

Optimal Coverage Analysis of Existing Public Facility using the Capacitated P-Center Model

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Abstract

P-centre problem algorithm has been one of the most famous ways of providing solutions in locating facilities. Despite the frequent usage of this algorithm, research on the integration of Geographical Information Systems (GIS) and the p-centre problem has not been much explored. This research uses the P-Centre problem approach in providing solution to determining optimal location of existing public facilities within Shomolu Local Government Area with the primary goal of minimizing the maximum travel distance from each demand node to the closest facility. The optimum and tolerable distances of Primary and Secondary Schools from the residential areas were determined. Spatial analysis was carried out on the sufficiency or otherwise of Primary and Secondary Schools within the Local Government Area. Multi-criteria queries were carried out and results revealed that between 2011 and 2015 the Local Government Area did not have sufficient number of schools to meet the demand, while from 2016 – 2020 schools will be grossly inadequate. Polynomials were derived for these deficiencies which can be used to predict future deficiencies if relevant government agencies fail to provide more schools as the population increases.

Keywords:

Geographical information systems (GIS), optimal location-allocation, facilities, spatial analysis, residential and service area.

1. Introduction and Concept

Facility location decisions are critical elements in strategic planning for a wide range of private and public firms: Consequent upon the high cost of locating or relocating facilities coupled with the numerous implications that a facility's location impact on the operationality and profitability of the organisation, facility location or relocation projects are generally long-term investments which if sited, it is expected to remain in place and in operation for a long time. Thus, decision makers must select sites that will not simply perform well according to the current system state, but that will continue to be profitable for the facility's lifetime, even as environmental factors change, populations shift and evolving market trends. Finding robust facility locations is thus a difficult task, demanding that decision makers account for uncertain future events (Daskin, 1996).

According to Snyder and Daskin (2005), the goal of the location-allocation problem's solution, is to find the best locations or location to fit one or more facilities which will make the highest utility value from criterion or multiple criteria. Bad location of the facility has negative effect in terms of providing services to the beneficiary. Distance from the area of supply and the area of demand should be optimal. If location of the facility is far from populated area (area of demand) beneficiary may not be able or interested to take the service from the facility. Location of the facility should be well distributed such that capacity of the facilities can meet all the demands. So with optimal distance, capacity of the facility needs to be considered when taking decision.

Generally, there are four components that characterize location problems: (1) customers are to be allocated to facilities, which are presumed to be located at points or on routes, (2) facilities to be located, (3) a space in which customers and facilities are located, and (4) a metric that indicates distances or times between customers and facilities (ReVelle, 2005). Where to locate a facility is a function of the sector concerned: private or public sector. Private sector seeks location of sites that maximize a profit, while the public sector seeks facility sites that optimize the population's access to those facilities or universality of the service (Marin et al, 2009).

Since it is impossible to locate facilities at every demand point because of limited resources; there is need to assess and address inadequacies in public facilities with available resources (NFPA, 2010). In essence, facility location

should be done objectively with a view of meeting demand rather than allowing unethical and political will to prevail while taking such important decisions (Rushton, 1984).

In more recent times when government espoused egalitarianism as a cornerstone of development (Nigeria, 1970), the location of public facilities has been seen more in terms of the equalization of their distribution within the six geopolitical zones into which Nigeria is divided and not usually in terms of sizes of facility and functionality. This political approach of geopolitical equity in distribution is subject to much bias. The nature of the problems created by political biasedness in allocation facilities are far from satisfying equity and egalitarianism. These problems are pronounced in many states in Nigeria especially in Oyo, Ogun, Ondo and Lagos States where some schools have been politically closed or merged.

In order to assist appropriate government agencies in Nigeria to determine areas where certain facilities are sufficient or otherwise, this research therefore seeks to find the optimal coverage of Primary and Secondary schools within Shomolu Local Government Area using the P-Center Model. Through the use of Geographical Information Systems (GIS), the conventional facility location models that employed concentric circle analysis can now be replaced with a more complex and realistic model. This will provide the right solution to assist decision makers in determining appropriate values for the physical suitability criteria in order to calculate the service area as the function of travel time zones. In this study, all major roads serving the commercial axis and inner areas of Shomolu Local Government Area were covered. Spatial (location of schools, street network map) and aspatial data were collected and integrated in a GIS environment. Facility locational criteria and P-centre Model were used for querying the coverage and service areas of the schools. This work did not attempt to investigate the structural stability of the facility or the road network, but investigated the appropriateness of the location wherein the facilities are sited and their corresponding spatial distribution (Euclidean distance) from potential users within the research area.

