Derivation of Orthometric Heights from GPS Measured Heights Using Geometric Technique and EGM96 Model

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DERIVATION OF ORTHOMETRIC HEIGHTS FROM GPS MEASURED HEIGHTS USING GEOMETRICAL TECHNIQUE AND EGM 96 MODEL.

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ABSTRACT

As a result of wide spread use of satellite based positioning techniques, especially Global Positioning System (GPS), a greater attention has been focused on precise determination of geoid models with an aim to replace the classical leveling with Global Navigation Satellite System (GNSS) measurements. In this research, geometric technique of deriving orthometric height from GPS survey along a profile and the use of EGM 96 geoid model for deriving orthometric height from GPS data (in GNSS solution software) are evaluated. The main focus of this research is to critically examine the potentials of these methods with a view to establishing the optimum technique as an alternative to classical differential levelling. From the results of the research, the standard erros of residuals are 1.453m and 1.450m respectively for EGM 96 model and the geometrical approach. From the graphical representation of the residuals from the two methods, it was observed that the two curves suddenly became sinusoidal from station 9 (corresponding to SB08 in the tables). This similarity pattern of the residuals makes it difficult to draw a conclusive judgment between the two methods examined; however, from the standard errors of residuals, it could be inferred that the geometrical technique gave a better result over EGM 96 model.

Key Words: Geometrical Interpolation, EGM 96, Orthometric Height, Ellipsoidal Height, Geoid Undulation, Static.

1.0 INTRODUCTION

The classical Vertical control is composed of several hierarchical networks which follows the principle of "working from the whole to the part". The primary Vertical Orthometric Control Network contains loops of first order precise leveling of some hundreds of kilometers in length. The accuracy of precise leveling (high precision leveling) should be at the millimeter-level per kilometer (Bomford, 1980). The other subnets in the network are densifications of the primary one, according to the needs - with decreasing accuracy. It is worth to note that it is reasonable to establish a third-order network, only in the densely populated area, however the difficulties involved in precise leveling is well-known; for example, Eriksson et al, (2002) observed that even with the most advanced technology of motorized leveling, it took some 25 years to accomplish the first-order network in Sweden. Due to these difficulties, it is actually impossible to get heights for lower-order networks with absolute accuracy (relative to the higher-order) better than 5-10 cm (Steinberg and Even-Tzur, 2005).

Because there was no alternative to precise levelling as a tool to achieve the objectives of primary levelling networks prior to the GPS era, it was naturally referred to as vertical control. However, current researches have shown that GNSS measurements are quite more effective for monitoring vertical tectonic changes over a wide area. Consequently, the ideas of Vertical Ellipsoidal Control, or 3-D Geodetic Control, based on Permanent GNSS Networks (called CORS – Continuous Operating Reference Stations) are becoming popular Steinberg and Papo (1996, 1998, 1999), Meyer et al (2004), Wonnacott (2005), etc. The advantage of GNSS networks over precise leveling is quite obvious. The major question therefore, is whether Ellipsoidal Height Control Networks can replace the Orthometric ones.

However, Steinberg and Even-Tzur (2005) observed that ellipsoidal Control is the imminent replacement for the orthometric control. For instance the Survey of Israel is already moving towards 3D Geodetic Control, based on the Israeli Permanent GPS Network. Generally speaking, Steinberg and Even-Tzur (2005) further observed that Vertical Ellipsoidal (Geometric) Control should be based on Permanent GNSS Network as one part of the 3-D Geodetic Control. The Permanent GNSS Network is the first order of the 3-D Control; by its nature (operating cost), Permanent Stations are quite far from each other. Due to the dependency of GPS accuracy on the length of the baselines (which can be compensated by longer measuring times), especially in the vertical direction, it is recommended to densify the first-order control by more orders, according to actual needs. This densification should be accomplished, of course, by GPS measurements. In Israel, it was decided that the accuracy of the Second-Order network will be 1cm (2σ), and that of the third-order, 2cm, relative to the nominal heights of the Permanent GPS stations.

In an effort towards realizing this goal, Steinberg and Even-Tzur, (2005) carried out GPS survey over eight points in Israel, and suggested that any available geoid model should be accepted as the official geoid model no matter the accuracy. The results of the research indicated that accuracy of 25ppm was realized for benchmarks of 1km apart in the fourth order leveling network, which is the same as the accuracy of the existing height difference of the benchmarks.

