)}$$

Where Temperature =T, TDS =T_1, BOD = B, COD = C, SO_4^{2-}= S, $Cl^- = C_1$, DO =D

By introducing coefficients i.e constants, equation 1 becomes

$$pH = f(a.T + b.T_1 + c.B + d.C + e.S + f.C_1 + g.D) \quad (2)$$

pH is the dependent variable, the letters a, b, c, d, e, f & g are the coefficients needed to be determined, and T, T_1, B, C, S, C_1 and D are the independent variables of the equation

Let the square of error between the observed pH and the predicted pH be I

$$\text{Hence } I = [P - pH]^2 \quad (3)$$

Where P is the observed pH and pH is the predicted pH

Now for the experimental data

$$I = [P - pH]^2$$

$$I = [P - (a.T + b.T_1 + c.B + d.C + e.S + f.C_1 + g.D)]^2 \quad (4)$$

"n" experiments carried out so for n experimental values of P and pH

$$nI = \sum \left[P_i - \begin{pmatrix} a.T_i + b.T_{1i} + c.B_i + d.C_i \\ +e.S_i + f.C_{1i} + g.D_i \end{pmatrix} \right]^2 \quad (5)$$

The error of this regression could be minimized by finding the derivative of nI with respect to the coefficients a, b, c, d, e, f and g, and then equating them to zero

$$\frac{\partial(nI)}{\partial a} = -2\sum T_i\,[P_i - (a.T_i + b.T_{1i} + c.B_i + d.Ci + e.Si + f.C1i + g.Di = 0 \quad (6)$$

$$\frac{\partial(nI)}{\partial b} = -2\sum T_{1i}\,[P_i - (a.T_i + b.T_{1i} + c.B_i + d.Ci + e.Si + f.C1i + g.Di = 0 \quad (7)$$

$$\frac{\partial(nI)}{\partial c} = -2\sum B_i\,[P_i - (a.T_i + b.T_{1i} + c.B_i + d.Ci + e.Si + f.C1i + g.Di0 \quad (8)$$

$$\frac{\partial(nI)}{\partial d} = -2\sum C_i\,[P_i - (a.T_i + b.T_{1i} + c.B_i + d.Ci + e.Si + f.C1i + g.Di = 0 \quad (9)$$

$$\frac{\partial(nI)}{\partial e} = -2\sum S_i\,[P_i - (a.T_i + b.T_{1i} + c.B_i + d.C_i + e.S_i + f.C_{1i} + g.D_i)] = 0 \quad (10)$$

$$\frac{\partial(nI)}{\partial f} = -2\sum C_{1i}\,[P_i - (a.T_i + b.T_{1i} + c.B_i + d.C_i + e.S_i + f.C_{1i} + g.D_i)] = 0 \quad (11)$$

$$\frac{\partial(nI)}{\partial g} = -2\sum D_i\,[P_i - (a.T_i + b.T_{1i} + c.B_i + d.Ci + e.Si + f.C1i + g.Di = 0 \quad (12)$$

Using Microsoft Excel to generate different values for the coefficients and then substituting we have:

$$pH = 0.154T - 0.001852T_1 + 0.03B - 0.0005803C - 0.0004041S - 0.012C_1 - 0.221D \quad (13)$$

$$pH = 0.345T + 0.002286T_1 + 0.014B - 0.0008247C - 0.052S + 0.047C_1 + 0.542D \quad (14)$$

$$pH = 0.075455T + 0.012113T_1 - 0.01481B - 0.00018C + 0.033036S - 0.07065C_1 - 0.00000000056D\ldots\ldots \quad (15)$$

Equations 13, 14 and 15 were modelled for rainy season, harmattan and dry seasons.

3. RESULTS AND DISCUSSION

The experimental data and their corresponding simulated results are given in Tables 1 to 6 below.

