
MATHEMATICS: THE LANGUAGE OF SCIENCE AND ENGINEERING

Israel Agwu Etu and Kufre Esenowo Jack

*Department of Electrical/Electronic Engineering Technology
Akanu Ibiam Federal Polytechnic Unwana, Afikpo, Ebonyi State, Nigeria*

Abstract: Experience shows that it is impossible to produce a tangible result in science and engineering without the application of some mathematical principles. It could be in the form of proof or outright calculation. The basic thing is that mathematics must come into play before any assertion is made and that is what this paper tries to establish. The paper x-rays the characteristics of language and demonstrates how mathematics exhibits some of the basic ones that qualify it as a natural language for science and engineering. Advice is also given to teachers in these fields to demystify mathematics for the students, and to students, to embrace mathematics just the way they do their favorite subjects and not be afraid of it since it is unavoidable in these fields.

Introduction

Science is the study of nature and natural phenomena (Bridgman and Holton, 1997). Moreover, nature and natural phenomena have some laws that guide their activities. Thus science cannot do without laws and principles; not to talk of calculations which provide the mathematical path through which these laws and principles supply numerical values to scientific quantities. Mathematics can therefore be seen as the language of science; and science cannot do without its language (Etu, 2013). Engineering, on the other hand, has been simply defined as the study of the art of directing the great sources of power in nature for the use and convenience of humans (Barker, 1997). It seeks to bring things into being; therefore it requires the creative imagination to innovate useful applications of natural phenomena (Etu and Jack, 2012). Therefore engineering has to do with the practical application of scientific laws and principles to solve human problems. Furthermore, mathematics is heavily involved here, as it considers

standards (that is, quality of product or service), specification (that is, accuracy of dimensions and parameters), time consumption and cost of production (Etu and Okekenwa, 2011). In the same vein, mathematics can also be said to be the language of engineering and engineering cannot do without its language.

Language

Osgood (1998) defined language as the faculty and ability possessed by normal human beings and by no other species, of using a spoken or written utterance to represent mental phenomena or events. The primary purpose of language is to communicate among persons. Language also applies to the product of the faculty, both spoken and written. This paper concerns itself with the written nature of language since mathematics is written and not spoken. Mathematics is a written form of language used to prove formulae and solve numerical problems. The subsequent section shows the properties of language and how

mathematics possesses those properties.

Some Properties of Language vis-a-vis Mathematics

According to Osgood (1998), language consists of a set of 13 properties. However, this paper concerns itself with only those properties that are inclined to the written and academic aspects of language:

- Language is transmitted by learning and stabilized by correction from parents, peers, teachers, and so on. In other words, it is not instinctive and automatic behaviour, like in the case of honeybees. Nevertheless, the human brain appears to incorporate an innate and perhaps partly instinctive language capability. This explains why teachers start by teaching kindergarten pupils how to count from 1 to 10 or 20 or 100 as the case may be, and whenever they make a mistake they correct them until they begin to get it right. However, the innate and instinctive language capability of the brain explains why an illiterate can count money correctly, take measurements correctly, etc.
- It is used regularly to produce effects on behaviour by presenting the mind and will of the speaker and therefore is not limited to mere self-expression or the automatic response to outer stimuli.

For instance, if the speaker has in mind to say that, for a force applied at an angle to an object to move the object, the angle of application of the force has to be well chosen; it

will be difficult for a layman to understand except by mathematical demonstration as shown below.

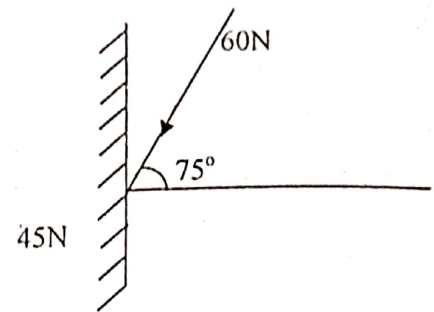


Fig.1. Force of 60N applied at an angle to an object of 45N weight.