The biggest constraint in using this alternative in Nigeria however; is the lack of an existing official geoid model as well as primary GPS (CORS) stations. Therefore, representing geoid heights as mathematically formulated surface and calculating the geoid heights in new measured points according to GPS technique constitutes the idea in this study.

1.1 Height Relationship

The procedure of geodetic leveling provides a height that is commonly known as a height above Mean Sea Level. The process gives level differences between two consecutive benchmarks, which are expressed by aligning the level bubble with the graded values on forward and backward levels staves. The orthometric heights so derived reflect local variation in gravity as well as topographic gradients.

The reference datum for orthometric heights, ellipsoid and the geoid, is approximated by Means Sea Level (MSL). Basically one has to establish a relationship between the Orthometric height obtained from geodetic leveling and GPS derived ellipsoidal heights using a common reference datum. The technique is often called geometrical approach for “height basis” estimation (Seker, Yildirim, 2002). The basic equation which relates the orthometric and ellipsoidal height is

$$h = N + H \qquad (1.0)$$

Where

H = Orthometric height, measured along curved plumbline.

h = Ellipsoidal height measured along the ellipsoidal normal

N = Geoid height, the separation between geoid and ellipsoid.

Theoretically, since the ellipsoidal height and orthometric height are measured along the normal to ellipsoid and along the direction of the plumbline respectively the relationship defined in equation (1.0) is only an approximation but serve the purpose for most of the engineering application.

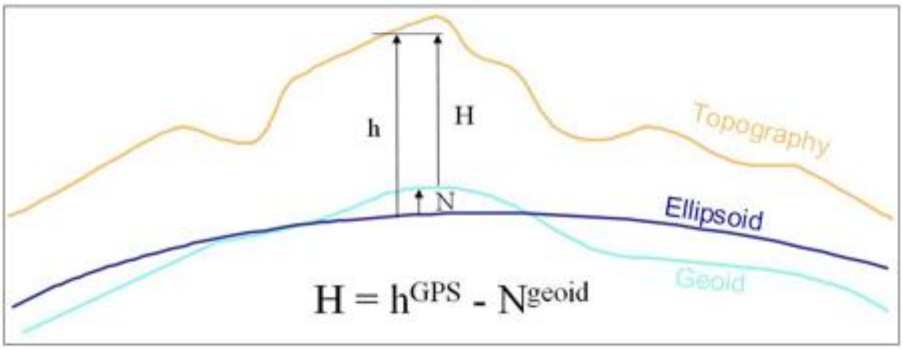

Figure1.0: **Relationship between Ellipsoidal height, Orthometric height, Geoidal height (NRC, 2008**).

1.2 Statement of Problem

The determination of orthometric heights from spirit levelling is known to be time consuming and cumbersome, especially in a large and very rough terrain. In fact, apart from the complexity in its field measurement, a lot of time and energy is spent in the stage of data reduction and adjustment thereby making it highly capital intensive to establish a country-wide high-resolution levelling network. Consequently, the availability of this data in most developing countries and particularly in Nigeria is inadequate; there is also lack of gravity data required to properly adjust the observed heights to yield the orthometric heights. However, advances in space technology has enhanced simultaneous determination of 3-D positioning referenced to the global (accurately determined) geocentric ellipsoid (WGS 84 ellipsoid), Unfortunately the geoid for Nigeria has not been accurately determined and the geopotential model geoid for Nigeria determined in 2006 to 1m accuracy is yet to officially receive wide acceptability for application. Therefore, there is a serious problem of data transformation/conversion between the local and global reference datums, hence, a serious limitation to GPS usage in Nigeria.

In order to proffer solution to these problems, this research critically examined the potentials of geometrical technique and EGM 96 geoid model for deriving orthometric heights from GPS, with a view to establishing the optimum technique to complement classical differential levelling.

1.3 Aim and Objectives of the Research

The aim of this research is to compare geometrical technique of deriving orthometric height from GNSS survey and the use of EGM96 model for processing orthometric height from GNSS survey.

The objectives of this research therefore are:

(i). To acquire 3-D positional data of some benchmarks with the GPS along a level profile

(ii). To compute the relative geometric geoid along the level profile by combining the ellipsoidal heights from GPS with the existing orthometric heights of observed benchmarks.

(iii) To compute orthometric height along the profile using linear interpolation technique and EGM 96 geoid model respectively.

