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Local Geoid Modelling of Lagos Island Area Using the Geometrical Interpolation Method

P. C. Nwilo*, Y. D. Opaluwa , Q. A. Adejare** ,**
E. G. Ayodele* and A.M. Ayeni*.

Abstract

The geoid as an equipotential surface which coincides on the average with the mean sea level has significant relevance in geodesy, surveying and other earth-related disciplines. It finds applications in geographic information systems (GIS), engineering, the transformation of ellipsoidal heights of points to the orthometric heights, etc. This paper focuses on the geoid modelling technique based on the geometrical interpolation approach by fitting a surface which depends on the reference points that are chosen in the critical and characteristic locations of the field to represent the trend of the geoid surface. Using the orthometric heights and the ellipsoidal heights, empirical geoidal undulations for all the points were computed. A multiple regression model was formulated as the required geometrical model to further adjust the derived geoid undulations from observation. Using a surface interpolation (kriging) approach, the coordinate and the computed geoidal heights of some well-selected points were utilised in Surfer8 for gridding. This was used as a model for generating the geoidal heights of any other arbitrary points whose coordinates are known. From the analysis, it was observed that the use of the lower-order polynomial (regression) to further model the geoid surface gave the mean square errors of 0.36cm and 5.58cm in self-and cross-validations respectively, with a smoothened geoid terrain, while the fundamental equation that relates the trio (orthometric, ellipsoidal and geoidal heights) gave the mean square errors of 7.35cm and 187.831cm in self-and cross-validations respectively. The modelled surface was generated at contour intervals of 0.5m for the two equations, while the digital terrain models were also generated in both cases.

Keywords: *Geoid modelling, Global Positioning System, Levelling, Geometrical Interpolation, Undulation, Equipotential, Orthometric Height, Ellipsoidal Height.*

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1.0 Introduction

One of the basic goals of geodesy is the determination of the geoid which is the equipotential surface of the earth gravity field and which coincides, on the average, with the mean sea level (Vanicerk and Krakiwsky, 1986). According to C. F. Gauss, geoid is the "mathematical figure of the earth" and the gravity field (Kiamehr, 2006). Furthermore, the geoid surface is considerably smoother than the physical surface of the earth but more irregular than the ellipsoid of revolution.

Therefore, geoid, as an equipotential surface of the earth's gravity field, has its conceptual importance in geodetic applications, because it is the datum upon which most height systems [orthometric height] are based. Also, all terrestrial measuring techniques are actually oriented relative to the geoid. The advent of satellite-based positioning techniques, especially Global Positioning System (GPS), which is currently used in a wide range of geodetic and surveying applications, has brought tremendous changes in the processes of precise geodetic control establishment: Data acquisition techniques have become more efficient, accuracies greatly improved with new areas of application opened up, orthometric heights can thus be acquired indirectly through geodetic heights from GPS if the geoid over the area is known (Moka and Agajelu, 2006). Since the ellipsoidal heights from GPS are basically geometric in nature and, therefore, do not reflect the direction of flow under the influence of gravity, heights from GPS are of little or no direct meaning in engineering construction and geodetic applications.

To utilise the opportunities provided by this technique, the need for the transformation between ellipsoidal heights and orthometric heights is very important. Using GPS technique, the positions are determined with reference to geocentric WGS84 (World Geodetic System, 1984) reference ellipsoid. This surface is the datum of ellipsoidal heights which are derived on any other ellipsoid such as the Minna datum. The orthometric heights are determined with reference to the geoid. There is therefore the need for accurate geoid model for transforming the geometrical (ellipsoidal) heights from GPS to the highly needed orthometric heights. Unfortunately, the geoid for Nigeria has not been accurately determined. Also because of uneven distribution as well as insufficient availability of gravity data in Nigeria, gravimetric geoid for Nigeria will be weak in some parts of the country. Therefore, the national / regional geoid model may not satisfy the accuracy which is necessary for most of the routine geodetic applications (Opaluwa, 2008). Local geoid model is advocated as an interim measure. This paper attempts a geoid modelling technique based on the geometrical interpolation approach by fitting a surface that depends on the reference points that are chosen in the critical and characteristic locations of the field to represent the trend of the geoid surface. This, according to Erol and Celik (2004), is the common method in small areas for local studies like the area under investigation.