The vertical component of the 60-N force is determined by resolving as follows:

$$\begin{aligned}F_v &= F \sin \theta \\ &= 60 \sin 75^\circ \\ &= 60 \times 0.9659 \\ F_v &= 57.96N\end{aligned}$$

The horizontal component of the 60-N force is determined by resolving as follows:

$$\begin{aligned}F_h &= F \cos \theta \\ &= 60 \cos 75^\circ \\ &= 60 \times 0.2588 \\ F_h &= 15.53N\end{aligned}$$

It is clear that this 60-N force cannot move the object in question because for the object to move, the horizontal component of the force (which is responsible for moving the object) must be greater than the weight of the object (in this case, 45N). Unfortunately, in this case, the horizontal component of the force is far less than the weight of the object that is 15.53N as against 45N weight).

By the above example, mathematics has helped the speaker to easily

express his mind. Again, for a speaker to summarily say that the $\cos x$ does not sound convincing except by going through the proof as shown below.

derivative of $\sin x$ is

$$\text{Let } \sin x = y$$

$$\text{ie, } y = \sin x$$

Taking increments in x and y , we have

$$y + \delta y = \sin(x + \delta x)$$

$$\delta y = \sin(x + \delta x) - y$$

$$= \sin(x + \delta x) - \sin x$$

$$\text{Recall, } \sin A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$$

$$\text{This implies, } \delta y = 2 \cos \frac{1}{2} (x + \delta x + x) \sin \frac{1}{2} (x + \delta x - x)$$

$$= 2 \cos \frac{1}{2} (2x + \delta x) \sin \frac{1}{2} (\delta x)$$

$$= 2 \cos \left(x + \frac{\delta x}{2}\right) \sin \frac{\delta x}{2}$$

Dividing both sides by δx , we have

$$\frac{\delta y}{\delta x} = \frac{2 \cos \left(x + \frac{\delta x}{2}\right) \sin \frac{\delta x}{2}}{\delta x}$$

Dividing both the numerator and the denominator by 2, we have

$$\frac{\delta y}{\delta x} = \cos \left(x + \frac{\delta x}{2}\right) \sin \frac{\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

Taking \lim of both sides

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \cos \left(x + \frac{\delta x}{2}\right) \lim_{\delta x \rightarrow 0} \sin \frac{\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$\text{Recall, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{\delta x \rightarrow 0} \sin \frac{\frac{\delta x}{2}}{\frac{\delta x}{2}} = 1$$

$$\therefore \frac{dy}{dx} = \cos x \times 1$$

$$= \cos x$$

- Language comprises rules of reference or word definition more or less agreed upon by the speakers. The use of rules of reference is everywhere in mathematics. Whenever any of the rules is violated, the result is a wrong result in practical terms. For instance, if someone is carrying a crate of 40 eggs and by accident he falls and breaks 5,

and yet again 12 fall off and break, to calculate the total number of broken eggs and the remainder, one would have to first of all add the broken ones. Mathematically it would be presented as $40 - 5 - 12$

$$\text{ie, } 40 - (5 + 12)$$

$$40 - 17$$

$$\text{ie, } 23 \text{ eggs}$$

the bracket would change when the bracket opens.

This implies that a total of 17 eggs were broken, remaining 23 eggs. The above example obeys the rule which states that if a minus sign precedes a bracket, the signs inside

Furthermore, let us see the rules involved in solving for the currents flowing in the circuit shown below.

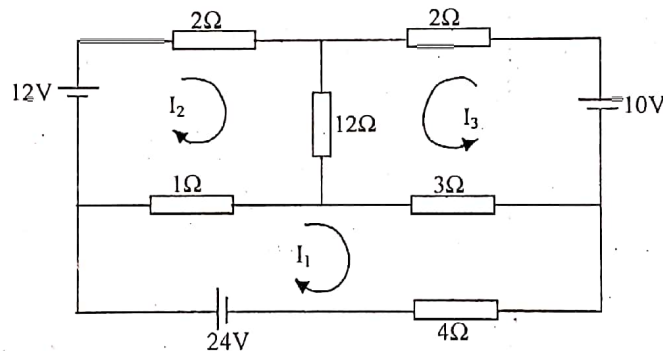


Fig.2: A D.C. circuit showing mesh currents

Rule 1: Transform the circuit to a system of linear equations using Kirchhoff's Voltage Law. From the 1st chamber,

