

Computational Analysis of a one-dimensional nonlinear reactive contaminant flow with an initial continuous point source by homotopy-perturbation method.

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Abstract

In this paper, a Homotopy-perturbation analysis of a non-linear reactive contaminant flow equation with initial continuous point source is provided. The equation is described by advection, diffusion and adsorption. We assume that the adsorption term is modeled by Freundlich Isotherm. We provide an approximation of this equation using homotopy-perturbation transformation and solve the resulting linear equations analytically. The graphs of the concentration against the distance, reaction parameter and time are presented and analyzed to determine the effects of increase in the reaction coefficient, time and distance on the concentration. Findings from this research show that the concentration of the contaminant decreases with time and decreases faster when the value of the reaction parameter α is high.

Keywords: Homotopy-perturbation, contaminant, advection, diffusion, adsorption, reaction.

1.0 Introduction

Worldwide, human Society has generated a large number of chemical substances that often find their way into the environment, either intentionally applied during agricultural practices or unintentionally released from leaking and municipal wastes disposal sites, or industrial related activities. Consequently, as many of these chemicals cause a significant health risk when they enter the food chain, contamination of both surface and subsurface water supplies has become a critical issue. A high percentage of drinking water in Nigeria comes from groundwater. The contamination of groundwater systems is a serious matter of discourse in assessing the hazards of public health. In order to predict the fate of such pollutants during their transport, a tasking job for hydro-geologist and scientists emerges. The problem involves defining the flow lines of groundwater in the aquifers, the travel time of the water along the flow lines and to predict the chemical reaction which alters the concentrations during transport.

Several researches have been carried out in the study of hydrodynamic dispersion with various initial and boundary conditions. Aiyesimi and co-workers [1-3] studied contaminant flow problems by method of perturbation, while some other authors [4, 5] employed numerical methods and Laplace Transform method [6, 7]. Another well-known method is the Homotopy perturbation method. The Homotopy perturbation method was used to solve wide range of physical problems, eliminating the limitations of perturbation method [8-11]. Most of the researches done in the past either neglects the non-linear term or considers it as a constant, and they are mostly nonreactive. In this paper, we provide an analytical solution for the non-linear reactive contaminant transport problem with initial continuous point source thereby obtaining a more accurate solution by homotopy-perturbation method.

2.0 Formulation of the problem

We consider an incompressible fluid flow through a homogeneous, saturated porous medium where the fluid is not solute-free, i.e. contaminated with solute of concentration $C(x,t)$. The following assumptions are made (i) The flow is steady (ii) the solute transport is described by advection, molecular diffusion and mechanical dispersion (iii) the flow is one dimensional and in x-direction.

Under these assumptions, mass conservation of the contaminant may be combined with a mathematical expression of the relevant process to obtain a differential equation describing flow [12]. This gives rise to the reactive contaminant flow equation below.

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Journal of the Nigerian Association of Mathematical Physics Volume 22 (November, 2012), 543 – 546

$$C_t + UC_x + \frac{pb}{n}S_t - DC_{xx} + \alpha C = 0, \quad 0 < x < \infty, t > 0 \tag{1}$$

where $S(x, t)$ is the mass of the contaminant absorbed on the solid matrix per unit mass of the solid, $p_b > 0$ is the bulk density of the porous medium, $n > 0$ is the porosity, $D > 0$ accounts for both molecular diffusion and mechanical dispersion, U is the fluid velocity, α is the reaction parameter and αC the reaction term.

Problem solution

We consider the non-linear flow equation (1):

$$C_t + UC_x + \frac{pb}{n}S_t - DC_{xx} + \alpha C = 0, \quad \alpha > 0, 0 < x < \infty \tag{2}$$

Equation (1) can be rewritten as

$$\frac{\partial C}{\partial t} + \frac{\partial \phi(C)}{\partial t} + U \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial x^2} + \alpha C = 0, \tag{3}$$

which can also be expressed in the form:

$$\frac{\partial C}{\partial t} + \frac{\partial \phi}{\partial C} \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial x^2} + \alpha C = 0 \tag{4}$$

This can again be expressed as

$$\frac{\partial C}{\partial t} + \varepsilon \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial x^2} + \alpha C = 0$$

where we define $\varepsilon = \frac{\partial \phi}{\partial C}$, the perturbation parameter.

That is,

$$(1 + \varepsilon) \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial x^2} + \alpha C = 0, C(x, 0) = Ae^{-\lambda x} \tag{5}$$

$$C(0, t) = A \text{ and } C(\infty, t) = 0, t > 0;$$

where $C(x,t)$ is the concentration of the contaminant, t is the time, x is distance of the contaminant from the point source, and αC is the reaction term.