1.1 The Figure of the Earth

The figure of the earth in geodesy refers to those surfaces used as an approximation to the physical shape/size of the earth for the purpose of computational convenience. Surfaces of computations are the referenced surfaces used in surveying, to which

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observations/ measurements are referred. Basically, there are 3 types of these surfaces which are used for the representation of the earth in surveying. They are the plane, ellipsoid and geoid. The surface that corresponds to the plane is the physical terrain of the earths surface. The terrain is a topographical surface characterised by hills and valleys. It is an irregular surface; thus it cannot be used for exact mathematical computations. In geodesy, the plane is not used as a computation surface due to the curvature of the earth.

An ellipsoid of revolution is a solid figure generated by rotating an ellipse about its minor axis. In its simplest form, an ellipsoid is a smooth elliptical model of the earth's surface. The geometrical figure used in geodesy, which most nearly approximates the shape of the earth, is an ellipsoid of revolution; thus it can be represented mathematically and analytically to be the closest to the earth. The ellipsoid is the computation surface used in geodesy for horizontal control network.

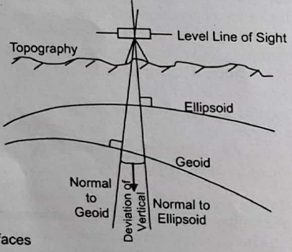

Fig 1.: Reference Surfaces

The geoid is the equipotential surface of the earth's gravity field which best fits, in the least squares sense, the mean sea level (Deakin, 1996). It has a definite physical interpretation, in the sense that it can be fixed by measurements over the ocean. The geoid is the figure that represents the actual shape of the earth, but it is not used as a computation surface because it cannot be represented mathematically. At every point, the geoid surface is perpendicular to the local plumb line and it is therefore a natural reference surface for height measured along the plumb line [that is the reference datum for orthometric height].

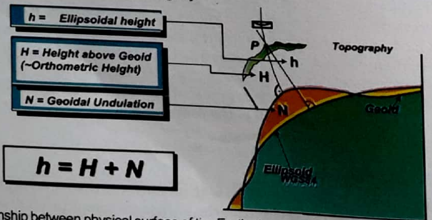

Fig. 2: Relationship between physical surface of the Earth, the geoid, and the ellipsoid **(Opaluwa, 2008)**

1.2 The Objective of Study

The geoid is known if the separation between it and the reference ellipsoid is known. The separation between the geoid and the reference ellipsoid is the geoid height (*N*). The fundamental relationship between ellipsoidal heights (h) obtained from GPS measurements and orthometric heights (H) with respect to a vertical geodetic datum established from spirit-levelling data with gravimetric corrections (referred to the geoid), is given by Heiskanen and Moritz (1967) and Moka and Agajelu (2006) as:

$$h \quad H \quad N = 0 \qquad (1.1)$$

Thus, $N = h - H$ (1.2)

Where, H = Orthometric height measured along the curved plumb line;
h = Ellipsoidal height measured along the ellipsoidal normal; and
N = Geoid undulation/Geoid height.

Fig. 2 shows the relationship between the geoid undulation, ellipsoidal height [h], orthometric height [H] and the vertical deflection.

The main focus of this research is on modelling the geoid of a local area using the geometrical interpolation approach to serve practical geodetic applications such as large- scale map production, GIS (Geographical Information Systems)-based studies, engineering applications, etc.

1.3 The Study Area

The study area is Lagos Island Local Government Area, in Lagos State, southwestern Nigeria. It covers an area of 8.66 sq. kilometres with a perimeter of about 11.52 kilometres Lagos Island Local Government Area can be described as a mixed settlement; because the area serves as a commercial centre and also as a residential area. The areas that constitute the Local Government include: Adeniji Adele, Obalende, CMS, Marina district, etc (Dauda, 2007). Fig. 3: shows the area of study.

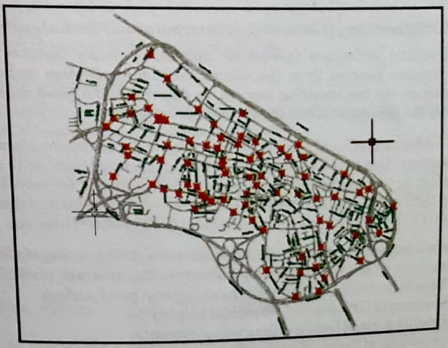

Fig. 3: Street Map of Lagos Island showing the distribution of points (adopted from Dauda, 2007).