(iv) To compare the computed orthometric height in each case with the provisional orthometric height from differential levelling of the observed points.

1.4 Study Area

The study area lies between the Standard Bench Mark (SBM) at Limawa primary school in Kpakungu and the Standard Bench Mark (SBM) at Garatu Primary School in Bosso Local Government Area of Niger State. it covers a total distance of 18km and it lies within latitude (φ) 9^0 29' 20'' and 09^0 36' 14.14'' North of the equator and longitude(λ) 6^0 26' 24.82'' and 6^0 31' 46.72''East of the meridian.

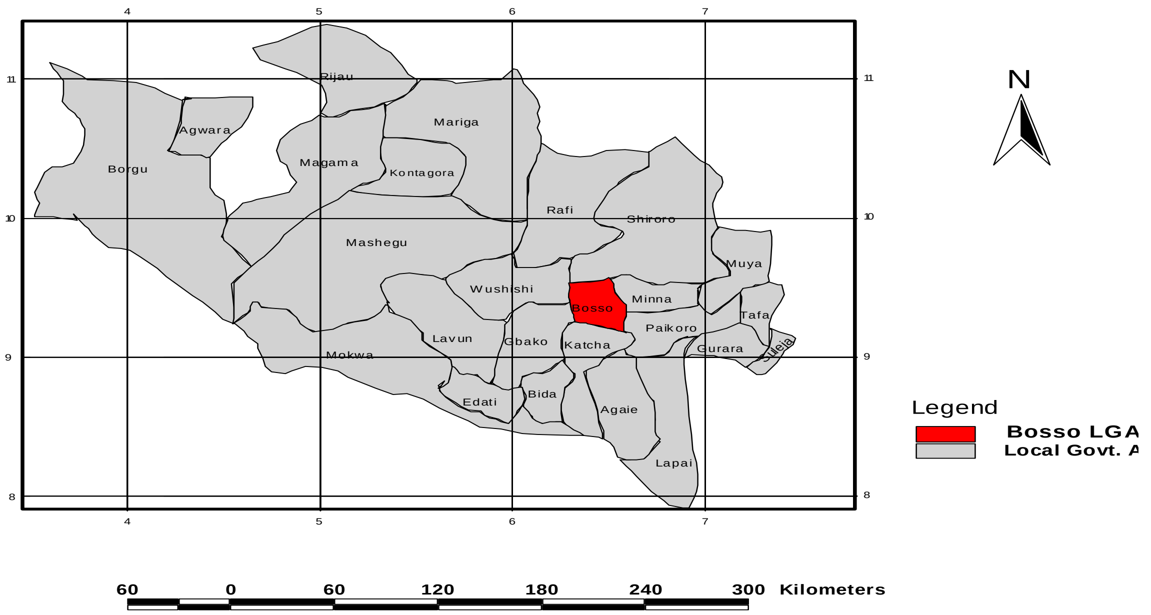

Fig. 2 Map of Niger State Showing the Study area.

2.0 GPS DERIVED ORTHOMETRIC HEIGHT

One of the primary applications of a gravimetric geoid model is for converting GPS-derived ellipsoid heights to orthometric height on the local vertical datum (Opaluwa, 2008).

The process of deriving elevations on a local height datum from GPS measurements has been well documented in Gilliland (1986); Kearsley (1988); Collier & Croft (1997); Featherstone *et al*. (1998). Since the main applications of a geoid model is to convert GPS-derived ellipsoid heights to gravity related elevations above a local height datum, central to this problem is the knowledge of geoid-ellipsoid separation relative to the GPS reference ellipsoid (Featherstone 1998, p.274). GPS-derived ellipsoidal heights can be converted to approximate height datum elevations in either an absolute or relative sense, depending on observation technique. The absolute case is a situation where an ellipsoidal height can be converted to an approximate orthometric height by algebraically subtracting the geoid-ellipsoid separation at a discrete point using equation;

$$H = h - N \qquad (2.0)$$

N is the geoid-ellipsoid separation (also known as geoid height) measured along the ellipsoid normal to the geoid. If the geoid is above the ellipsoid, N is positive. If the geoid is below the ellipsoid, N is negative. It is important to note that the ellipsoid height (*h*) and the geoid height (N) must refer to the same reference ellipsoid for the relationship to hold.