The P-Centre Problem Concept

The P-centre problem provides solution to the problems of locating and allocating facilities by providing information on some critical coverage distance or time within which demands need to be served if they are to be

counted as "covered" or "served adequately". Also, it ensures economic advantages in making sure that the total number of facilities sited is minimized within an area while still efficiently serving all the demand within the area. It is a model of locating p facilities on a network in order to minimize the maximum coverage distance between each vertex (demand point) and its closest facility.

In p -centre problems, we partition a set of demand points into exactly p subsets each associated with a centre. A centre is identified both by the location of its facility and by the set of demand points assigned to it. There is a given weight for assigning each demand point to each centre, and to minimize the maximum weighted distance among all demand points and their assigned facilities.

P -centre problems are discrete location problems, since the number of demand points that are served by the facilities is finite. Demands generally arise on the nodes and the facilities are restricted to a finite set of candidate locations. However, p -centre problem is a Covering based models assuming that there is some critical coverage distance or time within which demands need to be served if they are to be counted as "covered" or "served adequately." Such models are typically used in designing emergency services as there are both practical and, in many jurisdictions, legislative guidelines for coverage. Other application areas different from the emergency services could be considered for locating schools, industrial plants, warehouses, distribution centres and various service facilities in the telecommunication sector. Minimizing the maximum coverage distance in the case of locating industrial plants and warehouses is crucial in terms of the delivery cost.

For the most general form of the p -centre problem, centres can be located anywhere on the network. The location of the centres may be at vertices or at any point on edges. The general version of the problem is referred to as the absolute p -centre problem since there is no restriction on locating the centres on the network (Bonneu and Thomas-Agnan, 2009).

For a given location solution, the maximum distance which any user would have to travel to reach a facility would reflect the worst possible performance of the system. Two different surrogate measures which have received attention in location models are: (1) total weighted distance or time to travel to the facilities, and (2) the distance or time that the most distant user from a

facility would have to travel to reach that facility, that is, the maximal service distance (Toregas and ReVelle (1984); Toregas and ReVelle (1971). The maximal service distance concept, in the Location Set Covering Problem and in P-Centre problem, identifies the minimal number of facilities and the location of facilities, which ensures that no demand point will be farther than the maximal service distance from a facility (Toregas and ReVelle, 1984).

By solving the location set covering problem over a range of values of distances, it is possible to develop a cost-effectiveness curve from the pairs of numbers (maximal service distance S , minimum number of facilities to cover). Examination of the cost-effectiveness curve reveals that for a given number of facilities, there may be many location solutions which fulfill the requirement of coverage. However, for a desired level of expenditure then (e.g. fixed number of facilities), one may wish to determine the solution with the smallest maximal service distance (Hakimi, 1964). It could happen that the distance value obtained in this manner is much larger than the desired distance S . If it is, the decision maker may shift his attention to concern with the total population covered within S . Having realized that his resources (facilities) are insufficient to achieve total coverage within his distance goal, the decision maker may seek to cover as many people as possible within S using those limited resources.

In short, when a decision maker faces the reality of an insufficient number of facilities, he may want to cover the maximum population possible within the desirable distance but ensure at the same time that no one is farther than a distance T (where $T > S$) to his closest facility. Insistence on total coverage within T may be included in order to provide some degree of equity to those not served within the shorter and more desirable service distance S . The closer T can come to the desirable distance S , the fairer the solution is to the people not covered within S . (Hakimi, 1964).

2. Research Area

Shomolu Local Government is one of the premier local governments in Lagos State that was created at the time of colonial era. A residential suburb of Lagos with an estimated population of 1,162,773 (Lagos Bureau of Statistics Ministry of Economic Planning and Budget, 2012), the town is plagued with problems of overcrowding, poor housing, and inadequate sanitation. Shomolu Local Government is situated in the Mainland area of

Lagos State with boundary coordinates of 725590.327mN, 540290.617mE and 719750.770mN, 540284.798mE (Figure1).



Figure 1: Map of Shomolu Local Government Area.
(Office of the Surveyor General, Lagos State, 2009).

3. Methodology

ArcGIS was used to map the existing school locations and their demand points; along with other constrains and siting determinant factors (e.g. population, etc.). This thus served as a basis using the model builder for modelling the suitability or otherwise of the existing schools and the possible need for siting more schools at other locations.

3.1 Data Sources

Both spatial and other attribute data, primary and secondary data, necessary for the description and effective analysis of the spatial information were

collected. Table 1 gives an overview of the sources of the datasets used for this research.

Table 1: Data and their sources.