Table 1. Experimental analysis on effluent sample for rainy season

Run	PH	Temperature (°C)	TDS (mg/l)	BOD (mg/l)	COD (mg/l)	$SO4^{2-}$ (mg/l)	Cl^- (mg/l)	DO (mg/l)
1	7.11	27.1	1063	594.67	2412	834.22	314.12	34.42
2	7.65	28	1085	590.32	2302	839.12	295.38	33.13
3	7.42	28.9	1096	601	2511	837.33	299.45	35.4
4	7.6	29	997	597.2	2443	832.14	320.47	34
5	7.59	27.7	1126	622.1	2527	830	332.11	34.63
6	7.34	29.2	1017	585.57	2402	835.15	315	33.95

Table 2. Simulated result for rainy season

Run	PH	Temperature (°C)	TDS (mg/l)	BOD (mg/l)	COD (mg/l)	$SO4^{2-}$ (mg/l)	Cl^- (mg/l)	DO (mg/l)
1	7.11	27.1	1063	594.67	2412	834.22	314.12	34.42
2	7.65	28	1085	590.32	2302	839.12	295.38	33.13
3	7.4199	28.9	1096	601	2511	837.33	299.45	35.4
4	7.6	29	997	597.2	2443	832.14	320.47	34
5	7.59	27.7	1126	622.1	2527	830	332.11	34.63
6	7.399	29.2	1017	585.57	2402	835.15	315	33.95

Table 3. Experimental analysis on effluent sample for harmattan

Run	PH	T (°C)	TDS (mg/l)	BOD (mg/l)	COD (mg/l)	SO_4^{2-} (mg/l)	Cl^- (mg/l)	DO (mg/l)
1	7.32	28.7	996	585.67	2422	838.23	311	33.12
2	7.3	26	1005	593.13	2463	835.75	315.23	34.03
3	7.45	28.3	999	580.25	2394	837.23	309.1	33.75
4	7.72	27.6	1101	599	2399	839.95	326.71	32.54
5	7.54	27	1002	584.63	2497	840	330.05	33.25
6	7.63	29.3	987	583.12	2513	838.33	321.16	32.67

Table 4. Simulation result for harmattan

Run	PH	T (°C)	TDS (mg/l)	BOD (mg/l)	COD (mg/l)	SO_4^{2-} (mg/l)	Cl^- (mg/l)	DO (mg/l)
1	7.3248	28.7	996	585.67	2422	838.23	311	33.12
2	7.302	26	1005	593.13	2463	835.75	315.23	34.03
3	7.4455	28.3	999	580.25	2394	837.23	309.1	33.75
4	7.7193	27.6	1101	599	2399	839.95	326.71	32.54
5	7.5424	27	1002	584.63	2497	840	330.05	33.25
6	7.6259	29.3	987	583.12	2513	838.33	321.16	32.67

Table 5. Experimental analysis on effluent sample dry season

Run	PH	T (°C)	TDS (mg/l)	BOD (mg/l)	COD (mg/l)	SO_4^{2-} (mg/l)	Cl^- (mg/l)	DO (mg/l)
1	7.44	29.3	775	590.47	2428	841.12	322.21	32.23
2	7.47	28.6	797	579.43	2457	845.23	331.54	31.17
3	7.88	27.9	767	582.15	2363	849.55	320.05	33.15
4	7.52	29	782	594.32	5203	839.27	314.26	34.2
5	7.67	28.3	821	597.67	2475	843.14	326.17	30.56
6	7.92	27.3	794	602.14	2438	847.16	321.17	31.28

Table 6. Simulated result for dry season

Run	PH	T (°C)	TDS (mg/l)	BOD (mg/l)	COD (mg/l)	SO_4^{2-} (mg/l)	Cl^- (mg/l)	DO (mg/l)
1	7.4406	29.3	775	590.47	2428	841.12	322.21	32.23
2	7.4702	28.6	797	579.43	2457	845.23	331.54	31.17
3	7.8795	27.9	767	582.15	2363	849.55	320.05	33.15
4	7.5199	29	782	594.32	5203	839.27	314.26	34.2
5	7.6696	28.3	821	597.67	2475	843.14	326.17	30.56
6	7.92	27.3	794	602.14	2438	847.16	321.17	31.28

4. DISCUSSION

The simulated values from the modelled equations show little deviation from the experimental value as there was a mean deviation of 0.1617 between the experimental and simulated values for rainy season which is just 2.17%. For Harmattan, the deviation is 0.1359 which is 1.18% and 0.17306 (2.26%) for dry season. This is an improvement on the method adopted by [9] which was reported to have a percentage minimum deviation of 9%.