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2.0 Geoid Modelling

Several methods exist for modelling geoid, either local or global models derived as part of a global or regional geodetic infrastructure. Globally, the determination of the geoid has been carried out by various researchers. Different geoid modelling techniques have been detailed in Heiskanen and Moritz (1967), Nwilo (1980), Sideris et al. (1992), Featherstone et al (1998), Musa (2003), Kiamehr (2006), Forsberg and Madsen (1990), Fotopoulos et al. (1999b), Kearsley et al. (1993), Mainville et al. (1992), Erol and Celik (2004), Seker and Yildirim (2002), Andritsanos et al (2004), Krynski and Lyszkowicz (2006), Ezeigbo et al (1983 and 2004) etc.

In all the studies that have been carried out, it has been concluded that the combination of the geometrical interpolation method with other convectional geoid modelling methods gives better accuracy, especially for local geoid determination. However, Featherstone et al, (1998) observed that interpolation of the geometrically derived geoid can prove superior to a gravimetric geoid; for some survey areas that are smaller than the resolution of the gravimetric geoid; therefore caution must be taken in drawing judgment.

Basically, there are six major techniques by which the geoid can be determined. These are:

a. The GPS/levelling technique [Geometric approach];
b. The Gravimetric technique;
c. The Astrogeodetic technique;
d. The Satellite technique;
e. The Astro-gravimetric technique; and
f. The combination of the geometric approach and gravimetric technique.

This paper, however, focuses on geometrical (GPS/levelling) interpolation technique.

2.1 The GPS/Levelling (Geometrical Interpolation) Technique

The GPS/levelling technique [geometric approach] simply involves the use of ellipsoidal heights derived from the Global Positioning System and orthometric heights obtained via the levelling process, to determine the geoid undulation and subsequently the geoid model (Opaluwa, 2008).

One of the limitations of local geoid determination is datum inconsistency problem. But, in this study, this problem is not going to be considered because the focus is on testing surface fitting algorithms as a geometrical approach for modelling a local geoid using GPS and Levelling data. There are several other factors that affect the accuracy of GPS/ Levelling geoid model (Erol and Çelik, 2004). These are:

(i) Distribution and number of reference stations (GPS/Levelling stations). These points must be distributed homogeneously to the coverage area of the model and have to be chosen to figure out the changes of geoid surface;
(ii) The accuracy of GPS derived ellipsoidal heights (h);
(iii) The heights derived from levelling measurements;
(iv) Characteristic of the geoid surface in the area;

(v) The method used while modelling the geoid. Researches showed that there is no unique model that works properly for realising the geoid surface of different areas, Ibid (2004).

3.0 Methodology

This involves data acquisition, processing, data quality verification as well as the model formulations and the actual modelling/interpolation techniques.

3.1 Data Acquisition

The data utilised for the research were obtained from Dauda (2007). These include orthometric heights obtained from spirit levelling as well as positional data using Promark2 differential GPS receivers. A total of 75 points (benchmarks) were observed using both GPS and levelling. The GPS was used in rapid static differential mode for five minutes per station.

3.1.1 Data Quality Verification

The quality of data used in any research can be determined by the validity and reliability of such data. The reliability of data is determined by the accuracy of the data; which is the degree of closeness of the data sets to other data sets regarded as the true values. In addition, the validity of data can also be measured by the precision of the instrument used. In this study, no true value was available for all data obtained. However, the precision of the instruments used for data acquisition may be adequate for the purpose of this work. Further probe was performed using regression analysis. This was done by using the entire data to form a regression model of the form discussed in section 3.2 below. The residuals from the solution of the model were examined and points with residuals of 2m and above were considered for removal. 65 data points were utilised, while 30 of these points collocated with classical levelling benchmarks and were well distributed in the study area such that the trend properly representing the geoid surface of the area were carefully chosen as reference points for developing the general regression model required. The remaining 35 points were used as check observation for validation purpose.

3.2 Model Formulation

The fundamental relationship between the geoid undulation, orthometric heights and the ellipsoidal heights is given in equation 1.2; as:

$$\mathbf{N = h\ \ H}$$

Where;
N Geoidal Undulation;
H Orthometric Height; and
h Ellipsoidal height.