Data	Source
Primary and Secondary School locations.	Field observation using Garmin Handheld GPS.
Road Network	Office of the Surveyor General of Lagos – GIS Department.
Base Map	Google-Earth
Administrative Map	Office of the Surveyor General of Lagos – Cartographic Section
Primary and Secondary Schools Attribute Data (Number of Pupils in School, School Maximum Student Capacity)	Lagos Bureau of Statistics Ministry of Economic Planning and Budget, 2012; Lagos State Government Performance Management Report (PMR) and Education Sector, 2011)
Shomolu Local Government Population (Population/Estimated Population)	-ditto-

3.2 Formulation of Mathematical models

Figure 2 illustrated the mathematical formulation used in solving the P-Center location-allocation problem within the study area. with notations adopted to domesticate the original “set theory” based formulations in “Spatial Based” Algebraic form.

Location–Allocation solves the problems of matching supply and demand by using sets of objectives and constraints. A location-allocation analysis matches the distribution facilities and the demand while meeting both the objective and the constraint.

To obtain optimal location of facility:

$$OP(x, y) = P'(x', y') + S \quad (1)$$

where: $OP(x, y)$ = Optimal Spatial Location of Facility, $P'(x', y')$ = Demand Point Spatial Location, and S - Optimal Service Distance between Demand Point Spatial Location and Facility Spatial Location. However, if the total

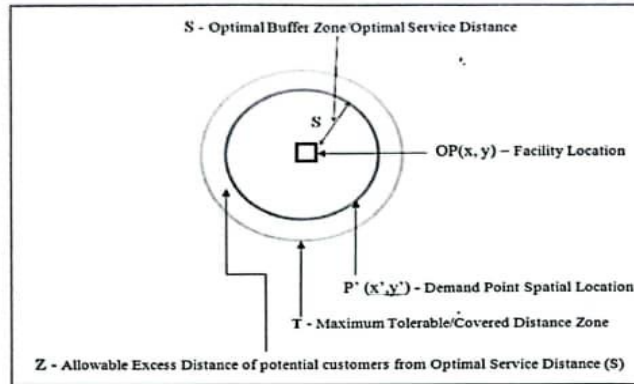


Figure 2: Formulating P-Centre algorithm.

number of existing facilities is such that Equation (1) cannot hold, i.e. insufficient number of facilities, then Maximum Tolerable/Covered Distance (T):

$$T = S + Z \quad (2)$$

where: Z - Allowable excess distance of potential customer from Optimal Service Distance (S). Therefore, the above models were used to determine whether the public facilities, schools, were optimally located to serve students located within S or T. Using a simple distance formula:

$$Speed = \frac{Distance}{Time} \quad (3)$$

Equation (3) was used as the underlying mathematical basis for computing the various distances and time impedance; then, the distance constrains computed were imposed on the facilities of interest.

3.2.1 P-Center Model application to Schools

To evaluate potential public facilities sites and support decisions concerning the location of additional public facilities areas, certain criteria must be identified. The following criteria were identified in respect to the public facilities, schools:

- a. Distance to residential buildings should be such that will facilitate easy access for the students. In the case of primary schools, it is expected that no pupil should walk more than 20mins to school while 30mins for secondary school students.
- b. In order to enhance easy assimilation of the students, the school should be located in an environment that is free from noise pollution i.e. industry areas (Seberang Perai Municipal Council, Malaysia 1989).

In this work, the Optimal Distance (S) and Tolerance Distance (T) were computed as:

(i) Optimal Distance (S)- the average speed of a primary school pupil was taken as 0.6m/s and that of a secondary school student was 0.8m/s, then the optimal distance (S) for location of primary and secondary school facilities away from the students' location is computed from Equation (3) as:

(a) For Primary Schools: $0.6\text{m/s} = \text{Distance}/20\text{minutes}$, Optimal Distance = 720m

(b) For Secondary Schools: $0.8\text{m/s} = \text{Distance} /30\text{minutes}$, Optimal Distance = 1,080m

(ii) Tolerance/Covered Distance (T)- an additional Allowable excess distance (Z) of 50m was approved for potential customer from Optimal Service Distance (S).

(iii) Noise pollution (e.g. industry areas, markets, commercial areas, etc.), an optimal distance of > 500m and a tolerable distance of 750m were used. Table 2 shows the distance factors that were obtained.