$$24 - 1(I_1 - I_2) - 3(I_1 + I_3) - 4I_1 = 0$$

$$24 - I_1 + I_2 - 3I_1 - 3I_3 - 4I_1 = 0$$

$$24 - 8I_1 + I_2 - 3I_3 = 0$$

$$-8I_1 + I_2 - 3I_3 = -24$$

$$8I_1 - I_2 + 3I_3 = 24 \dots \dots \dots (1)$$

From the 2nd chamber,

$$12 - 2I_2 - 12(I_2 + I_3) - 1(I_2 - I_1) = 0$$

$$12 - 2I_2 - 12I_2 - 12I_3 - I_2 + I_1 = 0$$

$$12 + I_1 - 15I_2 - 12I_3 = 0$$

$$I_1 - 15I_2 - 12I_3 = -12 \dots \dots \dots (2)$$

From the 3rd chamber,

$$10 - 3(I_1 + I_3) - 12(I_2 + I_3) - 2I_3 = 0$$

$$10 - 3I_1 - 3I_3 - 12I_2 - 12I_3 - 2I_3 = 0$$

$$10 - I_1 - 12I_2 - 17I_3 = 0$$

$$-3I_1 - 12I_2 - 17I_3 = -10$$

$$3I_1 + 12I_2 + 17I_3 = 10 \dots \dots \dots (3)$$

Rule 2: Express the 3 linear equations

$$8I_1 - I_2 + 3I_3 = 24$$

$$I_1 - 15I_2 - 12I_3 = -12$$

$$3I_1 + 12I_2 + 17I_3 = 10$$

Rule 3: Translate the linear equations to matrices

$$\begin{pmatrix} 8 & -1 & 3 \\ 1 & -15 & -12 \\ 3 & 12 & 17 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 24 \\ -12 \\ 10 \end{pmatrix}$$

$A \qquad \qquad B \qquad \qquad C$

Rule 4: Find the determinant of matrix A

$$|A| = \begin{vmatrix} 8 & -1 & 3 \\ 1 & -15 & -12 \\ 3 & 12 & 17 \end{vmatrix}$$

The rule of finding the determinant of a 3×3 matrix is given by

$$|A| = 8 \begin{vmatrix} -15 & -12 \\ 12 & 17 \end{vmatrix} - 1 \begin{vmatrix} 1 & -12 \\ 3 & 17 \end{vmatrix} + 3 \begin{vmatrix} 1 & -15 \\ 3 & 12 \end{vmatrix}$$

$$= 8(-15 \times 17 - 12 \times -12) + (1 \times 17 - 3 \times -12) + 3(1 \times 12 - 3 \times -15)$$

$$= 8(-255 + 144) + (17 + 36) + 3(12 + 45)$$

$$(8 \times -111) + 53 + (3 \times 57)$$

$$-888 + 53 + 171$$

$$|A| = -664$$

Rule 5: Replace the first column of matrix A with the voltage coefficients and find the determinant of the new matrix (i.e. matrix A_1)

$$|A_1| = \begin{vmatrix} 24 & -1 & 3 \\ -12 & -15 & -12 \\ 10 & 12 & 17 \end{vmatrix}$$

$$= 24 \begin{vmatrix} -15 & -12 \\ 12 & 17 \end{vmatrix} - 1 \begin{vmatrix} -12 & -12 \\ 10 & 17 \end{vmatrix} + 3 \begin{vmatrix} -12 & -15 \\ 10 & 12 \end{vmatrix}$$

$$= 24(-15 \times 17 - 12 \times -12) + (-12 \times 17 - 10 \times -12) + 3(-12 \times 12 - 10 \times -15)$$

$$\begin{aligned}
 &= 24(-255 + 144) + (-204 + 120) + 3(-144 + 150) \\
 &= (24 \times -111) + (-84) + 3(6) \\
 &= -2664 - 84 + 18 \\
 \therefore |A_1| &= -2730
 \end{aligned}$$

Rule 6: Calculate I_1 by dividing $|A_1|$ by $|A|$.