Using a base function f(e, n), we can functionally represent the geoid undulation (N) as a function of the coordinates of the points observed; i.e.

$$N = h - H = a_o + f(e, n) \qquad (3.1)$$

Where; $\mathbf{a_o}$ represents a bias; **e** and **n** are the eastings and northings in a plane coordinate system.

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If the geoid is approximated to a flat surface, which is correct over small areas (typically few kilometres), then we can write an expression for N at any point in terms of some base functions which depend on the coordinates of that point. Hence we have:

$$h_i - H_i = N_i = a_o + f_i\,(e, n) \qquad (3.2)$$

The function $f_i(e, n)$ can be expressed in terms of linear combination of some base functions as (Opaluwa, 2008):

$$f_i(e, n) = e_i\,x_1 + n_i\,x_2 \qquad (3.3)$$

Therefore, at any point where ellipsoidal height from GPS and orthometric height from levelling are known, we can solve for geoid undulation, N, using a least square regression model of the form (Featherstone et al, 1998):

$$N_i = h_i - H_i = a_o + e_i\,x_1 + n_i\,x_2 \qquad (3.4)$$

Where; $\mathbf{a_o}$ represents a bias; $\mathbf{x_1}$ and $\mathbf{x_2}$ represent tilt of the geoid plane with respect to WGS84 ellipsoid, while $\mathbf{e_i}$ and $\mathbf{n_i}$ are the eastings and northings in some plane coordinate system.

In order to determine the three parameters (a_o, x_1 and x_2) in 3.4 above, GPS observation must be made on a minimum of three benchmarks surrounding the survey area, so that equation 3.5 can be formed at each of the benchmarks and solved by matrix inversion as follows; (Featherstone et al, 1998):

$$\begin{bmatrix} a_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & e_A & n_A \\ 1 & e_B & n_B \\ 1 & e_C & n_C \end{bmatrix}^{-1} \begin{bmatrix} (h-H)_A \\ (h-H)_B \\ (h-H)_C \end{bmatrix} \qquad (3.5)$$

For an over-determined solution to equation 3.4 using least squares or multiple regression analysis, more than three (3) benchmarks will be required. Then equation 3.5 can be written for any number of control benchmarks for the area as:

$$\begin{bmatrix} a_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & e_i & n_i \\ 1 & e_{i+1} & n_{i+1} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \\ 1 & e_m & n_m \end{bmatrix}^{-1} \begin{bmatrix} (h-H)_i \\ (h-H)_{i+1} \\ \mathrm{M} \\ (h-H)_m \end{bmatrix} \qquad (3.6)$$

For the purpose of this study, thirty GPS data points collocated with classical levelling benchmarks and well distributed in the study area such that the trend properly representing the geoid surface of the area were selected as reference or control benchmarks. Equation 3.6 was then formed at the thirty points and solved using multiple regression analysis. The solution yields the values of the model parameters; a_0, x_1 and x_2, and subsequently, the adjusted undulations for the benchmarks.

These parameters are a_0 =3037.557, x_1 = -0.0015 and x_2 = -0.00308. Therefore, model 3.4 becomes:

$$\mathbf{h_i - H_i = N_i} = 3037.557 - 0.0015\,\mathbf{e_i} - 0.00308\,\mathbf{n_i} + \mathbf{v_i} \qquad (3.7)$$

Equation 3.7 is the required regression model to be solved for the other 35 points. From the solution, the geometrical interpolated undulations (or adjusted undulations) for the reference points were obtained.

R-square and F tests were then performed for significant test of fit for model 3.7 using SPSS 15.0 for windows. The R^2 obtained was 0.556; the computed F-statistics was 0.0232 while the value from F-distribution table is 3.35; hence, about 56% of the variations in the dependent variable N_i can be explained by the regression model with two independent variables x_1 and x_2 at 95% confidence interval.

In order to solve for other arbitrary points located within the area (selected such that they are surrounded by the control points), model 3.7 was then programmed using Spreadsheet and solved. Thus, their adjusted undulations were also derived. There are thirty-five (35) of such points for this research.