3.3 Population Estimation

The data obtained from the Lagos Bureau of Statistics Ministry of Economic Planning and Budget (2012) and Lagos State Government Performance Management Report (PMR) Education Sector (2011) were adapted and used in this work. 5% of the Local Government population are students out of which 45% are primary school students and 55% are secondary school students. These relationships are represented in Equation (4):

$$P = O + S \quad (4)$$

where: P - Total Population; O – Others (i.e. Non-Student), S - School Students. Based on this information, the population of primary and secondary school students were estimated for other years. Similarly, the estimated students' population, school capacities and sufficiency or otherwise within Shomolu Local Government Area were determined and shown in Table 2.

3.4 Regression Model Derivation of the Deficiencies

Polynomial fit of the data on the deficiencies in the number of schools available in Shomolu Local Government Area was carried out. The obtained models can be used to project future deficiencies.

3.5 Service Area Map Derivation

Maps were drawn showing the existing facilities in their spatial positions viz-a-viz the roads and wards. This map thus becomes a basis upon which the multi-criteria queries were based. To produce the service area map, buffers were created from the facilities based on the optimal and tolerable distances computed from the mathematical model (Equation (3) and Table 2). Thus, the buffer radius serves as the distance vector or service area covered by each facility. Each service area was then overlaid on the map of the Shomolu Local Government Area Wards and the Street layers.

To facilitate multi-criteria query and analysis, the buffered regions were clipped to the wards and thereafter a spatial join was carried out to join the attributes within both layers. The resulting map, called Service-Area Map, is a single layer that combines all attributes of both the facilities, the wards and streets layers. Multi-criteria queries were performed to identify facilities that fall within optimal distance to various residents.

4. Results and Discussion of Results

Results were obtained and analysis carried out as follows:

4.1 P-Centre Optimal and Tolerable Distances

Result obtained for optimal and tolerable distances using P-Centre Model is shown in Table 2.

Table 2: Shows the Distance Factors obtained from computation for each facility.

S/N	Facility	Optimal Distance to Commercial Areas (S)	Tolerable Distance to Commercial Areas (T)	Optimal Distance to Residential Areas (S)	Tolerable Distance to Residential Areas (T)
1.	Primary Schools	> 500m	> 750m	≤ 720m	≤ 770m
2.	Secondary Schools	> 500m	> 750m	≤ 1,080m	≤ 1,130m

4.2 Results of the Service Area Maps and Optimality Maps of all the Schools

4.2.1 Primary Schools-

Figure 3 is the spatial distribution of all Primary Schools within the study area.

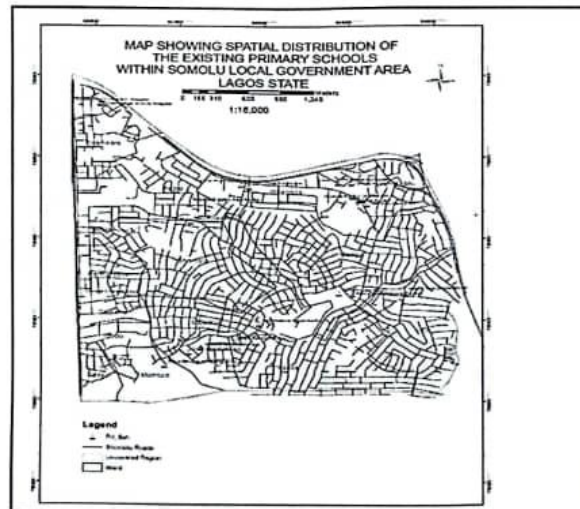


Figure 3 All Primary schools within the study area

The located Primary Schools were then queried to determine the service area maps (Figures 4 and 5) at optimal and tolerable distances respectively.

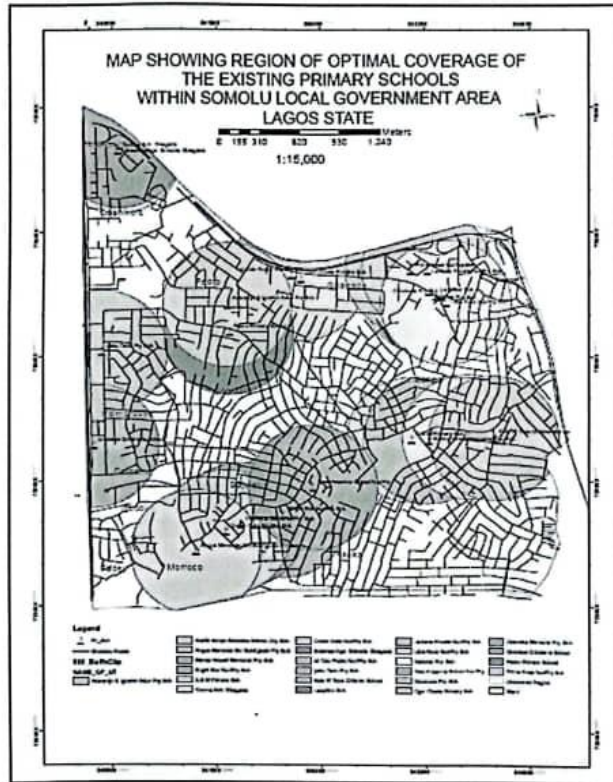


Figure 4 Service area map of primary schools covered within an optimum buffer distance of 720m.