Featherstone et al (1998, p.279) suggested that as the most accurate GPS applications are performed in the relative mode, equation (2.0) is not very practical for GPS height conversion. Rather, for the majority of surveying applications equation (2.0) can be rearranged to accommodate the relative situation, where an elevation is transferred from a known point, A, to an unknown point, B, via the following relationship Featherstone et al (1998):

$$H_B = H_A + (h_B - h_A) - (N_B - N_A) \qquad (3.0)$$

According to Kearsley (1988) and Featherstone *et al*. (1998), equation 3.0 can be reduced to:

$$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \qquad (4.0)$$

where Δ denotes ‘change in’.

Nevertheless, Sideris et al (1992) asserted that the determination of orthometric heights by traditional techniques, such as spirit levelling, is a difficult task, moreover levelling over areas with rough terrain is very strenuous and time consuming. On the other hand the combined use of GPS and geoid heights presents an alternative potential to the classic geometric levelling. Detail research on Geoid and GPS/Levelling differences can be found in Forsberg and Madsen (1990), Mainville et al. (1992), Kearsley et al. (1993), Featherstone and Kirby (1998), Erol and Celik (2004) and Fotopoulos et al. (1999b) e.t.c.

2.1 STRATEGY FOR GPS HEIGHT SURVEY

The accuracy specifications for GPS survey indicate in part, the GPS survey techniques and observables that must be used, Featherstone et al (1998). They further, affirmed that the accuracy of GPS techniques is near-proportional to the cost of GPS equipment and survey logistics involved. Therefore, the three main classes of GPS survey techniques and approaches to be used to model the geoid are as shown in table 1.0; while table 1.1 shows the list of the approximate accuracy of GPS survey mode.

Table 1.0: GPS survey modes and appropriate methods to determine the geoid with which to recover orthometric height (adopted from Featherstone *et al.* 1998).

S/NO.	GPS SURVEY METHOD	GEOID DETERMINATION METHOD
1	Single-point Code	There is no real need to use geoid heights because the max. geoid undulation is approx. 100m which is less than the error introduced by selective availability (i.e.±140m).
2	Code Differential	It is sufficient to use a global geopotential model such as EGM 96. (EGM96 alone has been estimated to provide Australian Height Datum (AHD) heights to less than 5m in many cases).
3	Carrier-phase Relative (integer fixed solution).	As this is the most accurate GPS survey method, the most accurate geoid modeling method should be used. The options include a gravimetric geoid alone, geometrical interpolation alone or combined method. It is strongly recommended that each is tested to determine optimal approach in each particular survey area.

Table 1.1: Summary of approximate accuracy of GPS Positioning (in metres), Featherstone et al, (1998).

GPS SURVEY METHOD	OBSERVABLES	HORIZONTAL (m)	VERTICAL (m)
Single Point	C/A	100	140
Static (differential)	C/A	0.5 – 2	1 – 3
Static (relative)	L1	0.02	0.03
Static (relative)	L1 & L2	0.005	0.02
Rapid (static)	L1 & L2	0.02	0.03
Pure kinematics	C/A	2 – 5	3 – 8
Pure kinematics	L1	0.03	0.05
Pure kinematics	L1 & L2	0.01	0.02
Semi-kinematics	L1 & L2	0.01	0.02
Real-time (differential)	C/A	3 – 5	4 – 8
Real-time(Pure kinematics)	C/A	2 – 5	3 – 8
Real-time(Pure kinematics)	L1	0.1	0.2
Real-time(Pure kinematics)	L1 & L2	0.05	0.1
Real-time(Semi-kinematics)	L1	0.03	0.05
Real-time(Semi-kinematics)	L1 & L2	0.02	0.03

3.0 METHODOLOGY

3.1 Geometrical Technique

Featherstone (2004) noted that the standard approach for gravimetric geoid model validation is by comparisons with GPS and levelling data observed at co-located points. Therefore, from the fundamental relationship between geoidal height, ellipsoidal height and orthometric height as shown in figure 1.0, discrete empirical geoid heights can be computed at each co-located point by re-arranging equation 1.0 to form:

$$N = h - H \qquad (5.0)$$

while relative empirical geoid height differences can be computed by re-arranging equations 1.0 and 3.0 to give:

$$N_B = N_A + (h_B - h_A) - (H_B - H_A) \qquad (6.0)$$

This can however, be reduced to:

$$\Delta N_{AB} = \Delta h_{AB} - \Delta H_{AB} \qquad (7.0)$$

The preceding calculations result is an empirical geoid model that can be used in comparisons with geoid heights interpolated from gravimetrically computed geoid models, subject to the errors in the GPS and levelling data McDonald, (2004).