The constituent parameter with the greatest influence on the simulated model was found to be dissolved oxygen (DO) for the rainy season and Harmattan with coefficients of 0.2210 and 0.5420 respectively. Temperature has the greatest effect during the dry season with a coefficient of 0.0755. The parameter with the least effect during rainy season was SO_4^{2-} with a coefficient of approximately 0.0004. COD has the least during Harmattan with an approximate coefficient of 0.0008 and dissolved oxygen in the dry season with a coefficient that can be approximated to zero.

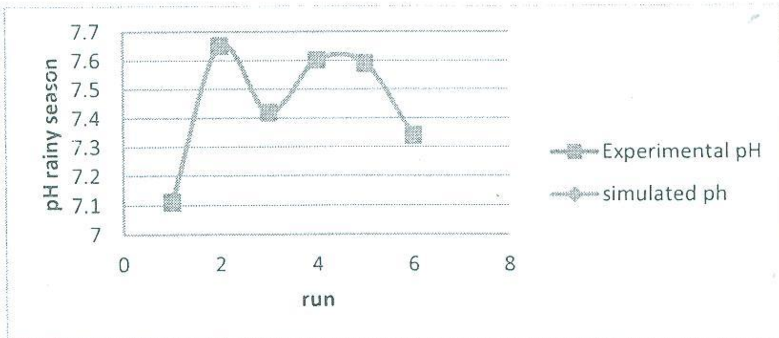

Fig. 1. Plot of experimental and simulated pH against run for rainy season

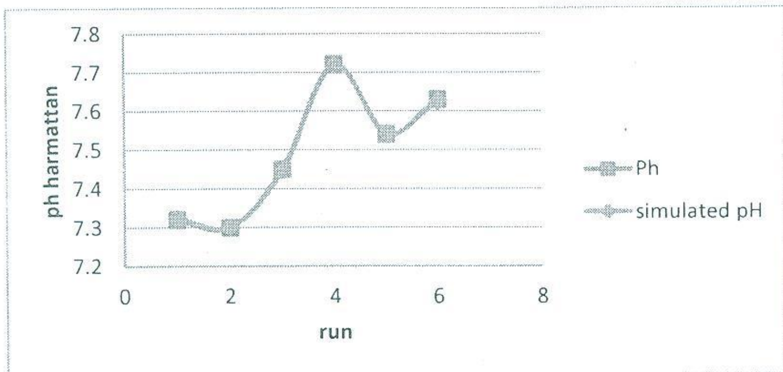

Fig. 2. Plot of experimental and simulated pH against run for harmattan

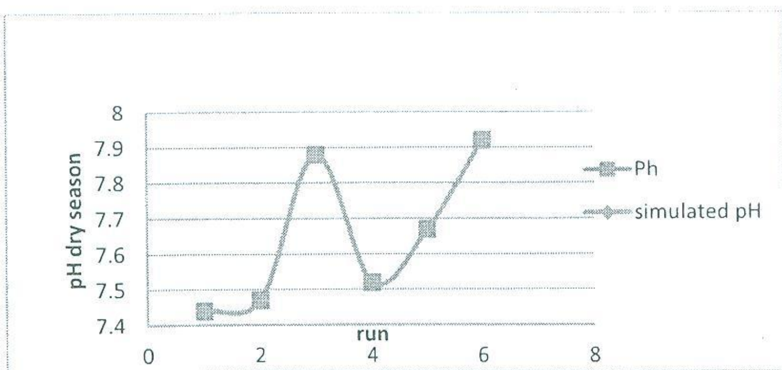

Fig. 3. Plot of experimental and simulated pH against run for dry season

A t-value of 1.581 was calculated for the rainy season, 0.011 for Harmattan and 0.202 for dry season. These gave corresponding p-values of 0.1805, 0.989 and 0.848 respectively.