In order to evaluate the performance of these models, geo-statistical kriging as a geospatial surface modelling technique was adopted for generating the grid map (using surfer 8 software) of the area utilising undulations obtained from equations 1.2 and 3.7 respectively. The use of geostatistical kriging and its reliability as a surface interpolation technique has been detailed in Erol and Celik (2003 and 2004) and Opaluwa (2008). A VBA ad-on programme was used on surfer 8 software such that it accepts the coordinate of any points and interpolates the geoid undulations of such points if a grid map enclosing such data set exists or is prepared and saved as surfer grid file. Using the coordinate and the undulations of the thirty (30) reference benchmarks, the grid maps were generated.

4.0 Results and Analysis

4.1 Results

The results from equations 1.2 and 3.7 for the benchmarks as well as the interpolated values for the same points as obtained using kriging method are shown in the Table 4.1, while similar results obtained for some arbitrarily selected points within the limit of the selected controls are in Table 4.2. The residuals of the interpolated undulations from both the observed (equation 1.2) and adjusted (equation 3.7) undulations using kriging were extracted from Tables 4.1 and 4.2 and graphically plotted as shown in Figs 4.1 and 4.2 respectively. The gridded contour maps and digital terrain models of the geoid surface obtained from both observation (eqn 1.2) and the adjusted undulations (using multiple regression, i.e. model 3.7) were also presented in Figs 4.3a, 4.3b, 4.4a and 4.4b respectively.

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Table 4.1: Comparing the recovered undulations for the control benchmarks using kriging with those from the equations.

Point ID	EASTING (metres)	NORTHING (metres)	N observed (h - H) m Eqn 1.2	N Interpol. (kriging) (metres)	Residuals of Interpol N (m) from Observation	N Adjusted (Regress) m Model 3.5	N Interpol. (Kriging) (metres)	Residuals of Interpol N (m) from Regression
SBM1	544502.545	712822.929	22.33998	22.32301	-0.01697	21.16176	21.15748	-0.00428
G002	545035.946	712889.525	20.75898	20.74579	-0.01319	20.15393	20.15626	0.002324
G004	544831.997	713185.406	18.83098	18.83831	0.007329	19.5488	19.55238	0.003583
G005	544496.282	713227.289	18.49598	18.52737	0.031394	19.92482	19.92413	-0.00068
G006	544425.152	713081.36	21.84398	21.7352	-0.10878	20.48164	20.48279	0.001159
G011	543935.865	712860.422	20.96598	21.01528	0.049304	21.89881	21.89743	-0.00138
G012	544088.142	712706.069	22.62498	22.60477	-0.02021	22.14546	22.13953	-0.00593
G015	544595.741	713482.211	17.35298	17.38505	0.032071	18.98942	18.99455	0.00513
G016	544477.771	713526.024	18.19798	18.18798	-0.01	19.03187	19.03412	0.002249
G020	544090.599	713590.58	20.08998	19.95419	-0.13579	19.41543	19.41592	0.000497
G022	544050.332	713615.553	18.06598	18.31715	0.251174	19.39904	19.40126	0.002224
G028	543620.352	713580.756	20.32298	20.24946	-0.07352	20.15324	20.15325	1.44E-05
G029	543511.579	713495.973	19.02798	19.13025	0.102273	20.57822	20.5788	0.000575
G032	543570.593	713220.441	21.73498	21.67653	- 0.05845	21.33871	21.33927	0.000564
G033	543470.111	713180.59	21.23098	21.25866	0.027676	21.61272	21.61218	-0.00055
G035	543694.334	713069.848	20.67098	20.71155	0.040566	21.6167	21.61562	-0.00108
G040	543058.034	713078.316	23.91498	23.85364	-0.06134	22.54797	22.53753	-0.01045
G043	542570.487	713393.193	21.91798	21.93893	0.020945	22.31098	22.30691	-0.00407
G044	542306.343	713559.222	21.61298	21.61402	0.001043	22.19666	22.18872	-0.00794
G047	542740.055	713934.31	20.05998	20.10528	0.045297	20.38796	20.389	0.001034
G049	542833.689	714159.63	19.80098	19.79093	-0.01005	19.55257	19.55166	-0.00091
G053	543428.357	714389.99	19.49498	19.48625	-0.00873	17.9478	17.953	0.005202
G054	543449.356	713964.278	19.70198	19.69357	-0.00841	19.22838	19.22763	-0.00076
G056	542187.196	714027.18	18.86998	18.89575	0.025772	20.93353	20.93359	5.54E-05
G057	542301.208	713888.63	21.47698	21.41266	-0.06432	21.18905	21.18933	0.000285
GA01	544958.924	712803.588	20.78198	20.79573	0.013751	20.53471	20.53011	-0.0046
T005	543864.577	713298.331	22.40198	22.29611	-0.10587	20.6563	20.65686	0.00056
T006	542888.113	714372.567	20.22698	20.19525	-0.03173	18.81435	18.81882	0.004472
T013	543041.287	714260.096	18.33598	18.44018	0.104202	18.93056	18.92824	-0.00232
T014	542873.521	713646.587	22.60198	22.56062	-0.04136	21.074	21.07532	0.001314