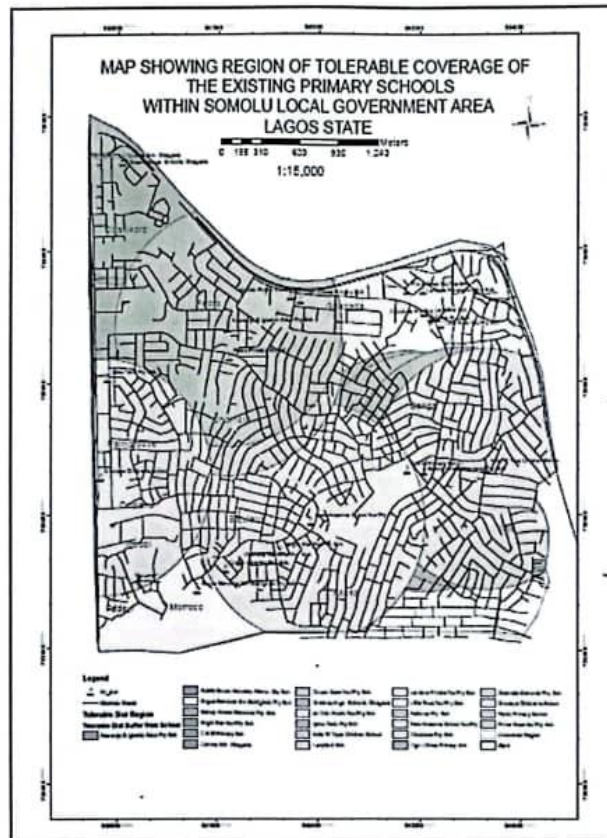


Figure 5 Service area map efficiently covered by primary schools at tolerable distance of 770m.

On the other hand, considering the population of students and the student capacity for each school, further query was carried out to determine if the primary schools available in Shomolu Local Government Area are sufficient

to accommodate the students' population within the study area. Table 3 and Figures 6 – 8 give an overview of results obtained from the queries.

4.2.2 Secondary Schools-

Figures 6 – 8 show the spatial location of Secondary Schools within the study area as well as Service Area map showing areas efficiently covered by the Optimum and Tolerable distances respectively.

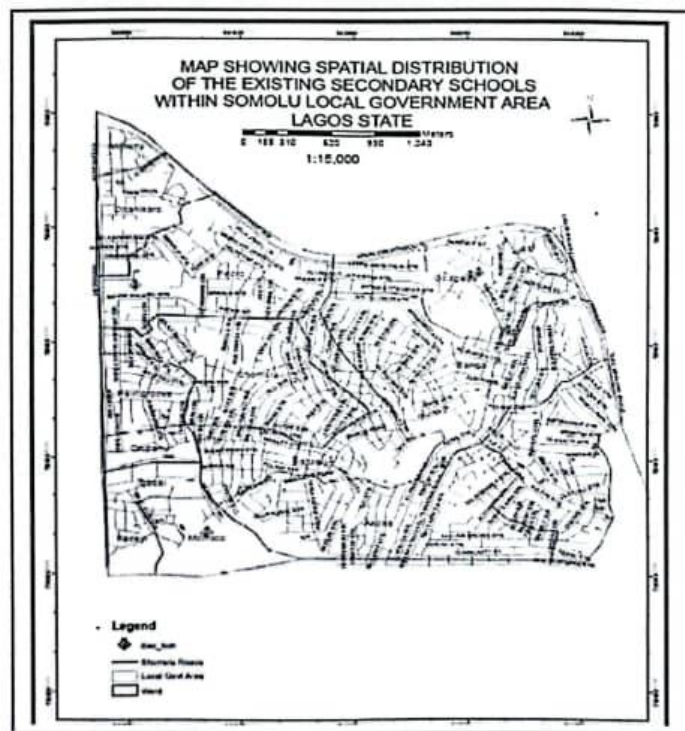


Figure 6 Spatial distribution of all secondary schools in Shomolu LGA.

The optimally located schools were then buffered at an optimal distance of 1,080m and tolerable distance of 1,130m respectively to determine the area efficiently serviced/covered within the study area.