If the geoid is assumed to be approximated by a flat surface, which is usually sufficient over small areas (typically few kilometers), linear interpolation can be used to estimate the geoid-ellipsoid separation, Featherston et al, (1998). Using two benchmarks which have both been occupied with GPS, the ellipsoidal height at an intermediate station X can be transformed to an orthometric height using (Featherstone et al, 1998):

$$H_i = H_A + \Delta h_{Ai} - (l_{Ai} / S_{AB}) * \Delta NAB \qquad (8.0)$$

Where; S_{AB} = separation (length) of benchmarks A and B

l_{Ai} = distance of the desired point X from point A.

3.2 Geopotential Geoid Model

The geopotential geoid model as one of the global geoid models, represents the long wavelength part of the gravity field and is obtained from global geopotential solutions which are given as a set of spherical harmonic coefficients (Opaluwa, 2008). Different datasets are used to determine these coefficients, ranging from satellite observations (which give the so-called satellite only solutions) to models which incorporate satellite altimetry and surface gravity data, thus usually containing more coefficients (Sideris et al, 1992). The expression for computing geoid undulation (N) from such set of spherical harmonic coefficients is given by Heiskanen and Moritz (1967) as;

$$N = R\sum_{n=2}^{n_{\max}}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\, m\lambda)\,\bar{P}_{nm}(\sin\phi) \qquad (9.0)$$

Where

n_{max} = maximum degree of expansion

$\bar{C}_{nm}, \bar{S}_{nm}$ = the fully normalized coefficients of the disturbing potential,

$\bar{P}_{nm}(\sin\phi)$ = fully normalized associated Legendre functions, R = mean radius of the earth

ϕ, λ = geodetic latitude and longitude.

Therefore, using the EGM96 geopotential coefficients, the geopotential model geoid undulation (N^{GM}) at any point on the earth’s surface can be computed using equation (9.0). This concept of geopontential model geoid has been programmed into GNSS solution software as a default vertical datum therefore, given the 3-D coordinate of any point using DGPS receivers, it automatically transform the ellipsoidal height to approximate orthometric height during post processing.

3.3 Procedure

For the purpose of this study as mentioned earlier, a level route bounded by two standard benchmarks (SBM Kpakungu and SBM Garatu) was identified along Minna-Bida road in Niger State. The route span for a distance of 18km with sixteen intermediate benchmarks and a primary cadastral point; these points were observed with a single frequency differential GPS using the cadastral point (CSN 128P) as the base for the observation. The stability of CSN 128P was first confirmed by observing on it using L40 as the base station. Each of the point including the SBMs was occupied for a minimum of 20 minutes in static survey mode.

As a precaution, the value for the epoch rate in a static survey must be the same for all receivers during the survey; this rate was set to be 5sec for this project, this was done to minimize the number of observations and thus the data storage requirements. All the receivers were connected to controllers that have internal memories as well as memory card (external device) for storing the observed data. However, the orthometric heights of the two Standard Bench Marks (SBMs) were obtained from the Office of the Surveyor General of the Federation, Minna Area office, the orthometric heights of the intermediate points were obtained from precise leveling exercise conducted by the Department of Surveying and Geoinformatics, FUT Minna in 2008; while the 3-D coordinate of the base (CSN 128P) was sourced from Niger State Surveyor General's Office.

3.4 DATA PROCESSING:

After GPS observation, the observed data were post processed using GNSS solution software in two phases. The first phase involved the post processing of acquired data to the reference ellipsoid without using any vertical datum. However, the second phase of data processing involved the selection of the default vertical datum (EGM96 geoid model).

The GNSS solution computes baseline vectors as changes in X, Y, Z between the base station and the rover stations. If base station (A) has known coordinates, then the coordinates of rover stations (B) can be computed according to Wolf and Ghilani (2006) as:

$$X_B = X_A + \Delta X$$
$$Y_B = Y_A + \Delta Y \qquad (10)$$
$$Z_B = Z_A + \Delta Z$$

Where (X_A,Y_A,Z_A) are the geocentric coordinates at the base station A, (X_B,Y_B,Z_B) are the unknown station B, while ΔX, ΔY, ΔZ are the computed baseline vector components. These yields the ellipsoidal 3-D coordinate of all the occupied stations. While the reduced level of the sixteen intermediate benchmarks as well as the primary traverse point (CSN128P) were adjusted using least square technique.