By the Homotopy-perturbation transformation equation:

$$H(v, p) = \left[(1 - p) \frac{\partial C}{\partial t} \right] + p \left[(1 + \varepsilon) \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} - D \frac{\partial^2 C}{\partial x^2} + \alpha C \right] = 0 \tag{6}$$

$$\text{Let, } C(x, t) = v^{(0)} + pv^{(1)} + p^2v^{(2)} + p^3v^{(3)} + \dots \tag{7}$$

By substituting (7) in (6) and comparing the coefficients of like terms on both sides, we have the following set of equations:

$$p^0: (1 + \varepsilon) \frac{\partial}{\partial t} v^{(0)}(x, t) = 0 \tag{8}$$

$$p^1: (1 + \varepsilon) \frac{\partial}{\partial t} v^{(1)}(x, t) + U \frac{\partial}{\partial x} v^{(0)}(x, t) - D \frac{\partial^2}{\partial x^2} v^{(0)}(x, t) + \alpha v^{(0)}(x, t) = 0 \tag{9}$$

$$p^2: (1 + \varepsilon) \frac{\partial}{\partial t} v^{(2)}(x, t) + U \frac{\partial}{\partial x} v^{(1)}(x, t) - D \frac{\partial^2}{\partial x^2} v^{(1)}(x, t) + \alpha v^{(1)}(x, t) = 0 \tag{10}$$

$$p^3: (1 + \varepsilon) \frac{\partial}{\partial t} v^{(3)}(x, t) + U \frac{\partial}{\partial x} v^{(2)}(x, t) - D \frac{\partial^2}{\partial x^2} v^{(2)}(x, t) + \alpha v^{(2)}(x, t) = 0 \tag{11}$$

$$p^4: (1 + \varepsilon) \frac{\partial}{\partial t} v^{(4)}(x, t) + U \frac{\partial}{\partial x} v^{(3)}(x, t) - D \frac{\partial^2}{\partial x^2} v^{(3)}(x, t) + \alpha v^{(3)}(x, t) = 0 \tag{12}$$

⋮
⋮
⋮

$$p^n: (1 + \varepsilon) \frac{\partial}{\partial t} v^{(n)}(x, t) + Lv^{(n-1)}(x, t) = 0; \quad L = \left(U \frac{\partial}{\partial x} - D \frac{\partial^2}{\partial x^2} + \alpha \right). \tag{13}$$

Equation (8) admits an initial solution of the form:

$$v^{(0)}(x, t) = Ae^{-\lambda x}, \tag{14}$$

such that

$$v^{(0)}(x, 0) = Ae^{-\lambda x}, A > 0.$$

$$v^{(0)}(0, t) = A, \text{ and } v^{(0)}(\infty, t) = 0.$$

By solving equation (9) which is of order 1, we have

$$v^{(1)}(x, t) = \frac{Ae^{-\lambda x}(-\alpha + U\lambda + D\lambda^2)t}{1 + \varepsilon}, \quad \therefore v^{(0)}(x, 0) = 0 \tag{15}$$

Similarly, the solutions of order 2 and 3 equations are respectively

$$v^{(2)}(x, t) = \frac{1}{2} \frac{Ae^{-\lambda x}t^2(-\alpha + U\lambda + D\lambda^2)^2}{(1 + \varepsilon)^2}, \quad \therefore v^{(2)}(x, 0) = 0 \tag{16}$$

$$v^{(3)}(x, t) = \frac{1}{6} \frac{Ae^{-\lambda x}t^3(-\alpha + U\lambda + D\lambda^2)^3}{(1 + \varepsilon)^3}, \quad \therefore v^{(3)}(x, 0) = 0 \tag{17}$$

The solution of the contaminant flow problem which is expressed in the form:

$$C(x, t) = \lim_{p \rightarrow 1} (v^{(0)} + pv^{(1)} + p^2v^{(2)} + p^3v^{(3)} + \dots) \tag{18}$$

is obtained as,

$$C(x, t) = Ae^{-\lambda x} + \frac{Ae^{-\lambda x}(-\alpha + U\lambda + D\lambda^2)t}{1+\varepsilon} + \frac{1}{2} \frac{Ae^{-\lambda x}t^2(-\alpha + U\lambda + D\lambda^2)^2}{(1+\varepsilon)^2} + \frac{1}{6} \frac{Ae^{-\lambda x}t^3(-\alpha + U\lambda + D\lambda^2)^3}{(1+\varepsilon)^3} + o(p^4)$$

$$= [1 + \frac{t(D\lambda^2 + U\lambda - \alpha)}{1+\varepsilon} + \frac{t^2}{2} (\frac{D\lambda^2 + U\lambda - \alpha}{1+\varepsilon})^2 + \frac{t^3}{6} (\frac{D\lambda^2 + U\lambda - \alpha}{1+\varepsilon})^3 + \dots] Ae^{-\lambda x} \tag{19}$$

$$= Ae^{\frac{t(D\lambda^2 + U\lambda - \alpha)}{1+\varepsilon} - \lambda x} \tag{20}$$

3.0 Results and Discussion

The analytical solution provided in equation (20) is discussed with the help of input data in graphs as follows. The concentration is plotted against distance with α varying as 0.2, 1, 5, 10 and the values $U=0.1, D=0.01, \lambda=1, t=1, \varepsilon = 0.001, A=1$ kept constant as we shall see in Figure 1. Next, the concentration is plotted against the time t as we vary x as 0.1, 0.4, 0.7, 1 and the values $U=0.1, D=0.01, \lambda=1, \alpha = 0.2, \varepsilon = 0.001, A=1$ are fixed. The graph is shown in Figure 2. Finally, the concentration is plotted against the reaction parameter α when the values $U=0.1, D=0.01, \lambda=1, t=1, \varepsilon = 0.001, A=1$ kept constant and α takes values from 0 to 1 and the graph is shown in Figure 3.