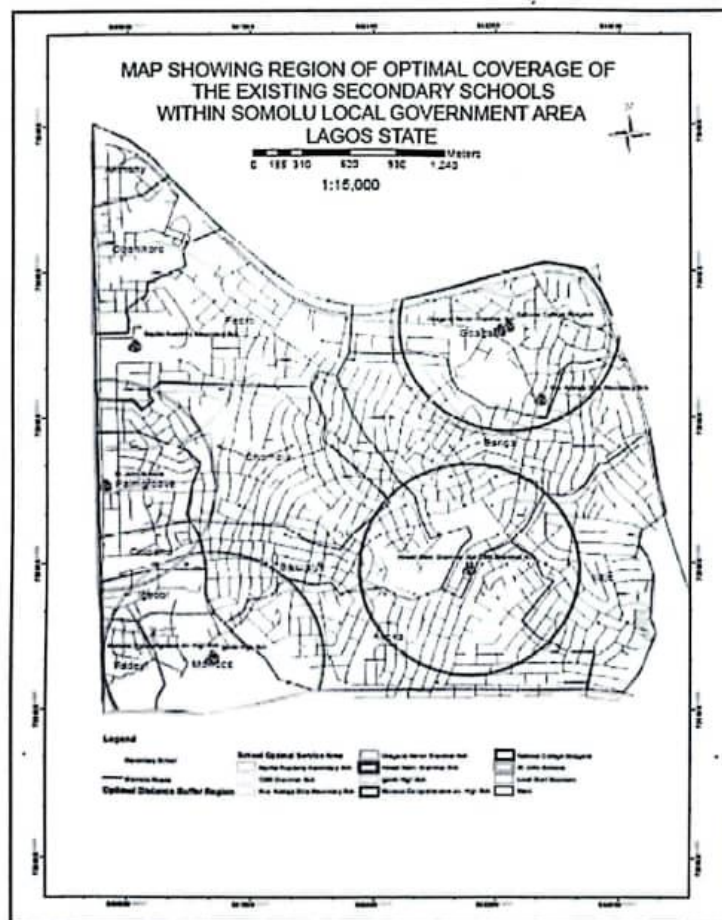


Figure 7 Service Area Map of secondary schools covered within an optimal distance of 1,080m.

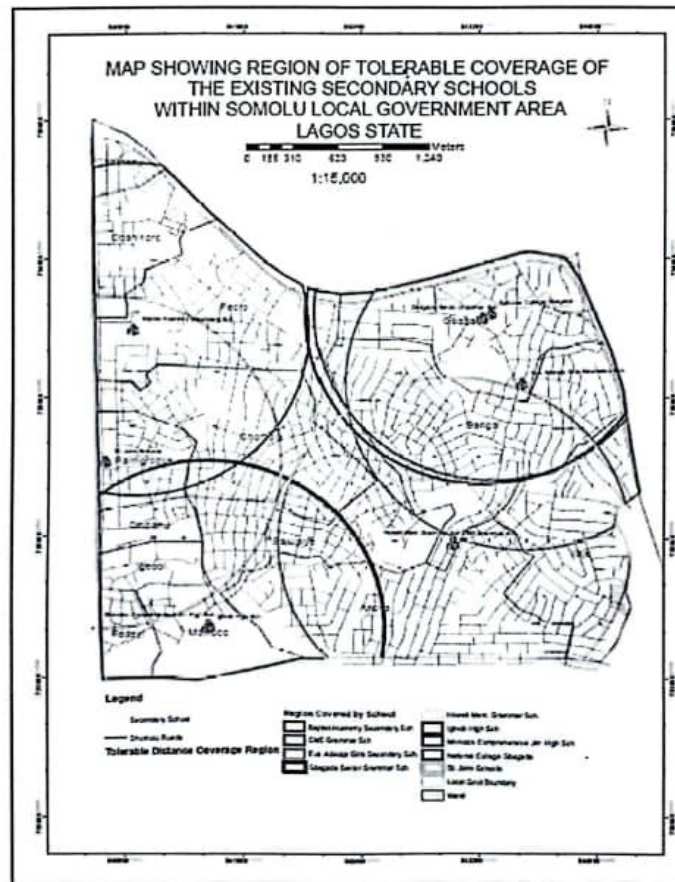


Figure 8 Service Area Map of Secondary Schools efficiently covered within the tolerable distance of 1,130m in the study area.

In addition to the previous queries, Figure 9 shows the Uncovered Region by the imposed Tolerable Distance within the study area by the existing Secondary Schools.