Table 4.2: Comparing the interpolation undulations (for other arbitrary benchmarks using kriging) with those from the equations.

Point ID	EASTING (metres)	NORTHING (metres)	N observed (h - H) m Eqn 1.2	N Interpol. (kriging) (metres)	Residuals of Interpol N (m) from Observation	N Adjusted (Regress) m Model 3.7	N Interpol. (Kriging) (metres)	Residuals of Interpol N (m) from Regression
4038	544431.229	712821.529	20.87998	22.44803	1.56805	21.27337	21.26507	-0.0083
4039	544471.125	712809.003	21.44998	22.41645	0.966468	21.25195	21.23808	-0.01388
4040	544497.193	712790.328	21.97898	22.38449	0.405511	21.27029	21.23936	-0.03093
G001	544777.957	712758.296	21.53998	21.40582	-0.13416	20.94659	20.84012	**-0.10647**
G003	544846.611	713095.24	20.06998	19.49429	-0.57569	19.80473	19.81267	0.007939
G008	544020.482	713077.591	18.62598	21.54668	2.920697	21.10212	21.02234	0.039769
G009	543910.166	713109.244	21.04998	21.52088	0.470898	21.17053	21.13897	0.036854
G017	544327.613	713577.06	19.33898	18.89004	-0.44894	19.10049	21.19644	0.025908
G018	544230.711	713479.339	16.55398	19.80701	3.253035	19.5475	21.58118	0.013083
G019	544137.918	713508.262	19.82798	20.16913	0.341154	19.59796	19.14531	0.044815
G021	544167.35	713734.374	20.58698	18.56175	-2.02523	18.85673	19.53774	-0.00976
G023	543989.986	713655.113	18.69798	18.42423	-0.27375	19.3679	19.5872	-0.01077
G024	543861.902	713640.066	21.74298	19.32206	-2.42092	19.60699	18.99392	**0.137194**
G025	543764.172	713773.9	20.72198	19.26103	-1.46095	19.34152	19.39151	0.023613
G026	543635.596	713700.635	22.30098	19.79313	-2.50785	19.7608	19.62165	0.014663
GO27	543565.343	713861.589	21.43898	19.57158	-1.8674	19.37039	19.38847	0.046956
G030	543580.731	713409.801	22.73398	20.29657	-2.43741	20.73979	19.77064	0.009846
G031	543462.34	713396.79	22.66698	20.18295	-2.48403	20.95802	19.3895	0.019107
G034	543507.498	713037.679	22.30198	21.49026	-0.81172	21.99697	20.74421	0.004423
G036	543658.61	712986.405	20.52598	21.00316	0.477177	21.92765	20.97238	0.014362
G038	543244.77	713119.803	21.77098	22.55287	0.781894	22.13914	21.92858	-0.06839
G039	543169.741	713200.854	23.09498	22.69007	-0.40491	22.0022	21.86912	-0.05852
G041	543004.027	713209.161	25.17298	23.26064	-1.91234	22.22593	22.12644	-0.01269
G042	542996.535	713410.277	20.18298	22.51761	2.334629	21.6173	22.03882	0.036623
G045	542407.164	713592.62	24.42298	21.69369	-2.72929	21.94202	22.2412	0.015272
G046	542676.74	713632.315	22.80598	21.97166	-0.83432	21.41407	21.65717	0.039874
G048	542843.314	713874.487	22.23398	20.62218	-1.6118	20.41699	21.98102	0.038994
M001	543298.97	713577.783	23.04	20.41104	-2.62896	20.64595	21.46253	0.048462
M002	544431.229	712821.529	25.454	21.49152	-3.96248	21.37305	20.40576	-0.01123
T003	544471.125	712809.003	19.05098	21.66198	2.610996	20.48166	20.65749	0.011536
T004	544497.193	712790.328	20.765	22.03679	1.271786	20.87223	21.41551	0.042458
T009	544777.957	712758.296	17.14598	19.42562	2.279642	19.70171	20.47766	-0.004
T010	544846.611	713095.24	18.77198	20.20548	1.433495	20.47616	20.88275	0.010521
T011	544020.482	713077.591	19.67098	21.80051	2.129525	22.70467	19.93548	**0.233779**
T015	543910.166	713109.244	22.0268	21.69227	-0.33453	20.95011	20.54443	0.068267