A significant difference is observed between the two values if the t value is greater than or equal to the critical t-value. A "critical t-value" is the minimum t-value needed to have $p < 0.05$. The critical t-value here (degree of freedom of 5) is 2.571.

Therefore the simulated pH values shows no significant difference from the experimental pH values since the t-values for rainy season, Harmattan and dry season are less than the critical t-value.

In addition, the p-values for the three seasons also show insignificant differences between the simulated pH values and the experimental pH values since the values are all greater than 0.05. In Figs. 1 to 3, the plot of pH plotted against number of runs in the experiments show no steady increase or decrease of pH versus runs. This indicates variations of volume and components of effluents discharge from time to time for the case under investigation.

Furthermore, the correlation coefficients of 0.999, 1.000 and 1.000 for rainy season, Harmattan and dry season show that the empirical equations developed are in close agreement and correlates perfectly with the experimental values.

Charts of the experimental values and simulated values were plotted against the number of runs and curves showing very close relationships were obtained

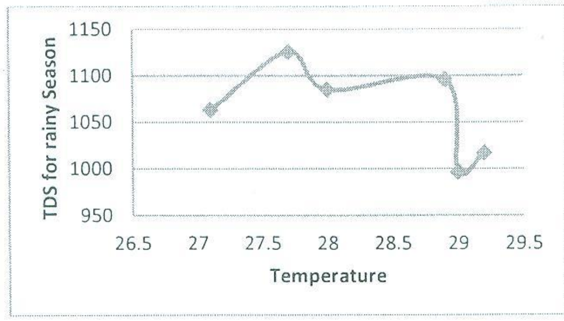

Fig. 4. TDS against temperature for rainy season

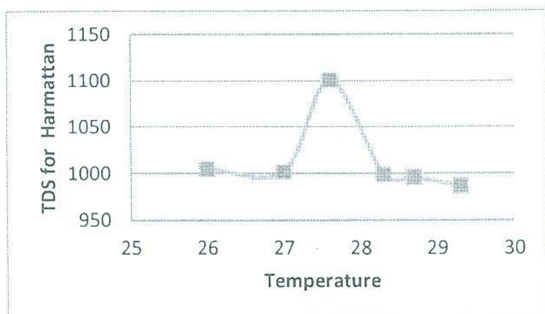

Fig. 5. TDS against temperature for harmattan

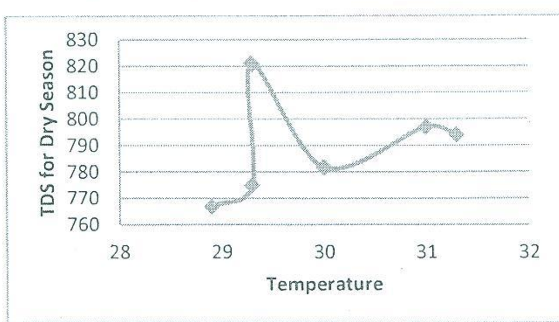

Fig. 6. TDS against temperature for dry season

from the plots. These charts and values obtained from the mean deviations show that the mathematical models developed represent the behaviour of the process parameter in the tannery effluent.

The graphs of TDS against Temperatures as shown in Figures 4 to 6 show that TDS of the effluent sample decreases with temperatures in some cases and increases with temperatures in some cases but it is not uniform in the graph since the volume of effluent discharges varies on daily basis.

5. CONCLUSION

The mean deviations of 0.1617, 0.135883 and 0.17306 for rainy season, Harmattan and dry season respectively, show that the empirical equations developed are in agreement with the experimental results. Hence it can be concluded that the model equations developed using the empirical method of the least squares can be used to investigate the effect of process parameters on a typical tannery effluent.

6. RECOMMENDATION

It is recommended that further study involving other process parameters from different tanneries should be carried out and then compared with this study. Other methods of modelling should be applied as well for comparison.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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