The processed ellipsoidal height from above was combined with the orthometric heights of the two SBMs (obtained from the Office of the Surveyor General of the Federation) to derive the relative geoid undulation along the profile using equation (7.0). Then, equation (8.0) was used to derive the approximate orthometric height of all the intermediate points by the combination of the derived relative geoid undulation with their respective ellipsoidal height. Since observation commenced from SBM Kpakungu, the computation originated from there.

In order to derive the height of the observed points directly from the GPS data, the observed vectors were re-processed by choosing EGM 96 Geoid model as the height datum.

4.0 NUMERICAL EVALUATIONS

In the numerical investigations, we closely examined the variations in the two sets of approximate orthometric heights from GPS by comparing each with the height of the respective points from classical (differential) levelling.

The heights as obtained from the various approaches discussed in this research are as shown in table 1.2 to 1.6, while table 1.7 shows their differences from classical levelling. The standard error of residuals was computed using SPSS 15.0 for windows; this is as shown in table 1.8 (the residual statistics).

Table 1.2 Ellipsoidal Heights from GPS

STATION ID	Easting (m)	Northing (m)	Ellips height(h) (m)
SBM KP	228934.946	1062478.392	270.580
SB01	227885.326	1061370.861	249.416
SB02	227020.63	1060830.955	252.749
SB03	226360.14	1060123.703	256.309
SB04	225716.18	1059425.578	236.345
SB05	225058.217	1058715.854	256.439
SB06	224363.139	1058076.3	251.509
SB07	223621.474	1057448.476	243.023
SB08	222899.033	1056814.499	255.405
SB09	222417.534	1055987.605	253.697
SB10	222031.132	1055113.772	249.728
SB11	221665.419	1054246.087	260.221
SB12	221283.485	1053355.461	266.261
SB13	220887.349	1052445.862	253.192
SB14	220508.178	1051550.971	261.658
SB15	219949.294	1050711.581	260.215
SB16	219348.684	1049910.498	253.401
SBM GA	219019.418	1049640.985	246.870
CSN 128P	222702.652	1056599.019	258.02

Table 1.3 Processed Orthometric Heights from GPS (using EGM 96 model)

STATION ID	Easting (m)	Northing (m)	Orthometric Height(H) from EGM96 model (m)
SBM KP	228934.946	1062478.392	242.152
SB01	227885.326	1061370.861	220.967
SB02	227020.63	1060830.955	224.292
SB03	226360.14	1060123.703	227.843
SB04	225716.18	1059425.578	207.872
SB05	225058.217	1058715.854	227.96
SB06	224363.139	1058076.3	223.027
SB07	223621.474	1057448.476	214.541
SB08	222899.033	1056814.499	226.923
SB09	222417.534	1055987.605	225.214
SB10	222031.132	1055113.772	221.244
SB11	221665.419	1054246.087	231.738
SB12	221283.485	1053355.461	237.781
SB13	220887.349	1052445.862	224.717
SB14	220508.178	1051550.971	233.189
SB15	219949.294	1050711.581	231.757
SB16	219348.684	1049910.498	224.957
SBM GA	219019.418	1049640.985	218.433
CSN 128P	222702.652	1056599.019	229.539

Table 1.4 Derived Orthometric Heights from Geometrical technique

STATION ID	Easting (m)	Northing (m)	Geometrical Interpolated height (H) (m)
SBM KP	228934.946	1062478.392	240.905
SB01	227885.326	1061370.861	220.3357
SB02	227020.63	1060830.955	224.0583
SB03	226360.14	1060123.703	227.9939
SB04	225716.18	1059425.578	208.399

SB05	225058.217	1058715.854	228.8695
SB06	224363.139	1058076.3	224.3076
SB07	223621.474	1057448.476	216.1996
SB08	222899.033	1056814.499	228.956
SB09	222417.534	1055987.605	227.607
SB10	222031.132	1055113.772	223.9873
SB11	221665.419	1054246.087	234.8264
SB12	221283.485	1053355.461	241.2266
SB13	220887.349	1052445.862	228.5297
SB14	220508.178	1051550.971	237.3612
SB15	219949.294	1050711.581	236.3103
SB16	219348.684	1049910.498	229.8865
SBM GA	219019.418	1049640.985	223.517
CSN 128P	222702.652	1056599.019	231.6843