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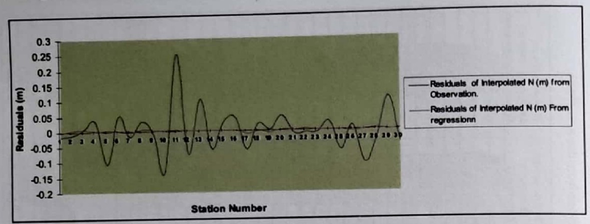

Fig. 4.1: Comparing residuals of the interpolated undulations (kriging) from the observed (N= h - H) and adjusted (regression) undulations (for the control points).

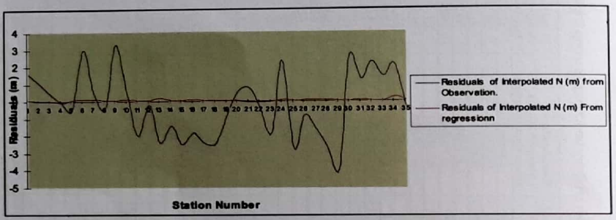

Fig. 4.2: Comparing residuals of the interpolated undulations (kriging) from the observed (N= h - H) and adjusted (regression) undulations (for arbitrary or validation points).

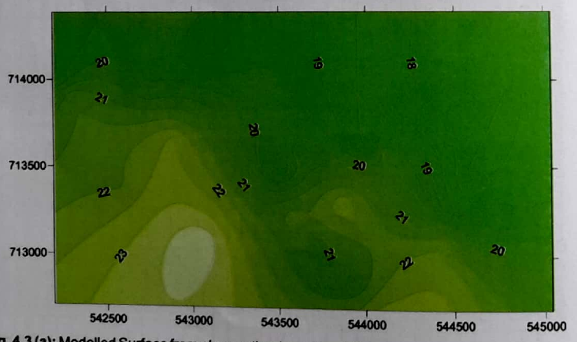

Fig. 4.3 (a): Modelled Surface from observation (equation 1.2)

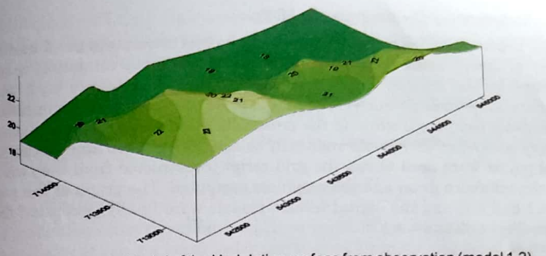

Fig. 4.3 (b): Digital Terrain Model of the Undulation surface from observation (model 1.2)

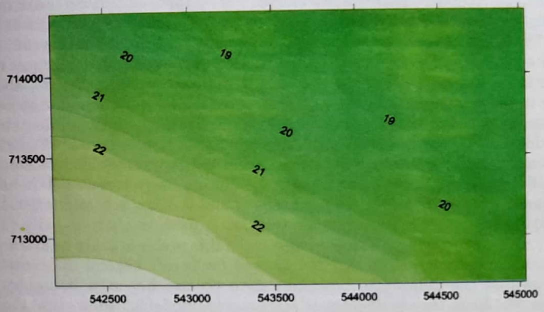

Fig. 4.4 (a): Modelled Surface using the adjusted undulations (model 3.7)

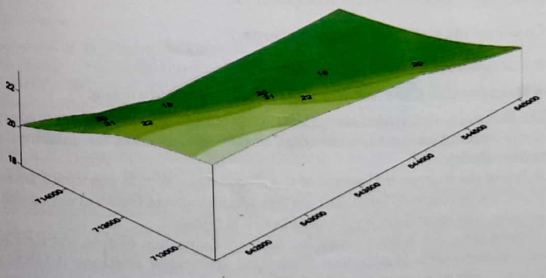

Fig. 4.4 (b): Digital terrain model using the adjusted undulations (model 3.7).