$$\begin{aligned}
 I_1 &= \frac{|A_1|}{|A|} \\
 &= \frac{-2730}{-664} \\
 \therefore I_1 &= 4.11A
 \end{aligned}$$

Rule 7: Solve for I_2 in the same manner; this time, replacing the second column of A with the voltage coefficients.

$$\begin{aligned}
 |A_2| &= \begin{vmatrix} 8 & 24 & 3 \\ 1 & -12 & -12 \\ 3 & 10 & 17 \end{vmatrix} \\
 &= 8 \begin{vmatrix} -12 & -12 \\ 10 & 17 \end{vmatrix} - 24 \begin{vmatrix} 1 & -12 \\ 3 & 17 \end{vmatrix} + 3 \begin{vmatrix} 1 & -12 \\ 3 & 10 \end{vmatrix} \\
 &= 8(-12 \times 17 - 10 \times -12) - 24(1 \times 17 - 3 \times -12) \\
 &\quad + 3(1 \times 10 - 3 \times -12) \\
 &= 8(-204 + 120) - 24(17 + 36) + 3(10 + 36) \\
 &= (8 \times -84) - (24 \times 53) + (3 \times 46) \\
 &= -672 - 1272 + 138 \\
 |A_2| &= -1806 \\
 I_2 &= \frac{|A_2|}{|A|} \\
 &= \frac{-1806}{-664} \\
 \therefore I_2 &= 2.72A
 \end{aligned}$$

Rule 8: Solve for I_3 in the same manner; this time, replacing the third column with the voltage coefficients.

$$\begin{aligned}
 |A_3| &= \begin{vmatrix} 8 & -1 & 24 \\ 1 & -15 & -12 \\ 3 & 12 & 10 \end{vmatrix} \\
 &= 8 \begin{vmatrix} -15 & -12 \\ 12 & 10 \end{vmatrix} - \begin{vmatrix} 1 & -12 \\ 3 & 10 \end{vmatrix} + 24 \begin{vmatrix} 1 & -15 \\ 3 & 12 \end{vmatrix} \\
 &= 8(-15 \times 10 - 12 \times -12) + (1 \times 10 - 3 \times -12) + 24(1 \times 12 - 3 \times -15) \\
 &= 8(-150 + 144) + (10 + 36) + 24(12 + 45) \\
 &= (8 \times -6) + 46 + (24 \times 57) \\
 &= -48 + 46 + 1368 \\
 \therefore |A_3| &= 1366 \\
 I_3 &= \frac{|A_3|}{|A|}
 \end{aligned}$$

$$= \frac{1366}{-664}$$

$$\therefore I_3 = -2.06A$$

(This means that the polarity of the 10V power supply unit should be reversed)

By the above calculation, it can be seen that mathematics is governed by rules from step to step just like a normal language. It is a system of interlocking systems, displaying a hierarchical organization in which, for example, units of sound at the lowest level combine to form units of word-meaning at the next, which in turn combine to form syntactic structures of more complex meaning at the next which combine into sentences, paragraphs and longer discourses at the next. Grammatical analysis seeks to define the units and describe their combination into sentences. In mathematics, numbers combine with other numbers and letters together with mathematical symbols to form expressions and equations, from where solutions could be derived. This can be seen in the following proof:

$$a. m \frac{dv}{dt} = mg - kv^2$$

$$b. mv \frac{dv}{dx} = mg - kv^2$$

'b' is more appropriate because it contains x (i.e. distance) which is given in the question.

$$mv \frac{dv}{dx} = mg - kv^2$$

$$\frac{dv}{dx} = \frac{1}{mv} (mg - kv^2)$$

$$\frac{mv}{mg - kv^2} dv = dx$$

$$\int \frac{mv}{mg - kv^2} dv = \int dx$$

Question: A

particle of mass, m falls from rest at a height, x above the ground and the retardation due to air resistance is kv^2 where k is a constant and v is the speed of the particle at any time, t . Show that the speed with which it reaches the ground is

$$V = \sqrt{\frac{mg}{k} \left(1 - e^{-\frac{2kx}{m}}\right)}$$

Solution: When the particle is released from rest, the weight (mg) acts downwards while the air resistance kv^2 acts in opposite direction. Thus, the effective or resultant force on the particle is

$$mg - kv^2$$

Applying Newton's second law of motion, the options of the expression to use are:

$$m \int \frac{v}{mg - kv^2} dv = \int dx$$

Since the differentiation of $mg - kv^2$ gives a multiple of v ($i \cdot e \cdot -2kv$), we accept the method below.