Table 1.5 Adjusted Orthometric Heights from Spirit (Geodetic) Levelling

STATION ID	East	North	Height(h) from classical Levelling (m)
SBM KP	228934.946	1062478.392	240.905
SB01	227885.326	1061370.861	219.616
SB02	227020.63	1060830.955	222.943
SB03	226360.14	1060123.703	226.508
SB04	225716.18	1059425.578	207.691
SB05	225058.217	1058715.854	227.69
SB06	224363.139	1058076.3	222.727
SB07	223621.474	1057448.476	214.229
SB08	222899.033	1056814.499	226.505
SB09	222417.534	1055987.605	229.207
SB10	222031.132	1055113.772	224.666
SB11	221665.419	1054246.087	220.475
SB12	221283.485	1053355.461	230.744
SB13	220887.349	1052445.862	236.737
SB14	220508.178	1051550.971	223.624
SB15	219949.294	1050711.581	231.97

SB16	219348.684	1049910.498	230.442
SBM GA	219019.418	1049640.985	223.517
CSN 128P	222702.652	1056599.019	216.964

Table 1.6 The three orthometric heights

Station ID	classical Height (m)	Height from EMG96 model (m)	Geometrical Interpolated Height (m)
SBM KP	240.905	242.152	240.905
SB01	219.616	220.967	220.3357
SB02	222.943	224.292	224.0583
SB03	226.508	227.843	227.9939
SB04	207.691	207.872	208.399
SB05	227.69	227.96	228.8695
SB06	222.727	223.027	224.3076
SB07	214.229	214.541	216.1996
SB08	226.505	226.923	228.956
SB09	224.666	225.214	227.607
SB10	220.475	221.244	223.9873
SB11	230.747	231.738	234.8264
SB12	236.737	237.781	241.2266
SB13	223.624	224.717	228.5297
SB14	231.97	233.189	237.3612
SB15	230.442	231.757	236.3103
SB16	223.517	224.957	229.8865
SBM GA	216.964	218.433	223.517
CSN 128P	229.207	229.539	231.6843

Table 1.7: Variation of the two GPS derived orthometric heights from Classical levelling.

STATION ID	classical Height (m) [1]	Height from EGM96 model (m) [2]	Geometrical Interpolated Height [3]	Residual $[R_1]$=[2-1]	Residual $[R_2]$ = [3-1]
SBM KP	240.905	242.152	240.905	1.247	0
SB01	219.616	220.967	220.3357	1.351	0.719706

SB02	222.943	224.292	224.0583	1.349	1.115339
SB03	226.508	227.843	227.9939	1.335	1.485879
SB04	207.691	207.872	208.399	0.181	0.707997
SB05	227.69	227.96	228.8695	0.27	1.17955
SB06	222.727	223.027	224.3076	0.3	1.580569
SB07	214.229	214.541	216.1996	0.312	1.970617
SB08	226.505	226.923	228.956	0.418	2.450993
SB09	229.207	225.214	227.607	-3.993	-1.60005
SB10	224.666	221.244	223.9873	-3.422	-0.6787
SB11	220.475	231.738	234.8264	11.263	14.3514
SB12	230.744	237.781	241.2266	7.037	10.48263
SB13	236.737	224.717	228.5297	-12.02	-8.20734
SB14	223.624	233.189	237.3612	9.565	13.73718
SB15	231.97	231.757	236.3103	-0.213	4.340326
SB16	230.442	224.957	229.8865	-5.485	-0.55546
SBM GA	223.517	218.433	223.517	-5.084	0
CSN 128P	216.964	229.539	231.6843	12.575	14.72029

Table 1.8 Residuals Statistics

	Number of points	Minimum	Maximum	Std. Error
Residuals from EGM 96	18	-12.02	12.575	1.453236
Residuals from Interpolation	18	-8.20734	14.72029	1.450058

4.1 ANALYSIS OF RESULTS

From the numerical evaluations and the table of results (table 1.2 to 1.7) displayed above, it could be seen from the residuals in table 1.7 and the residual statistics in table 1.8 that the maximum and minimum residuals from EGM 96 model are 12.575m and -12.02m respectively, while the geometrical interpolation method (equation 8.0) gave a maximum and minimum residual values of 14.720m and -8.207m respectively. However, the standard errors of residuals for the two cases under investigation are 1.453m and 1.450m respectively for EGM 96 model and the geometrical approach. Also, the correlation statistics (pearson) gave a value of 95 which is an indication that the two sets of orthometric height derived are significantly correlated at 0.05 significant level.