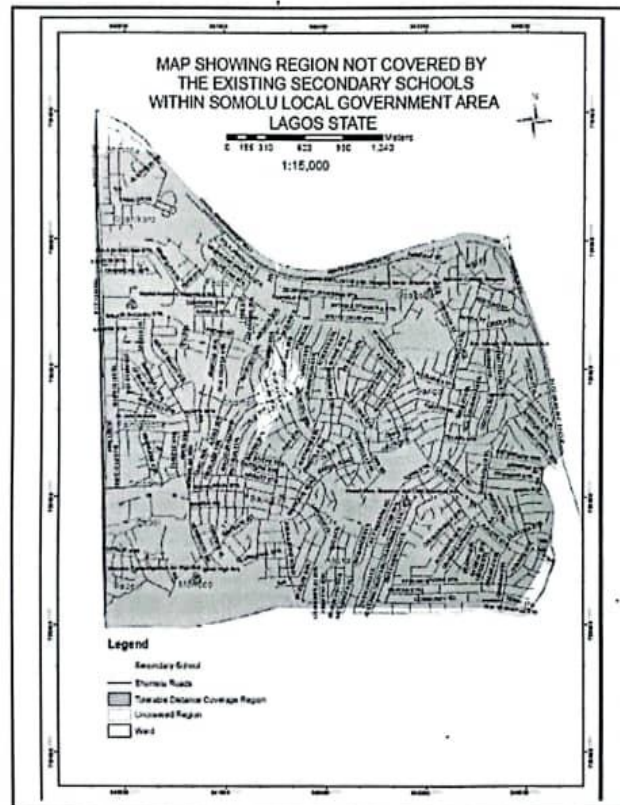


Figure 9 The uncovered region by the imposed tolerable distance.

Table 3 is the summary of the Local Government population, students' population, school capacity and sufficiency or otherwise of schools within the Shomolu Local Government area.

Table 3 Summary of schools in Somolu, Lagos (A)

Y (1)	EP (2)	EPP (3)	ESP (4)	TESP (5)	EP (6)	AP/S (7)
2010	1,162,773	15,611	19,357	34,968	48	325
2011	1,199,982	15,867	19,675	35,542	48	325
2012	1,238,381	16,127	19,997	36,125	48	325
2013	1,278,009	16,392	20,325	36,717	49	325
2014	1,318,906	16,661	20,659	37,319	49	325
2015	1,361,111	16,934	20,997	37,931	49	325
2016	1,404,666	17,211	21,342	38,553	49	325
2017	1,449,615	17,494	21,692	39,186	49	325
2018	1,496,003	17,781	22,048	39,828	49	325
2019	1,543,875	18,072	22,409	40,481	49	325
2020	1,593,279	18,369	22,777	41,145	49	325

Legend

Y- Year	EP- Estimated Local Govt Population	EPP- Estimated Priy Sch. Population	ESP- Estimated Sec. Sch. Population	TESP- Total Estimated Sch. Population	EP- Existing No. of Priy Sch.	APS- Ave. No. Priy Pupils/Sch.
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Table 3 contd

EPC (8)	ES (9)	AS/S (10)	ESC (11)	PD (12)	SD (13)	R (14)
15,611	20	957	19,135	Nil	222	Fairly Adequate
15,611	20	957	19,135	256	540	Deficient
15,611	20	957	19,135	516	862	Deficient
15,936	21	957	20,092	455	234	Deficient
15,936	21	957	20,092	724	567	Deficient
15,936	21	957	20,092	998	906	Deficient
15,936	21	957	20,092	1,275	1,250	Grossly Deficient
15,936	21	957	20,092	1,558	1,600	Grossly Deficient
15,936	21	957	20,092	1,844	1,956	Grossly Deficient
15,936	21	957	20,092	2,136	2,317	Grossly Deficient
15,936	21	957	20,092	2,432	2,685	Grossly Deficient

Legend:

EPC- Existing Priy Sch. Capacity	ES- Existing No. of Sec. Sch.	AS/S- Ave. No. Sec. Pupils/Sch.	ESC- Existing Sec. Sch. Capacity	PD- Priy Sch. Deficiency	SD- Sec Sch. Deficiency	R- Remark
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The estimated primary, secondary and total students' population from 2010 to 2020 are shown in Figure 10.