$$\frac{m}{-2k} \int \frac{-2kv}{mg - kv^2} dv = \int dx$$

$$\frac{-m}{2k} \ln(mg - kv^2) = x + c \dots \dots \dots (1)$$

when $t = 0, v = 0$ and $x = 0$, eqn (1) will result to

$$c = \frac{-m}{2k} \ln(mg)$$

substitute c in eqn (1) above

$$\frac{-m}{2k} \ln(mg - kv^2) = x - \frac{m}{2k} \ln(mg)$$

$$\frac{m}{2k} [\ln(mg) - \ln(mg - kv^2)] = x$$

$$\frac{m}{2k} \ln\left(\frac{mg}{mg - kv^2}\right) = x$$

$$\ln\left(\frac{mg}{mg - kv^2}\right) = \frac{2kx}{m}$$

Recall, $e^{\ln a} = a$

This is got by saying

$$y = e^x$$

$$\log_e y = x$$

$$e^{\ln y} = e^x = y$$

$$\therefore e^{\ln y} = y$$

Applying it to the problem, we have

$$e^{\ln\left(\frac{mg}{mg - kv^2}\right)} = e^{\frac{2kx}{m}}$$

$$\frac{mg}{mg - kv^2} = e^{\frac{2kx}{m}}$$

$$mg = mge^{\frac{2kx}{m}} - kv^2 e^{\frac{2kx}{m}}$$

$$kv^2 e^{\frac{2kx}{m}} = mge^{\frac{2kx}{m}} - mg$$

$$v^2 = \frac{mge^{\frac{2kx}{m}}}{ke^{\frac{2kx}{m}}} - \frac{mg}{ke^{\frac{2kx}{m}}}$$

$$= \frac{mg}{k} - \frac{mg}{k} e^{-\frac{2kx}{m}}$$

$$v^2 = \frac{mg}{k} \left(1 - e^{-\frac{2kx}{m}}\right)$$

$$\therefore V = \sqrt{\frac{mg}{k} \left(1 - e^{-\frac{2kx}{m}}\right)}$$

Language displays an infinite productivity or creativity as a result; that is, using relatively small means, a normal language-user to utter or understand what is constantly and endlessly new. This property is demonstrated in the next example where Gauss-Seidel iteration method is used to solve a system of

linear equations. In this solution procedure, it is possible to produce infinite answers or results and each of the answers can be useful to the user. Question: Solve, correct to 4 decimal places, the system of linear equations using Gauss-Seidel iteration method.

$$0.9411x_1 - 0.0175x_2 + 0.1463x_3 = 0.631 \dots \dots \dots (1)$$

$$-0.8641x_1 - 0.4243x_2 + 0.0711x_3 = 0.2581 \dots \dots \dots (2)$$

$$0.364x_1 + 0.1573x_2 + 0.8642x_3 = -0.7551 \dots \dots \dots (3)$$

Solution:

$$\text{From (1): } x_1 = \frac{0.631 + 0.0175x_2 - 0.1463x_3}{0.9411}$$

$$\text{From (2): } x_2 = \frac{0.2501 + 0.8641x_1 - 0.0711x_3}{-0.4243}$$

$$\text{From (3): } x_3 = \frac{-0.7551 - 0.3641x_1 - 0.1573x_2}{0.8642}$$

Assume $x_1 = x_2 = x_3 = 0$

From the 1st iteration formula

$$x_1 = \frac{0.631 + 0 + 0}{0.9411}$$

$$= 0.670492$$

From the 2nd iteration formula

$$x_1 = 0.670492; x_3 = 0$$

$$x_2 = \frac{0.2501 + (0.8641)(0.670492) - 0}{-0.4243}$$

$$x_2 = -1.954919$$

From the 3rd iteration formula

$$x_1 = 0.670492; x_2 = -1.754919$$

$$x_3 = -\frac{[0.7551 + (0.3641)(0.670492) + (0.1573)(-1.954919)]}{0.8642}$$

$$\therefore x_3 = -0.800414$$

With $x_2 = -1.954919$ and x_3

$$= -0.800414, \text{ from the 1st iteration formula}$$

$$x_1 = \frac{0.631 + (0.0175)(-1.954919) - (0.1463)(-0.800414)}{0.9411}$$

$$x_1 = 0.758569$$

With $x_1 = 0.758569$ and x_3

$$= -0.800414, \text{ from the 2nd iteration formula}$$

$$x_2 = \frac{0.2501 + (0.8641)(0.758569) - (0.0711)(-0.800414)}{-0.4243}$$

$$\therefore x_2 = -2.268416$$

With $x_1 = 0.758569$ and $x_2 = -2.268416$, from 3rd iteration formula

$$x_3 = \frac{-[0.7551 + (0.3641)(0.758569) + (0.1573)(-2.268416)]}{0.8642}$$

$$\therefore x_3 = -0.780459$$

With $x_2 = -2.268416$ and x_3

$$= -0.780459, \text{ from the 1st iteration formula}$$

$$x_1 = \frac{0.631 + (0.0175)(-2.268416) - (0.1463)(-0.780459)}{0.9411}$$

$$\therefore x_1 = 0.749638$$

With $x_1 = 0.749638$ and x_3

$$= -0.780459, \text{ from the 2nd iteration formula}$$

$$x_2 = \frac{0 \cdot 2501 + (0 \cdot 8641)(0 \cdot 749638) - (0 \cdot 0711)(-0 \cdot 780460)}{-0 \cdot 4243}$$

$$\therefore x_2 = -2 \cdot 246884$$

With $x_1 = 0 \cdot 749638$ and x_2

$= -2 \cdot 246884$, from the 3rd iteration formula

$$x_3 = - \frac{[0 \cdot 7551 + (0 \cdot 3641)(0 \cdot 749638) + (0 \cdot 1573)(-2 \cdot 246884)]}{0 \cdot 8642}$$

$$\therefore x_3 = -0 \cdot 780616$$

Below are the results so far

x_1	x_2	x_3
0.0000	0.0000	0.0000
0.6705	-1.9549	-0.8004
0.7586	-2.2684	-0.7805
0.7496	-2.2469	-0.7806

More iteration would generate more results and the process is capable of generating infinite new results just like a normal language is capable of generating infinite words, sentences, paragraphs, etc. Language operates in time, so that each utterance has a linear or "left-to-right", property. This requires, in the normal substratum of language,

the capacity for temporary and rapid non-linear integration of information. This property is seen in the following example that has to do with object acting under the influence of time. The simple integration of expression helps to derive a system of equation for determining the distance covered by the object after a time, t.

Question: A body moves in a straight line and its velocity after t seconds is given by $36 - 4t$. The distance of the body from a fixed point on the time after t seconds is s metres and $s = 40$. When $t = 1$, find s in terms of t.

Solution: $V = 36 - 4t$

but velocity $v = \frac{\text{distance, } s}{\text{time, } t}$

$$\frac{ds}{dt} = 36 - 4t$$

$$ds = (36 - 4t)dt$$

$$\int ds = \int (36 - 4t)dt$$

$$s = 36t - \frac{4t^2}{2} + c$$

$$s = 36t - 2t^2 + c$$

$$s = 40 \text{ when } t = 1$$

$$40 = 36t - 2t^2 + c$$

$$= 36(1) - 2 + c$$

$$40 = 36 - 2 + c$$

$$\therefore c = 40 - 34$$

$$= 6$$

$$\therefore S = 36t - 2t^2 + 6$$

Conclusion

By the foregoing, this paper has tried to show that mathematics possesses some key characteristics of normal language and that it is indispensable in the world of science and engineering. All the calculations done in the paper are addressing pure science cases or outright engineering cases, showing that it is really the day-to-day tool of these two fields of study. This is akin to the way language is being used as a day-to-day tool in normal living and communication. It is also the advice of this paper that science- and engineering-based students should embrace mathematics since they cannot do without it, seeing that it is only by so doing that they can be well grounded in their area of specialization.

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