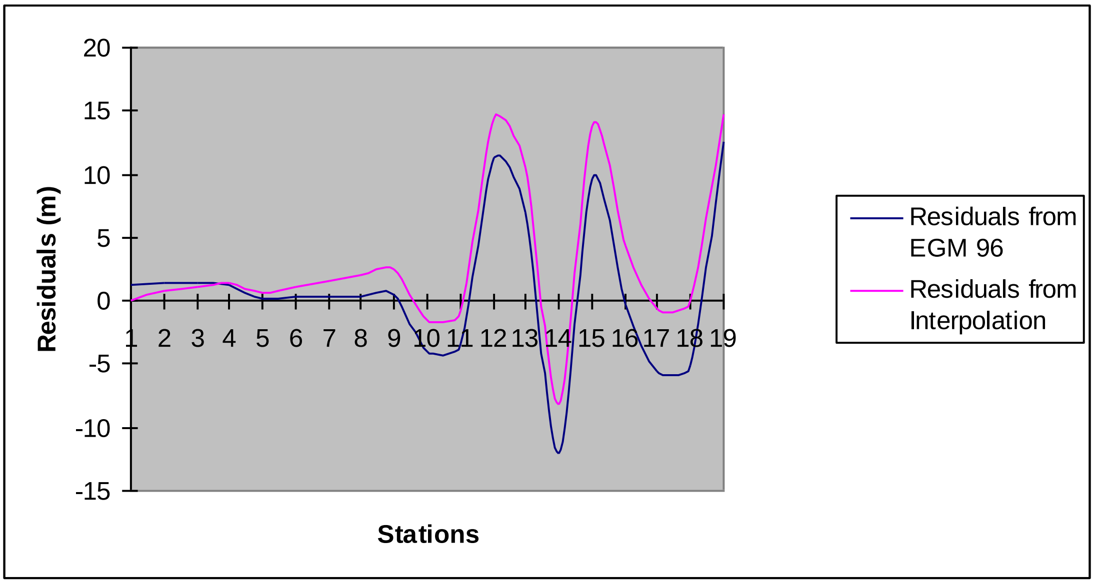

Fig. 3.0 Residuals plot for EGM96 model and geometrical interpolated heights

The graphical representation of the two residuals is as shown in figure 3.0. Furthermore, a close look at table 1.7 indicated that the geometric technique gave a zero residual for SBM GA which is one of the higher order height controls, while very large residuals values were noticed at points SB11, SB12, SB14 and CSN128P with fairly large ones at SB08, SB13 and SB15. Similarly for EGM96 model, very large residuals were observed at SB11, SB13, SB14 and CSN128P with fairly large ones at SB09, SB10, SB 12, SB16 and SBM GA. These can also be clearly discerned from the graph in figure 3.0 as the two curves suddenly became sinusoidal from station 9 (corresponding to SB08 in the tables). This similarity pattern of the residuals as observed from the graph further confirm the results of the correlation statics above and this makes it difficult to draw a conclusive judgment between the two methods examined; however, from the standard error of residuals (table 1.8), it could be inferred that the geometrical method gave a better result over EGM 96 model.

5.0 CONCLUSION

Derivation of orthometric height from GPS Survey along a level profile has been discussed; the possibility of adopting geometrical interpolation procedure for profile GPS survey in relative static mode and the use of EGM 96 model as a reference datum for height in GPS data post processing was also examined. The two techniques were evaluated with a view to identifying the preferred approach for GPS leveling in the study area. From the performance evaluation of the two methods, arriving at a conclusive judgment was difficult because they are significantly correlated having almost equal standard errors of residuals (about 1.5m), however, the geometrical technique whose standard error was slightly less (1.450m) was seen to have an edge over EGM96 model with standard error of (1.453m).

Therefore, the two methods showed no significant difference in the situation examined, but in view of our observation of the result from this exercise, it will be worthwhile to further examine the integrity of the EGM96 model as height datum (in GNSS solution software) for converting ellipsoidal height from GPS to height above the geoid and the use of geometrical technique, considering a network situation rather than profile; this is expected to yield a better result that will lead to a conclusive remark.

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