4.1.1 Validation of Results from the Models

In order to determine the model that will give the best estimate of geoid undulations for any arbitrary point in the study area, self-and cross-validation tests were performed on the results. To perform the self-validation test in each case, the coordinate of the same points were used to recover their respective undulations and the residuals computed, while in the cross-validation test, the coordinates of the arbitrary thirty-five (35) points randomly selected within the limit of the selected control points were used to run the grid script (constructed from the two models using the reference data) and the residuals computed. The graph of the residuals (Figs 4.1 and 4.2) and the digital terrain models were finally generated from the exercise (Figs 4.3b and 4.4b).

4.2 Analysis

The residuals computed from the two models were compared and the result from the polynomial model (3.7) proved to be more accurate (see Tables 4.1 and 4.2) in both cases. The graph of the residuals (Figs 4.1 and 4.2) and the digital terrain models generated from the exercise (Fig. 4.3b and 4.4b) further confirmed these results. Furthermore, the statistics of the results gave a mean square error of 7.35cm and 0.36cm in the self-validation test using the observed ($N = h - H$) and the adjusted (regression) undulations respectively for the reference points, while the cross-validation test (i.e. the arbitrary points) gave mean square errors of 187.831cm for equation 1.2 (observed) and 5.58cm for model 3.7 (adjusted) at 95% confidence interval. It was however observed that the residuals of the interpolated undulations (using kriging) from the adjusted (regressions) undulations at stations G001, G024 and T011 in Table 4.2 (i.e. the arbitrary data) are still relatively large, with values of 10.647cm, 13.719cm and 23.378cm respectively; this problem could be attributed to the weak extrapolation ability of this method for points beyond the control data limit, because the points are boundary data points; hence, a boundary value problem. Similarly, the result of the R^2 statistics for model 3.7 shows that the model cannot account for about 44% of the variations in the interpolated undulations since only about 56% of the variations in the dependent variable N_i can be explained by the regression model at 95% confidence interval.

5.0 Conclusions and Recommendations

5.1 Recommendations

In view of the foregoing, the following recommendations are made:

(i) The Federal Government should intensify efforts at ensuring early completion of the ongoing nationwide coverage of GPS observation for data availability and reliability.

(ii) Office of the Surveyor-General of the Federation should put up an action plan (if not yet in place) to facilitate the commencement of GPS survey of the entire primary and secondary levelling network in Nigeria.

(iii) Nigeria does not have to wait for African Reference Frame (AFREF) before establishing GPS Continuous Operating Reference Stations (CORS) in the country. We are aware that the Office of the Surveyor General is currently in the process of establishing COR Stations in some universities such as the University of Lagos, University of Nigeria, Enugu Campus, Federal University of Technology, Yola and a host of others. This will aid research and reduce cost of carrying Global Navigation Satellite System (GNSS) Survey campaigns.

(iv) Other state governments should also emulate the ongoing efforts of Lagos State Government by establishing GPS COR Stations in their respective states. The availability of a well-distributed CORS will, in the near future, provide a means of providing primary horizontal and vertical control networks in Nigeria.

5.2 Conclusions

Geoid modelling using the geometrical interpolation technique has been discussed; the possibility of using a lower-order polynomial for modelling geoid as postulated by Featherstone et al (1998) and Sideris et al (1992) was examined, while the potentials of geostatistical kriging as a robust surface modelling technique was also presented. From the foregoing analysis, a regression model which accounts for about 56% of the variations in the dependent variables is not a poor model for large scale-engineering applications; although it may not be very good for precise geodetic applications such as space research, geodynamic applications, etc. The research, however, is still ongoing as data from precise levelling benchmarks collocating with full static mode GPS observations are being considered for use as reference points. Nevertheless, the overall result from the model indicates that the objective of this research, which is meeting the needs of GIS and engineering applications, has been achieved. We therefore conclude that the geometrical interpolation technique of geoid modelling is a promising alternative to the age-long problem of insufficient gravity data for national geoid determination in Nigeria.

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