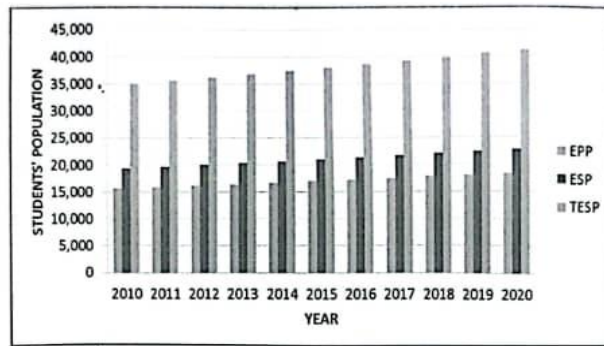


Figure 10 Estimated primary (EPP),sSchool (ESP) and total students' population (TESP) from 2010 to 2020.

Similarly, figure 11 shows the deficiencies of primary and secondary schools from 2010 to 2020.

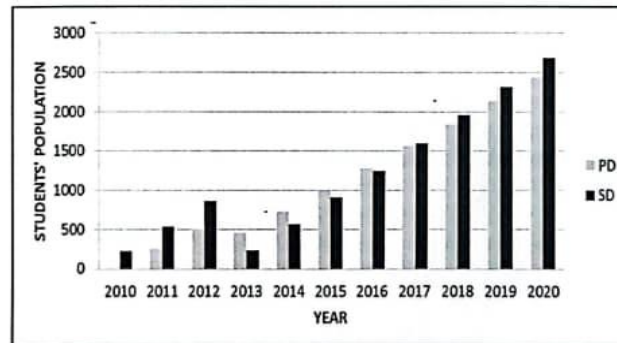


Figure 11 Shows the deficiencies of primary (PD) and secondary (SD) schools from 2010 to 2020.

4.3 Model Derivation of the Deficiencies

Models derived from the polynomial fitting of the data in Table 3 are shown in Figure 12 and Equations (5a) and (5b).

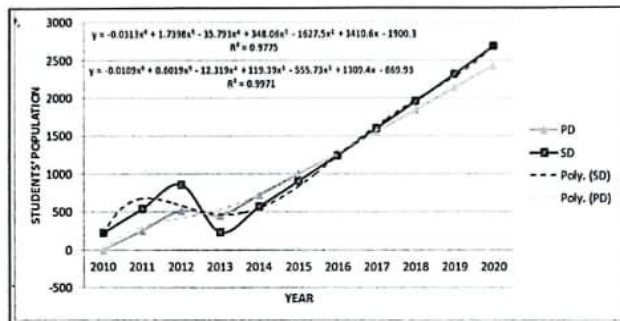


Figure 12 Polynomial fitting of the deficiencies in schools' sufficiency from 2010 to 2020.

The fitted models are:

(i) for Primary Schools-

$$y = -0.0109x^6 + 0.6019x^5 - 12.319x^4 + 119.39x^3 - 555.73x^2 + 1309.4x - 869.93 \quad (5a)$$

$$R^2 = 0.9971$$

(ii) for Secondary Schools-

$$y = -0.0313x^6 + 1.7398x^5 - 35.793x^4 + 348.06x^3 - 1627.5x^2 + 3410.6x - 1900.3 \quad (5b)$$

$$R^2 = 0.9775$$

4.4 Discussion of Results

From the results, the following inferences were drawn on the facilities: Since there are limited funds available for funding education generally and schools may not be located everywhere optimally, therefore, from Figure 3.3 more areas are efficiently covered by Primary Schools at Tolerable Distance of

770m than at Optimal distance of 720m. Similarly, the secondary schools service delivery also revealed that more are covered with Tolerable Distance than Optimal Distance. Despite this Tolerable Distance option, Figure 7 showed that some areas in the Local Government are still under. Uncovered Region by the existing Secondary Schools. This means that school pupils in this area will suffer more in trekking longer distances to school.

Table 3 and Figure 12 revealed that up to 2015 there is inadequate number of both primary and secondary schools in Shomolu Local Government, whereas from 2016 – 2020 if nothing is done by the government, there will be over 1,000 pupils without any school to attend, i.e. gross inadequacy, of both schools. At 2016, there will be equivalent level of inadequacies for both primary and secondary schools.

5. Conclusion

The P-centre location-allocation model was used to map the optimal and tolerable service areas of both public Primary and Secondary Schools within Shomolu Local Government Area with a primary goal of minimising the maximum distance from each demand node to the closest facility. This method of location-allocation when integrated with GIS will provide an optimal location of facilities.

Based on capacity and demand information and queries generated in this work, it is evident that there is need for more secondary and primary schools. This work has proved that both the Local Government Area and Lagos State Government must quickly increase their developmental projects in establishing more educational facilities in order to curb the upsurge in school inadequacy in this Local Government and to avoid breeding future hoodlums.

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