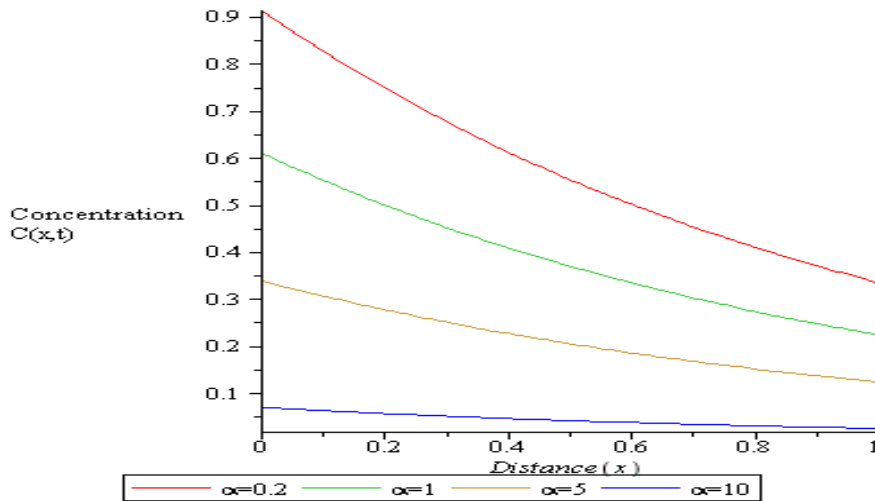


Figure 1: Graph of concentration against distance.

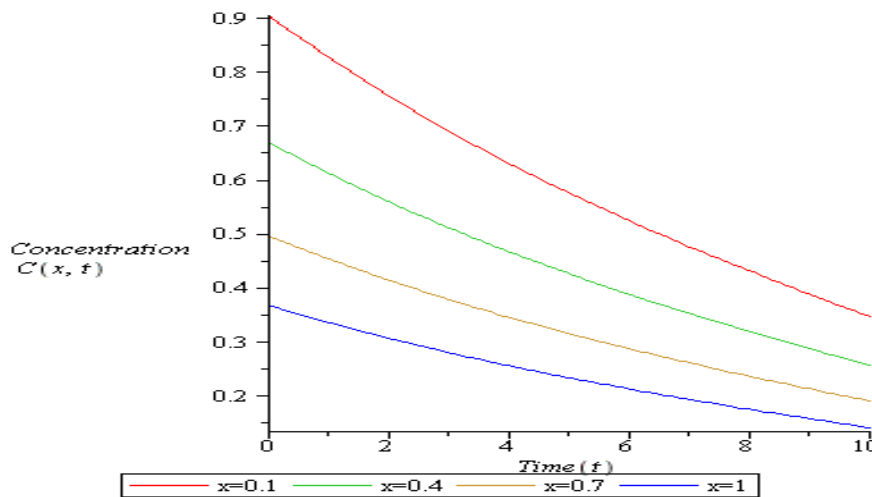


Figure 2: Graph of concentration against time.

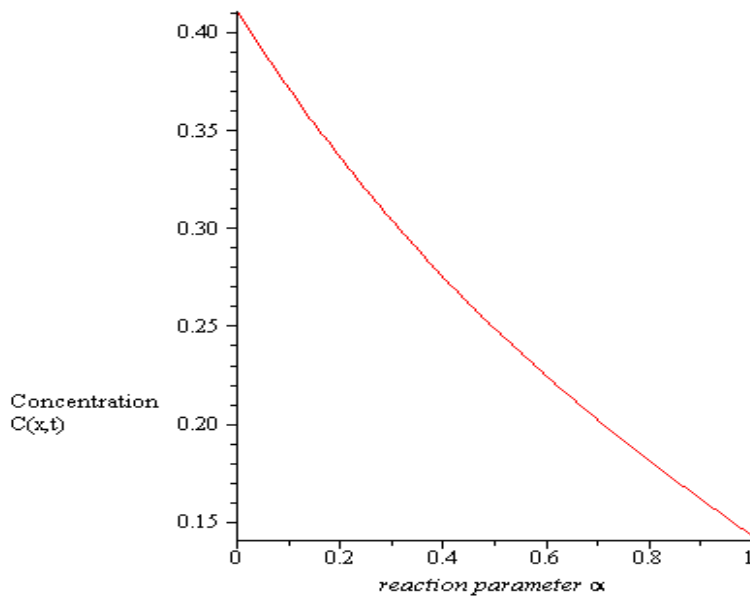


Figure 3: Graph of Concentration against reaction parameter α .

4.0 Conclusion

In this work, the analytical solution with the input data shows that the concentration drops as time increases and the concentration decreases as the distance increases. Finally, the higher the reaction parameter α , the faster the concentration of contaminant decreases.

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